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A

HIGH SCHOOL

ARITHMETIC

(WENTWORTH & HILL'S PRACTICAL ARITHMETIC).

BY

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FOR HIGH SCHOOLS AND ACADEMIES.

TEACHERS' EDITION.

BOSTON:
PUBLISHED BY GINN & COMPANY.
1888.

Typography by J. S. Cushing & Co., Boston.

Presswork by Ginn & Co., Boston.

PREFACE.

THIS edition is intended for teachers, and for them only. The publishers will make every effort to keep the book from pupils; and teachers are urged to exercise the utmost care not to lose their copies, or to leave them where pupils can have access to them.

It is hoped that young teachers will derive great advantage from studying the systematic arrangement of the arithmetical work, for such attention has been paid to this as the limitation of the page would allow.

It is also expected that many teachers, who are pressed for time, will find great relief by not being obliged to work out every problem in the Arithmetic.

G. A. WENTWORTH.

PHILLIPS EXETER ACADEMY,
August, 1888.

ARITHMETIC.

45. Find the following sums: 231 + 764; 341 + 57.8; 430.31 + 58.61; 512.87 + 36.84 + 12.78 + 711.56 + 415.86.

			512.87
			36.84
			12.78
231	341.	430.31	711.56
764	57.8	58.61	415.86
995	398.8	488.92	1689.91

46. Add 1543.1 to 164.7; to 1728; to 402.56; to 1897.3; to 475.34; to 6897.65.

164.7	1728.	402.56	1897.3	475.34	6897.65
1543.1	1543.1	1543.1	1543.1	1543.1	1543.1
1707.8	3271.1	1945.66	3440.4	2018.44	8440.75

47. Add 1897.3 to 475.34; to 6897.65; to 1728; to 402.56; to 164.7; to .5236; to 2.71828.

475.34	6897.65	1728.	402.56	164.7
1897.3	1897.3	1897.3	1897.3	1897.3
2372.64	8794.95	3625.3	2299.86	2062.

0.5236	2.71828
1897.3	1897.3
1897.8236	1900.01828

48. Find the following sums: .7854 + 3.1416 + 2.71828; .7854 + 3.1416 + 30,103; 2.71828 + 402.56 + 1897.3; 2.7113 + 27.53 + 341.586.

0.7854	0.7854	2.71828	2.7113
3.1416	3.1416	402.56	27.53
2.71828	30,103.	1897.3	341.586
6.64528	30,106.937	2284.57828	371.8273

49. Add 737.87 to each of the following numbers: 111; 1011; 2304; 222; 263; 373; 262.13; 561.2; 32.35; 604.3.

111.	1011.	2304.	222.	263.
737.87	737.87	737.87	737.87	737.87
848.87	1748.87	3041.87	959.87	1000.87

373.	262.13	561.2	32.35	604.3
737.87	737.87	737.87	737.87	737.87
1110.87	1000.	1299.07	770.22	1342.17

50. Find the five sums: 230.8 + 223 + 2.63 + 373.8 + 56.123; 32.358 + 821.9 + 23.04 + 73.7; 202.3031 + 71.575 + 65.813 + .0078 + 7.377; 653.03 + 65.303 + 6.5033; 939.303 + 65.746 + 8.2794 + 681.28.

230.8		202.3031		
223.	32.358	71.575		939.303
2.63	821.9	65.813	653.03	65.746
373.8	23.04	0.0078	65.303	8.2794
56.123	73.7	7.377	6.5033	681.28
886.353	950.998	347.0759	724.8363	1694.6084

52.	2.7182818	0.4342945	3.2808693
	3.1415927	4.8104774	2.5399772
	0.7853982	2.5399772	4.8104774
	6.6452727	7.7847491	10.6313239
	1.6093295	0.3047973	0.3937043
	15.4323487	0.3010300	0.3819660
	3.785	0.6213768	0.4342945
	20.8266782	1.2272041	1.2099648

53.	0.3010300	0.6180340	0.3819660
	0.6180340	2.2360680	1.7320508
	0.3819660	1.7320508	1.4142136
	1.30103	4.5861528	3.5282304
	2.2360680	15.4323487	3.785
	1.4142136	0.8450980	0.6213768
	15.4323487	0.264	1.6093295
	19.0826303	16.5414467	6.0157063

	0.6213768	1.6093295	0.3047973
	3.2808693	3.2808693	0.3937043
	0.3047973	0.3937043	2.5399772
	4.2070434	5.2839031	3.2384788
	0.3937043	0.3047973	2.7182818
	3.2808693	0.4342945	3.1415927
	4.8104774	0.5235988	0.5235988
	8.485051	1.2626906	6.3834733
54.	2.7182818	3.1415927	0.7853982
	3.1415927	0.5235988	0.4342945
	0.7853982	4.8104774	4.8104774
	0.5235988	2.5399772	0.3937043
	0.4342945	0.3937043	0.3047973
	7.603166	11.4093504	6.7286717
	0.3937043	1.6093295	1.6093295
	3.2808693	0.3047973	0.6213768
	1.6093295	0.3937043	3.785
	0.4342945	0.5235988	0.264
	4.8104774	0.4342945	15.4323487
	10.528675	3.2657244	21.712055
	0.6213768	3.785	15.4323487
	0.264	0.264	1.4142136
	15.4323487	15.4323487	2.2360680
	0.3937043	1.4142136	0.3819660
	3.2808693	1.7320508	0.6180340
	19.9922991	22.6276131	20.0826303
55.	0.4771213	2.7182818	0.3010300
	0.2908882	3.1415927	0.6180340
	1.6093295	0.7853982	0.3819660
	0.8450980	0.4342945	2.2360680
	0.3819660	4.8104774	1.7320508
	0.6180340	2.5399772	1.4142136
	0.3010300	0.3937043	15.4323478
	4.523467	14.8237261	22.1157111

1.6093295	0.3937043	3.785
0.6213768	0.3047973	15.4323487
3.785	3.2808693	0.6213768
0.264	1.6093295	1.4142136
15.4323478	0.6213768	3.2808693
1.4142136	3.785	0.3047973
1.7320508	0.264	4.8104774
24.8583194	10.2590772	29.6490831

57. Add by double columns:

45.68	154.31	73.86
73.91	296.85	453.71
78.54	736.48	137.64
534.69	345.19	98.87
134.70	782.34	643.48
581.43	78.43	462.71
1448.95	2393.60	1870.27

498.50	65.42	621.65
17.37	638.34	167.32
684.29	763.43	856.96
231.56	809.31	718.83
210.10	798.83	501.49
671.54	835.78	315.72
643.53	356.47	768.44
2956.89	4267.58	3950.41

791.52	32.54	763.89
504.83	254.63	78.23
879.26	63.27	345.61
243.97	131.56	26.73
732.86	506.72	489.56
47.95	283.54	812.35
856.43	345.83	607.28
497.65	643.46	219.07
541.26	708.91	68.72
616.72	463.73	216.78
857.94	67.74	436.74
6570.39	3501.93	4064.96

69. $8-3-2=3$; $(8-3)-2=5-2=3$; $8-(3-2)=8-1=7$.
$8-2-3=3$; $(8-2)-3=6-3=3$; $8-(2-3)=8-(-1)=9$.
$18-(7-3)=18-4=14$; $18-(3-7)=18-(-4)=22$;
$18-3+7=22$.

70. The following questions will illustrate the meaning of minus numbers:

Starting 90 miles south of Chicago, I go 50 miles due north; and the next day 80 miles, still north. How far from Chicago am I now?
$-90+50+80=-90+130=40$, the number of miles north. *Ans.*

With only $67 I undertake to pay three bills, of $47, of $13, and of $11. Can I pay the bills? How much shall I lack?
$67-(47+13+11)=67-71=-4$. I shall lack $4. *Ans.*

73. Subtract 123 from each of the numbers: 234, 343, 424, 555, 676, 725, 839, 999, 1000, 10,101, 5120.

234	343	424	555	676	725
123	123	123	123	123	123
111	220	301	432	553	602

839	999	1000	10101	5120
123	123	123	123	123
716	876	877	9978	4997

74. Subtract 456 from each of the numbers: 789, 879, 978, 6378, 6855, 6853, 7797, 7006, 3542, 4334, 9790, 3455.

789	879	978	6378	6855	6853
456	456	456	456	456	456
333	423	522	5922	6399	6397

7797	7006	3542	4334	9790	3455
456	456	456	456	456	456
7341	6550	3086	3878	9334	2999

75. What is the difference between 779 and 974? 368 and 249? 479 and 2301? 2731 and 929? 708 and 394? 1123 and 1072? 891 and 773? 8103 and 5621? 19,001 and 3456? 792 and 2180?

974	368	2301	2731	708
779	249	479	929	394
195	119	1822	1802	314
1123	891	8103	19001	2180
1072	773	5621	3456	792
51	118	2482	15545	1388

76. Subtract:

$183.45	$716.43	$647.51	$270.04
76.47	628.74	549.64	128.31
$106.98	$87.69	$97.87	$141.73
$125.	$247.93	$641.87	$56.27
101.50	129.47	333.95	29.89
$23.50	$118.46	$307.92	$26.38

77. Subtract from 7854 each of the numbers: 788, 879, 567, 5006, 6107, 578, 867, 894, 463, 4603.

7854	7854	7854	7854	7854
788	879	567	5006	6107
7066	6975	7287	2848	1747
7854	7854	7854	7854	7854
578	867	894	463	4603
7276	6987	6960	7391	3251

78.

3.1415927	0.7853982	4.8104774
2.7182818	0.5235988	0.4342945
0.4233109	0.2617994	4.3761829
2.5399772	0.3937043	3.2808693
0.3937043	0.3047973	0.3047973
2.1462729	0.088907	2.976072
3.2808693	3.785	15.4323487
1.6093295	0.6213768	0.264
1.6715398	3.1636232	15.1683487

	1.7320508	2.2360680	0.6180340
	1.4142136	0.3819660	0.3010300
	0.3178372	1.854102	0.317004
	2.2360680	2.2360680	0.3819660
	1.7320508	0.6180340	0.3010300
	0.5040172	1.618034	0.080936
79.	3.1415927	3.1415927	
	0.7853982	0.5235988	
	2.3561945	2.6179939	1.5707963
	0.7853982	0.5235988	0.5235988
	1.5707963	2.0943951	1.0471975
	0.7853982	0.5235988	0.5235988
	0.7853981	1.5707963	0.5235987
80.	1.	1.4142136	0.6180340
	0.3819660	0.6180340	0.3819660
	0.6180340	0.7961796	1.

81. In a school of 83 pupils, 37 are girls; the rest, boys. How many boys are there? $83 - 37 = 46$. *Ans.*

82. Take 1787 from 21,205, and what is the remainder?
$21{,}205 - 1787 = 19{,}418$. *Ans.*

83. Into a bowl containing 338 fine shot I poured a handful more, and the bowl then contained 720. How many did I pour in?
$720 - 338 = 382$. *Ans.*

84. From a box containing 209 oranges I took a basketful, and left 163 oranges. How many did I take in the basket?
$209 - 163 = 46$. *Ans.*

85. The minuend being 1718.754, and the subtrahend 1389.328, what is the remainder? $1718.754 - 1389.328 = 329.426$. *Ans.*

86. If the minuend was 6532.18, and the remainder 1916.47, what was the subtrahend? $6532.18 - 1916.47 = 4615.71$. *Ans.*

87. How many must be taken from 729,434 in order to leave 613,488? $729{,}434 - 613{,}488 = 115{,}946$. *Ans.*

88. How many must be taken from 1,000,000 to leave 817,259?

$1{,}000{,}000 - 817{,}259 = 182{,}741$. *Ans.*

89. Subtract 4187.94 from 8010.101.

$8010.101 - 4187.94 = 3822.161$. *Ans.*

90. Find the difference between 8,765,420 and 9,873,210.

$9{,}873{,}210 - 8{,}765{,}420 = 1{,}107{,}790$. *Ans.*

91. In a till are \$391 in bills, \$67.50 in gold, \$39.75 in silver, and \$2.77 in copper and nickel. How much money is in the till?

$\$391 + \$67.50 + \$39.75 + \$2.77 = \$501.02$. *Ans.*

92. Starting out with \$315.75 in one wallet and \$54.37 in another, I pay the grocer \$127.38; the butcher, \$64.17; the shoemaker, \$21.40; the landlord, \$50; the tailor, \$35. What ought I to have left?

$$\begin{array}{r} \$127.38 \\ 64.17 \\ 21.40 \\ 50. \\ 35. \\ \hline \$297.95 \end{array} \qquad \begin{array}{r} \$315.75 \\ 54.37 \\ \hline \$370.12 \\ 297.95 \\ \hline \$72.17 \end{array} \ \textit{Ans.}$$

93. On a bill of \$753.43, I pay \$517.87. How much do I still owe? If I owe \$817.87, and have but \$637.50, how much do I lack of being able to pay?

$$\begin{array}{r} \$753.43 \\ 517.87 \\ \hline \$235.56 \end{array} \qquad \begin{array}{r} \$817.87 \\ 637.50 \\ \hline \$180.37 \end{array}$$

94. If a man was born January 1, 1812, how old was he January 1, 1878? How old December 31, 1857?

$$\begin{array}{rrr} 1878 & 1 & 1 \\ 1812 & 1 & 1 \\ \hline 66 & & \end{array} \qquad \begin{array}{rrr} 1857 & 12 & 31 \\ 1812 & 1 & 1 \\ \hline 45 & 11 & 30 \end{array}$$

95. America was discovered in 1492. How many years after its discovery was each of the following events?

Settlement of Florida, 1565; of Virginia, 1607; of Massachusetts, 1620; of Quebec, 1608; French and Indian War, 1756; Declaration

of Independence, 1776; inauguration of Washington, 1789; war with England, 1812; Mexican War, 1846; Civil War, 1861.

1565	1607	1620	1608	1756
1492	1492	1492	1492	1492
73	115	128	116	264
1776	1789	1812	1846	1861
1492	1492	1492	1492	1492
284	297	320	354	369

96. How many days in common years, and in leap-years, between January 1 and March 1? January 4 and April 4? February 5 and May 5? February 7 and October 7? January 4 and July 4? March 4 and July 4?

Between January 1 and March 1, 58 days; 59 in a leap-year.
Between January 4 and April 4, 89 days; 90 in a leap-year.
Between February 5 and May 5, 88 days; 89 in a leap-year.
Between February 7 and October 7, 241 days; 242 in a leap-year.
Between January 4 and July 4, 180 days; 181 in a leap-year.
Between March 4 and July 4, 121 days; 121 in a leap-year.

97. The sum of two numbers is 3; their difference, 1. What are the numbers? The sum of two numbers is 5; their difference, 1. Required the numbers. What two numbers added together make 8, if the difference of the numbers is 2? If the difference is 0? if 4? if 6?

$$\left.\begin{matrix}(3+1)\div 2=2\\(3-1)\div 2=1\end{matrix}\right\}. \quad \left.\begin{matrix}(5+1)\div 2=3\\(5-1)\div 2=2\end{matrix}\right\}. \quad \left.\begin{matrix}(8+2)\div 2=5\\(8-2)\div 2=3\end{matrix}\right\}.$$

$$\left.\begin{matrix}(8+0)\div 2=4\\(8-0)\div 2=4\end{matrix}\right\}. \quad \left.\begin{matrix}(8+4)\div 2=6\\(8-4)\div 2=2\end{matrix}\right\}. \quad \left.\begin{matrix}(8+6)\div 2=7\\(8-6)\div 2=1\end{matrix}\right\}.$$

98. If the minuend is 9874, and remainder 3185, what is the subtrahend? The subtrahend being 7659, and remainder 675.68, what is the minuend?

9874	minuend.	7659.	subtrahend.
3185	remainder.	675.68	remainder.
6689	subtrahend.	8334.68	minuend.

99. The smaller of two numbers is 7.95764328; their difference is .00087692. What is the larger number?

$$7.95764328 + 0.00087692 = 7.9585202. \quad \textit{Ans.}$$

100. The larger of two numbers is 7.95764328, and their difference is 7.153485. What is the smaller number?

$$7.95764328 - 7.153485 = 0.80415828. \quad \textit{Ans.}$$

101. A hired man pumps out of my cistern in one hour 243.75 gallons; in the next hour, 227.5 gallons; in 45 minutes more, an additional 137.75 gallons; and the cistern is empty. How much was in it?

$$243.75 + 227.5 + 137.75 = 609.$$

609 gals. *Ans.*

102. From what number must I subtract 5 to leave 7? 8 to leave 9? From what number must I subtract 5.1736 to leave 8.1964? 6.231 to leave 9.6648? 74.213 to leave 25.787?

5	8	5.1736	6.231	74.213
7	9	8.1964	9.6648	25.787
12	17	13.37	15.8958	100.

103. What must be subtracted from 1 to leave .5? to leave .53? to leave .532? to leave .5236? to leave .5235988?

1.	1.	1.	1.	1.
0.5	0.53	0.532	0.5236	0.5235988
0.5	0.47	0.468	0.4764	0.4764012

104. I start on a journey of 3433 miles. The first day I make 428 miles; the second day, 511 miles; the third, 497 miles; the fourth, 513. How many miles of my journey remained for me at the close of each day? How many miles had I gone at the close of each day?

3433			
428			
3005	after first day.	428	end of first day.
511		511	
2494	after second day.	939	end of second day.
497		497	
1997	after third day.	1436	end of third day.
513		513	
1484	after fourth day.	1949	end of fourth day.

105. Subtract 76,343 from the sum of 61,932, 51,387, 5193, 4674, and 8199; then subtract 23,657 from the remainder.

$$61{,}932 + 51{,}387 + 5193 + 4674 + 8199 = 131{,}385.$$

$$131{,}385 - 76{,}343 = 55{,}042;\ 55{,}042 - 23{,}657 = 31{,}385.\ \textit{Ans.}$$

106. J. bought a farm and stock for \$7633.90; sold off the stock for \$305.75; then sold the farm for \$7325. What did he lose?

$$\$305.75 + \$7325 = \$7630.75;$$

$$\$7633.90 - \$7630.75 = \$3.15.\ \textit{Ans.}$$

107. If I gave \$4375 for my land, and paid for house, barn, sheds, and fences, \$2789.50; also \$973.75 for horses, cattle, tools, etc.; what did my farm and stock cost?

If I sold part of the land for \$675, and some cattle, etc., for \$217.50, what may I estimate as the cost of what I have left?

$$\$4375 + \$2789.50 + \$973.75 = \$8138.25.$$

$$\$675 + \$217.50 = \$892.50.$$

$$\$8138.25 - \$892.50 = \$7245.75.\ \textit{Ans.}$$

108. Alfred the Great died at the age of 52, A.D. 901. In what year was he born? William the Conqueror began to reign A.D. 1066, and reigned 21 years. In what year did he die? Socrates was born B.C. 469, and died at the age of 70. In what year did he die? Plato was born B.C. 429, and died at the age of 82. In what year did he die? Demosthenes died at the age of 60, B.C. 322. In what year was he born? The battle of Marathon was fought B.C. 490; 560 years later Jerusalem was destroyed by Titus. In what year was Jerusalem destroyed?

901	1066	469	429	322	560
52	21	70	82	60	490
A.D. 849	A.D. 1087	B.C. 399	B.C. 347	B.C. 382	A.D. 70

109. John has 158 cents, James has 271 cents; James gives John 56 cents. Which has more than the other, and how many more?

158	271	215
56	56	214
214 John's cents.	215 James's cents.	1 James's excess.

116. Multiply 111 by 5; 123 by 3; 231 by 2; 114 by 3; 421 by 4; 512 by 5; 4328 by 4; 1187 by 6; 1782 by 8; 8.287 by 7; 9.6198 by 3; 62.818 by 7; 9.2758 by 8; 52.134 by 9.

111	123	231	114	421
5	3	2	3	4
555	369	462	342	1684
512	4328	1187	1782	8.287
5	4	6	8	7
2560	17312	7122	14256	58.009
9.6198	62.818	9.2758	52.134	
3	7	8	9	
28.8594	439.726	74.2064	469.206	

117. Multiply 0.5235988 by 6; 0.7853982 by 4; 3.14159265 by 5, and the product by 5.

		3.14159265
		5
0.5235988	0.7853982	15.70796325
6	4	5
3.1415928	3.1415928	78.53981625

118. Multiply 3.1416 by 11; by 12; by 10 and by 3, and add the two results; by 10 and by 4, and add the results; by 9 and by 6, and add the results. Multiply 2.236068 by 11; by 6 and by 7, and add the results; by 8 and by 9, and add the results; by 10 and by 7, and add the results (compare the sum of these two products with the sum of the last two products); by 10 and by 8, and add the results; by 12 and by 7, and add the results.

3.1416	3.1416	3.1416	3.1416	31.4160
11	12	10	3	9.4248
34.5576	37.6992	31.4160	9.4248	40.8408
3.1416	3.1416	31.4160	3.1416	3.1416
10	4	12.5664	9	6
31.4160	12.5664	43.9824	28.2744	18.8496

28.2744	2.236068	2.236068	2.236068	13.416408
18.8496	11	6	7	15.652476
47.1240	24.596748	13.416408	15.652476	29.068884
2.236068	2.236068	17.888544	2.236068	2.236068
8	9	20.124612	10	7
17.888544	20.124612	38.013156	22.360680	15.652476
22.360680	2.236068	2.236068	22.360680	2.236068
15.652476	10	8	17.888544	12
38.013156	22.360680	17.888544	40.249224	26.832816
2.236068	26.832816			
7	15.652476			
15.652476	42.485292			

120. How much is 10 times 3.14159265? 100 times? a million times? What will 10 barrels of apples cost, at \$3.75 a barrel? at \$2.17? at \$5.875? How much will 100 barrels cost at each of these prices, and at \$3.375? at \$5.125?

$10 \times 3.14159265 = 31.4159265$; $100 \times 3.14159265 = 314.159265$; $1{,}000{,}000 \times 3.14159265 = 3{,}141{,}592.65$; $10 \times \$3.75 = \$37.50$; $10 \times \$2.17 = \$21.70$; $10 \times \$5.875 = \$58.75$; $100 \times \$3.75 = \$375$; $100 \times \$2.17 = \$217$; $100 \times \$5.875 = \$587.50$; $100 \times \$3.375 = \$337.50$; $100 \times \$5.125 = \$512.50$.

122. What is a tenth of 2.36? a hundredth of 2.36? a thousandth of 0.63. Write the second members of the following equations, and then read them:

$0.01 \times 7.8 =$ $\quad 0.001 \times 4.31 =$ $\quad 0.0001 \times 23.31 =$

$0.1 \times 0.065 =$ $\quad 0.01 \times 0.012 =$

$0.1 \times 2.36 = 0.236$; $0.01 \times 2.36 = 0.0236$; $0.001 \times 0.63 = 0.00063$; $0.01 \times 7.8 = 0.078$; $0.001 \times 4.31 = 0.00431$; $0.0001 \times 23.31 = 0.002331$; $0.1 \times 0.065 = 0.0065$; $0.01 \times 0.012 = 0.00012$.

123. Find the cost of 30 barrels of flour, at \$3.27 a barrel; of 70 barrels, at \$4.58; of 90 barrels, at \$6.76; of 100 barrels, at \$7.84; of 120 barrels, at \$8.57.

$30 \times \$3.27 = \$98.10$; $70 \times \$4.58 = \$320.60$; $90 \times \$6.76 = \$608.40$; $100 \times \$7.84 = \$784$; $120 \times \$8.57 = \$1028.40$.

124. Find the cost of 0.03 of a barrel of oil, at \$27.875 a barrel; of 0.7; of 0.009; of 0.17; of 0.019; of 0.13; of 0.8; of 0.83; of 0.014 of a barrel?

$0.03 \times \$27.875 = \$0.83625$; $0.7 \times \$27.875 = \$19.5125$;

$0.009 \times \$27.875 = \$0.250875$; $0.17 \times \$27.875 = \$4.73875$;

$0.019 \times \$27.875 = \$0.529625$; $0.13 \times \$27.875 = \$3.62375$;

$0.8 \times \$27.875 = \$22.30$; $0.83 \times \$27.875 = \$23.13625$;

$0.014 \times \$27.875 = \$0.39025$.

125. What is the numerical value of the expressions:

30×8.75? 700×7.81? 300×7.85?

0.07×6.975? 8000×65.432? 0.0009×10356.78?

$30 \times 8.75 = 262.5$; $700 \times 7.81 = 5467$; $300 \times 7.85 = 2355$;

$0.07 \times 6.975 = 0.48825$; $8000 \times 65.432 = 523,456$;

$0.0009 \times 10356.78 = 9.321102$.

129. Multiply 0.785398 by each of the following numbers: 2; 20; 3; 300; 5; 0.5; 0.005; 737; 7.37; 856; 85.6; 0.0856; 10; 1001; 1.001; 954; 0.00954.

$$\begin{array}{r}0.785398\\2\\\hline 1.570796\end{array}\qquad\begin{array}{r}0.785398\\20\\\hline 15.707960\end{array}\qquad\begin{array}{r}0.785398\\3\\\hline 2.356194\end{array}\qquad\begin{array}{r}0.785398\\300\\\hline 235.619400\end{array}$$

$$\begin{array}{r}0.785398\\5\\\hline 3.926990\end{array}\qquad\begin{array}{r}0.785398\\0.5\\\hline 0.3926990\end{array}\qquad\begin{array}{r}0.785398\\0.005\\\hline 0.003926990\end{array}\qquad\begin{array}{r}0.785398\\737\\\hline 5497786\\2356194\\5497786\\\hline 578.838326\end{array}$$

$0.785398 \times 7.37 = 5.78838326$.

```
  0.785398
       856
  --------
   4712388
  3926990
 6283184
 ----------
 672.300688
```

$0.785398 \times 85.6 = 67.2300688.$

$0.785398 \times 0.0856 = 0.0672300688.$

```
 0.785398
       10
 --------
 7.853980
```

```
  0.785398
      1001
  --------
    785398
 785398
 ----------
 786.183398
```

```
  0.785398
       954
  --------
   3141592
  3926990
 7068582
 ----------
 749.269692
```

$0.785398 \times 1.001 = 0.786183398.$

$0.785398 \times 0.00954 = 0.00749269692.$

130. Multiply 2150.42 by 0.1; by 0.001; by 0.75; by 0.075; by 0.083.

```
 2150.42    2150.42      2150.42      2150.42      2150.42
     0.1      0.001         0.75        0.075        0.083
 -------    -------    ---------    ---------    ---------
 215.042    2.15042      1075210      1075210       645126
                        1505294      1505294      1720336
                       ---------    ---------    ---------
                       1612.8150    161.28150    178.48486
```

131. Multiply 1.4142136 by 0.7; by 0.707; by 0.7071; by 0.707107. Multiply 1.41421 by 1.4; by 1.4142; by 1.41422. Multiply 1.732 by 1.732; 2.23607 by 2.236; 0.618 by 618; 0.618034 by 0.618035. Subtract this last product from 1.

```
  1.4142136       1.4142136         1.4142136          1.4142136
        0.7           0.707            0.7071           0.707107
 ----------    ------------     -------------    ---------------
 0.98994952        98994952          14142136           98994952
                 98994952          98994952         14142136
               ------------      98994952          98994952
               0.9998490152     -------------     98994952
                                0.99999043656    ---------------
                                                 1.0000003360552
```

1.41421	1.41421	1.41421	1.732
1.4	1.4142	1.41422	1.732
565684	282842	282842	3464
141421	565684	282842	5196
1.979894	141421	565684	12124
	565684	141421	1732
	141421	565684	2.999824
	1.999975782	141421	
		2.0000040662	

2.23607	0.618	0.618034	1.
2.236	618	0.618035	0.38196664319
1341642	4944	3090170	0.61803335681
670821	618	1854102	
447214	3708	4944272	
447214	381.924	618034	
4.99985252		3708204	
		0.381966643190	

133. Find the value of the expressions: 88×718.54; 96×6.8193; 6.3×71.569; 1.32×234.769.

718.54	6.8193	71.569	234.769
11	12	0.9	0.12
7903.94	81.8316	64.4121	28.17228
8	8	7	11
63231.52	654.6528	450.8847	309.89508

134. Multiply 291.47 by 16, and the product by 625. In like manner, find the continued products:

$8 \times 125 \times 278.56$; $8 \times 3.75 \times 3.33333$; $8 \times 625 \times 1.5708$.

$$8 \times 125 \times 278.56 = 1000 \times 278.56 = 278{,}560.$$

$$8 \times 3.75 \times 3.33333 = 30 \times 3.33333 = 99.9999.$$

$$8 \times 625 \times 1.5708 = 5000 \times 1.5708 = 7854.$$

291.47
16
174882
29147
4663.52
.625
2331760
932704
2798112
2914700.00

135. One mile measures 5280 feet. How many feet in 3 tenths of a mile? in 0.7? in 0.17? in 0.573? in 0.846 of a mile?

$0.3 \times 5280 = 1584$; $0.7 \times 5280 = 3696$; $0.17 \times 5280 = 897.6$;

$0.573 \times 5280 = 3025.44$; $0.846 \times 5280 = 4466.88$.

138. Multiply (using complements) 0.7854 by 9.9; by 0.99; by 0.099. Multiply 0.5236 by 99.7; by 9.989; by 9.87. Multiply 8537 by 0.0097; by 0.9995.

$$\begin{array}{ll} 0.7854 \times 10 & = 7.854 \\ 0.7854 \times 0.1 & = \underline{0.07854} \\ 0.7854 \times 9.9 & = 7.77546 \end{array} \qquad \begin{array}{ll} 0.7854 \times 1 & = 0.7854 \\ 0.7854 \times 0.01 & = \underline{0.007854} \\ 0.7854 \times 0.99 & = 0.777546 \end{array}$$

$$\begin{array}{ll} 0.7854 \times 0.1 & = 0.07854 \\ 0.7854 \times 0.001 & = \underline{0.0007854} \\ 0.7854 \times 0.099 & = 0.0777546 \end{array} \qquad \begin{array}{ll} 0.5236 \times 100 & = 52.36 \\ 0.5236 \times 0.3 & = \underline{0.15708} \\ 0.5236 \times 99.7 & = 52.20292 \end{array}$$

$$\begin{array}{ll} 0.5236 \times 10 & = 5.236 \\ 0.5236 \times 0.11 & = \underline{0.0057596} \\ 0.5236 \times 9.989 & = 5.2302404 \end{array} \qquad \begin{array}{ll} 0.5236 \times 10 & = 5.236 \\ 0.5236 \times 0.13 & = \underline{0.068068} \\ 0.5236 \times 9.87 & = 5.167932 \end{array}$$

$$\begin{array}{lll} 8537 & \times 0.01 & = 85.37 \\ 8537 & \times 0.0003 & = \underline{2.5611} \\ 8537 & \times 0.0097 & = 82.8089 \end{array} \qquad \begin{array}{lll} 8537 & \times 1 & = 8537 \\ 8537 & \times 0.0005 & = \underline{4.2685} \\ 8537 & \times 0.9995 & = 8532.7315 \end{array}$$

139. Multiply 0.61803 by 147; by 373; by 7.56; by 8.93; by 9.93. Multiply 0.5236 by 5.99; by 7.99; by 8.997; by 699.98.

0.61803	0.61803	0.61803	0.61803	0.61803
147	373	7.56	8.93	9.93
432621	185409	370818	185409	185409
247212	432621	309015	556227	556227
61803	185409	432621	494424	556227
90.85041	230.52519	4.6723068	5.5190079	6.1370379

0.5236	0.5236	0.5236	699.98
5.99	7.99	8.997	0.5236
47124	47124	36652	419988
47124	47124	47124	209994
26180	36652	47124	139996
3.136364	4.183564	41888	349990
		4.7108292	366.509528

140. Multiply 0.7854 by 0.618; by 0.382; by 0.7854; by 0.302. Multiply 2.718 by 0.618; by 0.382; by 0.7854; by 0.607.

0.7854	0.7854	0.7854	0.7854
0.618	0.382	0.7854	0.302
62832	15708	31416	15708
7854	62832	39270	23562
47124	23562	62832	0.2371908
0.4853772	0.3000228	54978	
		0.61685316	

2.718	2.718	2.718	2.718
0.618	0.382	0.7854	0.607
21744	5436	10872	19026
2718	21744	13590	16308
16308	8154	21744	1.649826
1.679724	1.038276	19026	
		2.1347172	

141. Find the continued products: $0.477 \times 101 \times 0.708$; $15.43 \times 0.4343 \times 3$; $4 \times 0.175 \times 3.28$; $0.615 \times 0.771 \times 10$; $3.2809 \times 5 \times 0.71$; $0.785 \times 0.7 \times 0.202$; $0.471 \times 0.807 \times 22$; $3.28 \times 25 \times 0.909$.

0.477	15.43	0.175	0.615
101	0.4343	4	0.771
477	4629	0.7~~00~~	615
477	6172	3.28	4305
48.177	4629	2.296	4305
0.708	6172		0.474165
385416	6.701249		10
337239	3		4.74165
34.109316	20.103747		

3.2809	0.785	0.471	3.28
5	0.7	0.807	25
16.4045	0.5495	3297	1640
0.71	0.202	3768	656
164045	10990	0.380097	82.$\cancel{0}\cancel{0}$
1148315	10990	22	0.909
11.647195	0.1109990	760194	738
		760194	738
		8.362134	74.538

144. Find the product, to the fifth fractional place, of 3.14159265 by 2.236. Find 1414.2136 × 14142.136, to the second place; 0.618034 by 0.618034, to the sixth place: 2.236068 by 2236.068, to the third place; 1.73205 by 1732.0508, to the second place.

3.14159265	1414.2136000	0.6180340
6322	63124141	4308160
6283185	14142136000	3708204
628318	5656854400	61803
94248	141421360	49442
18849	56568544	185
7.02460$\cancel{9}$	2828427	24
	141421	0.3819658
	42426	0.381966. *Ans.*
	8485	
	20000001.06$\cancel{3}$	

2.2360680	1.732050
8606322	80502371
44721360	1732050
4472136	1212435
670820	51962
134163	3464
1341	87
178	1
4999.9998	2999.999
5000. *Ans.*	3000. *Ans.*

147. 1. What will a man earn in a year if he has $2 a day, omitting Sundays? Suppose that the year begins on Sunday? Suppose the year to be leap-year, and not begin on Sunday? Suppose it leap-year, and to begin on Saturday?

$(365-52)\times \$2=\$626$; $(365-53)\times \$2=\$624$;
$(366-52)\times \$2=\$628$; $(366-53)\times \$2=\$626$.

2. If a field of corn averages 2 ears to a stalk, how many ears on 673 stalks?

$$\begin{array}{r l}
673 & \\
\underline{2} & \text{real multiplicand.} \\
1346 &
\end{array}$$

1346 ears. *Ans.*

3. At 27 bushels an acre, how much wheat to the square mile of 640 acres, deducting 47 acres for roads and waste land?

$$\begin{array}{r l}
640 & \\
\underline{47} & \\
593 & \\
\underline{27} & \text{real multiplicand.} \\
4151 & \\
\underline{1186} & \\
16011 &
\end{array}$$

16,011 bu. *Ans.*

4. How much money would be required to give $7000 to each of 7568 men?

$$\begin{array}{r l}
7568 & \\
\underline{7000} & \text{real multiplicand.} \\
52976000 &
\end{array}$$

$52,976,000. *Ans.*

5. In a certain book of 378 pages, the words average 7 letters to a word, and 10 words to a line. There are, on an average, 29 lines to a page. How many letters in the book?

$$\begin{array}{r l}
378 & \\
\underline{29} & \text{real multiplicand.} \\
3402 & \\
\underline{756} & \\
10962 & \\
\underline{10} & \text{real multiplicand.} \\
109620 & \\
\underline{7} & \text{real multiplicand.} \\
767340 &
\end{array}$$

767,340 letters. *Ans.*

6. How many bushels of wheat in a township of 37 square miles, if we deduct 47 acres to the square mile for roads and waste, and suppose that half the remainder is in wheat averaging 23 bushels to an acre?

```
    47               640
    37                37
   ----             -----
   329              4480
  141              1920
  ----             -----
  1739             23680
                    1739
                   -----
                   21941
                     0.5
                 -------
                 10970.5
                      23 real multiplicand.
                 -------
                  329115
                 219410
                --------
                252321.5
```

252,321.5 bu. *Ans.*

7. If 5700 persons, each paying 1 cent toll, and 324 carriages, each paying 5 cents toll, pass over a bridge in a day, how much money will be received?

```
  5700                          324
  0.01 real multiplicand.      0.05 real multiplicand.
  -----                        -----
  57.00                        16.20
                               57
                               -----
                               73.20
```

$73.20. *Ans.*

8. A merchant bought 960 pounds of cheese at 7 cents a pound, and 147 pounds of butter at 20 cents. He gave in payment 12.5 yards of cloth at 1 dollar a yard, 2 barrels of sugar, each weighing 226 pounds, at 9 cents a pound, and the remainder in cash. How much money had he to pay?

960		226	
0.07	real multiplicand.	2	real multiplicand.
67.20		452	
		0.09	real multiplicand.
		40.68	
		12.50	
147		53.18	
0.20	real multiplicand.		
29.40		96.60	
67.20		53.18	
96.60		43.42	\$43.42. *Ans.*

148. 1. Express the product of $7^5 \times 7^3$; $8^2 \times 8$; $2^8 \times 2$; $5^4 \times 5^2$.

$7^5 \times 7^3 = 7^8$; $8^2 \times 8 = 8^3$; $2^8 \times 2 = 2^9$; $5^4 \times 5^2 = 5^6$.

2. Express the product of: $3.01^2 \times 3.01$; $0.67^2 \times 0.67^8$; 0.208×0.208^3.

$3.01^2 \times 3.01 = 3.01^3$; $0.67^2 \times 0.67^8 = 0.67^{10}$;
$0.208 \times 0.208^3 = 0.208^4$.

3. Express the product of:

$2.003^2 \times 2.003^4$; $20.03^3 \times 20.03$; 20.03×20.03^2.

$2.003^2 \times 2.003^4 = 2.003^6$; $20.03^3 \times 20.03 = 20.03^4$;
$20.03 \times 20.03^2 = 20.03^3$.

153. Divide 963 by 3; 846 by 2; 846 by 3; 846 by 6; 848 by 4; 52.05 by 5; 84.028 by 7; 13.31 by 11; 1.728 by 12.

3) 963	2) 846	3) 846
321	423	282
6) 846	4) 848	5) 52.05
141	212	10.41
7) 84.028	11) 13.31	12) 1.728
12.004	1.21	0.144

158. 1. Divide 0.003 by 0.07; 0.003 by 110; 110 by 0.003.

7) 0.30000	11) 0.000300	3) 110000.00000
0.04286	0.000027	36666.66667

2. Divide 0.07 by 0.003; 110 by 0.07; 1.3 by 0.07.

3) 70.00000	7) 11000.00000	7) 130.00000
23.33333	1571.42857	18.57143

3. Divide 1.7 by 0.07; 0.07 by 110; 1.3 by 110.

7)170.00000	11)0.00700	11)0.13000
24.28571	0.00064	0.01182

4. Divide 1.7 by 110; 0.07 by 1.2; 0.003 by 1.2.

11)0.17000	12)0.70000	12)0.0300
0.01545	0.05833	0.0025

5. Divide 110 by 1.2; 1.7 by 1.2; 17 by 1.2.

12)1100.00000	12)17.00000	12)170.00000
91.66667	1.41667	14.16667

6. Divide 136 by 0.06; 136 by 0.12; 136 by 1100.

6)13600.00000	12)13600.00000	11)1.36000
2266.66667	1133.33333	0.12364

7. Divide 256 by 0.8; 2.56 by 0.08; 0.0256 by 0.008.

8)2560	8)256	8)25.6
320	32	3.2

8. Divide 256 by 8000; 1.06 by 0.9; 1.06 by 9000.

8)0.256	9)10.60000	9)0.00106
0.032	1.17778	0.00012

160. 1. Divide 1.6093295 by 0.479; by 0.917; by 0.017; by 0.0087.

3.35977	1.75499	94.66644	184.98040
479)1609.3295	917)1609.3295	17)1609.3295	87)16093.2950
1437	917	153	87
1723	6923	79	739
1437	6419	68	696
2862	5042	113	433
2395	4585	102	348
4679	4579	112	852
4311	3668	102	783
3685	9115	109	699
3353	8253	102	696
332	862	75	350
		68	348
		7	2

2. Divide 3 by 1.7; by 1.73; by 1.732; by 1.7321.

```
      1.76471                1.73410
 17)30.0000            173)300.0000
    17                     173
    ---                    ----
    130                    1270
    119                    1211
    ---                    ----
     110                    590
     102                    519
     ---                    ---
       80                    710
       68                    692
       --                    ---
      120                     180
      119                     173
      ---                     ---
        1                       7

       1.73210                 1.73200
 1732)3000.0000        17321)30000.0000
      1732                   17321
      -----                  ------
      12680                  126790
      12124                  121247
      -----                  ------
       5560                   55430
       5196                   51963
       ----                   -----
        3640                   34670
        3464                   34642
        ----                   -----
         1760                     280
         1732
         ----
           28
```

3. Divide 1.6093295 by 5280, and the quotient by 12.

```
      0.0003048                0.0000254
 528)0.16093295            12)0.0003048
       1584
       ----
        2532
        2112
        ----
         4209
```

4. Divide 2 by 1.4142; 5 by 2.236.

$$
\begin{array}{r}
1.41423 \\
14142\overline{)20000.00000} \\
14142 \\
\hline
58580 \\
56568 \\
\hline
20120 \\
14142 \\
\hline
59780 \\
56568 \\
\hline
32120 \\
28284 \\
\hline
3836
\end{array}
\qquad\qquad
\begin{array}{r}
2.23614 \\
2236\overline{)5000.0000} \\
4472 \\
\hline
5280 \\
4472 \\
\hline
8080 \\
6708 \\
\hline
13720 \\
13416 \\
\hline
3040 \\
2236 \\
\hline
804
\end{array}
$$

165. Perform the work in the following questions by the use of reciprocals:

1. $8 \times 0.25 = 8 \div 4 = 2.$
2. $171 \div 0.25 = 171 \times 4 = 684.$
3. $876 \times 1.25 = 876 \div 0.8 = 8760 \div 8 = 1095.$
4. $132 \times 2.5 = 132 \div 0.4 = 1320 \div 4 = 330.$
5. $591 \div 2.5 = 591 \times 0.4 = 236.4.$
6. $756 \div 0.125 = 756 \times 8 = 6048.$
7. $268 \times 25 = 268 \div 0.04 = 26800 \div 4 = 6700.$
8. $753 \div 25 = 753 \times 0.04 = 30.12.$
9. $567 \div 625 = (567 \div 5) \times 0.008 = 113.4 \times 0.008 = 0.9072.$
10. $1764 \times 0.025 = 1764 \div 40 = 44.1.$
11. $5381 \div 0.025 = 5381 \times 40 = 215{,}240.$
12. $7452 \div 0.875 = 7452 \times 8 \div 7 = 59{,}616 \div 7 = 8516.6.$
13. $651 \times 0.33333 = 651 \div 3 = 217.$
14. $456 \times 6.66667 = 456 \div 0.15 = 45{,}600 \div 15 = 3040.$
15. $1554 \times 0.16667 = 1554 \div 6 = 259.$

16. $432 \div 1.33333 = 432 \times 0.75$
$= 324.$

17. $375 \div 16.667 = 375 \times 0.06.$
$= 22.50.$

18. $225 \div 6.6667 = 225 \times 0.15$
$= 33.75.$

167. 1. Taking 7 as unity, what would be the value of 14? of 28? of 35? of 3.5? of 2.8105? of 6.31415?

$$\frac{7)\,14}{2} \qquad \frac{7)\,28}{4} \qquad \frac{7)\,35}{5} \qquad \frac{7)\,3.5}{0.5} \qquad \frac{7)\,2.8105}{0.4015} \qquad \frac{7)\,6.31415}{0.90202}$$

2. If the side of a square is 10 inches, and its diagonal 14.14214, express the side in terms of the diagonal as unity.

```
             0.70710
1414214)1000000.00000
         9899498
         -------
          10050200
           9899498
          --------
            1507020
            1414214
            -------
             928060
```

3. If the diagonal of a square is one foot, what decimal of a foot must its side be? $1 \div 1.414214 = 0.70710.$ (See **167**, 2.)

4. If the diameter of a circle is 11.3 inches, and its circumference 35.5 inches, what is the circumference in terms of the diameter? What is the diameter in terms of the circumference?

```
      3.14159             0.31831
113)355.00000       355)113.00000
    339                 1065
    ---                 ----
     160                 650
     113                 355
     ---                 ----
      470                2950
      452                2840
      ---                ----
       180                1100
       113                1065
       ---                ----
        670                350
        565
        ---
        1050
        1017
        ----
```

5. What decimal fraction of 87 is 47? 53? 43.5? 29?

0.54023
87)47.00000
435
350
348
200
174
260

0.60919
87)53.00000
522
800
783
170
87
830
783

0.5
87)43.5
43.5

0.333
87)29.000
261
290
261
290
261
29

6. How many times 393 is 587? 7857? 131? 196.5?

1.49
393)587.00
393
1940
1572
3680
3537

19.99
393)7857.00
393
3927
3537
3900
3537
363

0.33
393)131.00
1179
1310
1179
131

0.5
393)196.5
196.5

7. How many 684's are there in 1368? in 1760? in 342? in 77? in 6.84? in 0.0785?

2
684)1368
1368

0.5
684)342.0
342 0

0.01
684)6.84
6 84

2.57
684)1760.00
1368
3920
3420
5000
4788

0.1126
684)77.000
684
860
684
1760
1368
392

0.00011
684)0.0785
684
101

168. 1. Divide 11.4285285 by 3.1415927 to six decimal places.

$$\begin{array}{r} 3.637813 \\ 31\cancel{4}\cancel{1}\cancel{5}\cancel{9}\cancel{2}\cancel{7})\overline{114285285} \\ 94247781 \\ \hline 20037504 \\ 18849556 \\ \hline 1187948 \\ 942478 \\ \hline 245470 \\ 219911 \\ \hline 25559 \\ 25132 \\ \hline 427 \\ 314 \\ \hline 113 \\ 94 \\ \hline \end{array}$$

2. Divide 0.004239239 by 3.2783278 to five decimal places.

$$\begin{array}{r} 0.00129 \\ 32783\cancel{2}\cancel{7}\cancel{8})\overline{42392.39} \\ 3278328 \\ \hline 960911 \\ 655675 \\ \hline 305236 \\ 295049 \\ \hline \end{array}$$

3. Divide 437 by 215.253 to three decimal places.

$$\begin{array}{r} 2.030 \\ 21\cancel{5}\cancel{2}\cancel{5}\cancel{3})\overline{437000} \\ 43051 \\ \hline 649 \\ 646 \\ \hline \end{array}$$

4. Divide 0.0053 by 72.654 to eight decimal places.

$$\begin{array}{r} 0.00007294 \\ 72\cancel{6}\cancel{5}\cancel{4})\overline{5.30000} \\ 508578 \\ \hline 21422 \\ 14531 \\ \hline 6891 \\ 6539 \\ \hline 352 \\ 300 \\ \hline \end{array}$$

5. Divide 6 by 0.1573 to three decimal places.

$$\begin{array}{r} 38.144 \\ 15\cancel{7}\cancel{3})\overline{60000.0} \\ 4719 \\ \hline 12810 \\ 12584 \\ \hline 2260 \\ 1573 \\ \hline 687 \\ 629 \\ \hline \end{array}$$

6. Divide 0.11 by 1937.43 to eight decimal places.

$$\begin{array}{r} 0.00005677 \\ 19\cancel{3}\cancel{7}\cancel{4}\cancel{3})\overline{11.0000} \\ 96872 \\ \hline 13128 \\ 11624 \\ \hline 1504 \\ 1356 \\ \hline 148 \\ 133 \\ \hline \end{array}$$

7. Divide 46 by 0.00751515151 to three decimal places.

```
                        6120.968
75̸1̸5̸1̸5̸1̸5̸1̸) 4600000000000
            450909091
            ---------
              9090909
              7515152
              -------
              1575757
              1503030
              -------
                72727
                67636
                -----
                 5091
                 4509
                 ----
                  582
```

169. 1. Find the value of 10^0; 10^1; 10^2; 10^3; 10^4; 10^5; 10^6.

$10^0 = 1$; $10^1 = 10$; $10^2 = 10 \times 10 = 100$; $10^3 = 10 \times 10 \times 10 = 1000$; $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$; $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$; $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$.

2. Find the value of 10^0; 10^{-1}; 10^{-2}; 10^{-3}; 10^{-4}; 10^{-5}; 10^{-6}.

$10^0 = 1$; $10^{-1} = 0.1$; $10^{-2} = 0.1^2 = 0.01$; $10^{-3} = 0.1^3 = 0.001$; $10^{-4} = 0.1^4 = 0.0001$; $10^{-5} = 0.1^5 = 0.00001$; $10^{-6} = 0.1^6 = 0.000001$.

3. Find the value of $10^0 \times 10^0$; $10^1 \times 10^{-1}$; $10^2 \times 10^{-2}$; $10^3 \times 10^{-3}$; $10^4 \times 10^{-5}$.

$10^0 \times 10^0 = 10^0 = 1$; $10^1 \times 10^{-1} = 10^0 = 1$; $10^2 \times 10^{-2} = 10^0 = 1$; $10^3 \times 10^{-3} = 10^0 = 1$; $10^4 \times 10^{-5} = 10^{-1} = 0.1$.

4. Find the value of $10^3 \div 10^{-1}$; $10^{-2} \div 10^2$; $10^{-1} \div 10^{-3}$; $20^{-2} \div 10^{-4}$.

$10^3 \div 10^{-1} = 10^4 = 10{,}000$; $10^{-2} \div 10^2 = 10^{-4} = 0.1^4 = 0.0001$; $10^{-1} \div 10^{-3} = 10^2 = 100$; $10^{-2} \div 10^{-4} = 10^2 = 100$.

5. Find the value of $10^{-3} \times 10^2$; $10^3 \div 10^2$; $10^{-3} \div 10^2$; $10^{-2} \div 10^{-3}$.

$10^{-3} \times 10^2 = 10^{-1} = 0.1$; $10^3 \div 10^2 = 10^1 = 10$; $10^{-3} \div 10^2 = 10^{-5} = 0.1^5 = 0.00001$; $10^{-2} \div 10^{-3} = 10^1 = 10$.

6. Find the value of $10^2 \div 10$; $10^2 \div 10^3$; $10^0 \div 10^{-1}$; $10^{-1} \div 10^{-1}$.
$10^2 \div 10 = 10$; $10^2 \div 10^3 = 10^{-1} = 0.1$; $10^0 \div 10^{-1} = 10$;
$10^{-1} \div 10^{-1} = 10^0 = 1$.

7. Find the value of $1.01^2 \div 1.01^{-1}$; $1.01^2 \times 1.01^{-1}$; $1.01^{-2} \div 1.01^{-1}$.
$1.01^2 \div 1.01^{-1} = 1.01^3 = 1.030301$; $1.01^2 \times 1.01^{-1} = 1.01$;
$1.01^{-2} \div 1.01^{-1} = 1.01^{-1} = 1 \div 1.01 = 0.99009$.

Exercise I.

1. Express in words, 327.244.
Three hundred twenty-seven and two hundred forty-four thousandths.

2. Express in words, 80.9056.
Eighty and nine thousand fifty-six ten-thousandths.

3. Express in words, 0.390012.
Three hundred ninety thousand twelve millionths.

4. Express in words, 20000.002.
Twenty thousand and two thousandths.

5. Express in words, 0.0000008.
Eight ten-millionths.

6. Express in words, 41.27105.
Forty-one and twenty-seven thousand one hundred five hundred-thousandths.

7. Write in figures, two hundred thirty-five and eight hundred thirty-five thousandths.

235.835.

8. Write in figures, seventy-four and two hundred three thousand six millionths.

74.203006.

9. Write in figures, twelve hundred and eight thousand three ten-millionths.

1200.0008003.

10. Write in figures, five thousand sixty-four millionths.

0.005064.

11. Write in figures, one million and four tenths.

1000000.4.

12. Write in figures, six hundred-millionths.

0.00000006.

13. Multiply and divide 789.365 by 10; by 100; by 100,000.

7893.65; 78.9365; 78936.5; 7.89365; 78,936,500; 0.00789365.

14. Multiply and divide 0.004 by 100; by 10,000; by 1000.

0.4; 0.00004; 40; 0.0000004; 4; 0.000004.

15. Multiply and divide 436 by 1,000,000; by 1000; by 10.

436,000,000; 0.000436; 436,000; 0.436; 4360; 43.6.

16. Multiply and divide 0.1 by 10; by ten millions.

1; 0.01; 1,000,000; 0.00000001.

17. Find the value of 21.3706 + 15.243 + 1.8954 + 0.26891 + 5.328 + 29.74.

$$\begin{array}{l}
21.3706 \\
15.243 \\
1.8954 \\
0.026891 \\
5.328 \\
29.74 \\
\hline
73.603891
\end{array}$$

18. Find the value of 57 + 0.0057 + 6.8 + 1200 + 0.847 + 159.2 + 3.

$$\begin{array}{l}
57. \\
0.0057 \\
6.8 \\
1200. \\
0.847 \\
159.2 \\
3. \\
\hline
1426.8527
\end{array}$$

19. Find the value of $0.0012 + 10 + 5.8281 + 5 + 39.43 + 0.6827 + 1$.

$$\begin{array}{r} 0.0012 \\ 10. \\ 5.8281 \\ 5. \\ 39.43 \\ 0.6827 \\ 1. \\ \hline 61.942 \end{array}$$

20. Find the value of $23.9875 - 12.4764$; $35.14732 - 27.62815$.

$$\begin{array}{r} 23.9875 \\ 12.4764 \\ \hline 11.5111 \end{array} \qquad \begin{array}{r} 35.14732 \\ 27.62815 \\ \hline 7.51917 \end{array}$$

21. Find the value of $102.1274 - 83.072$; $39.801 - 17.9645$.

$$\begin{array}{r} 102.1274 \\ 83.072 \\ \hline 19.0554 \end{array} \qquad \begin{array}{r} 39.801 \\ 17.9645 \\ \hline 21.8365 \end{array}$$

22. Find the value of $30 - 5.2817$; $1.7 - 0.8469$.

$$\begin{array}{r} 30.0000 \\ 5.2817 \\ \hline 24.7183 \end{array} \qquad \begin{array}{r} 1.7000 \\ 0.8469 \\ \hline 0.8531 \end{array}$$

23. Find the value of $1 - 0.54237$; $100 - 0.00176$.

$$\begin{array}{r} 1.00000 \\ 0.54237 \\ \hline 0.45763 \end{array} \qquad \begin{array}{r} 100.00000 \\ 0.00176 \\ \hline 99.99824 \end{array}$$

24. Find the value of $24.271 - 3.5485 + 15.271 - 13.256 - 14.125$.

$$\begin{array}{r} 24.271 \\ 15.271 \\ \hline 39.542 \end{array} \qquad \begin{array}{r} 3.6485 \\ 13.256 \\ 14.125 \\ \hline 31.0295 \end{array} \qquad \begin{array}{r} 39.542 \\ 31.0295 \\ \hline 8.5125 \end{array}$$

25. Find the value of $52 + 0.52 - 17.8946 - 30.254 - 0.5 + 21.12$.

52.	17.8946	
0.52	30.254	73.64
21.12	0.5	48.6486
73.64	48.6486	24.9914

26. Find the value of 41.289×0.5; 0.268×0.9; 0.112×0.2.

41.289	0.268	0.112
0.5	0.9	0.2
20.6445	0.2412	0.0224

27. Find the value of 2.435×4.23; 71.651×3.37; 0.251×0.04.

2.435	71.651	0.251
4.23	3.37	0.04
7305	501557	0.01004
4870	214953	
9740	214953	
10.30005	241.46387	

28. Find the value of 0.0012×0.005; 2.26823×200; 5.6125×0.0768.

0.0012	2.26823	5.6125
0.005	200	0.0768
0.000006	453.646	449000
		336750
		392875
		0.43104

29. Find the value of $0.7 \times 7 \times 0.07$; $0.15625 \times 23.7 \times 0.00192 \times 5$.

0.7	0.15625	3.703125
7	23.7	0.00192
4.9	109375	7406250
0.07	46875	33328125
0.343	31250	3703125
	3.703125	0.00711
		5
		0.03555

30. Find the value of $(2.465 + 1.121) \times (3.2 - 2.89)$.

$$\begin{aligned} &(2.465 + 1.21) \times (3.2 - 2.89) \\ &= 3.675 \times 0.31 \\ &= 1.13925. \end{aligned}$$

31. Find the value of $(3.01)^2$; $(0.045)^2$; $(0.0081)^2$; $(5.1004)^3$; $(0.76)^3$.

$$\begin{array}{r} 3.01 \\ 3.01 \\ \hline 301 \\ 903 \\ \hline 9.0601 \end{array} \qquad \begin{array}{r} 0.045 \\ 0.045 \\ \hline 225 \\ 180 \\ \hline 0.002025 \end{array} \qquad \begin{array}{r} 0.0081 \\ 0.0081 \\ \hline 81 \\ 648 \\ \hline 0.00006561 \end{array}$$

$$\begin{array}{r} 5.1004 \\ 5.1004 \\ \hline 204016 \\ 51004 \\ 255020 \\ \hline 26.01408016 \\ 5.1004 \\ \hline 10405632064 \\ 2601408016 \\ 13007040080 \\ \hline 132.682214448064 \end{array} \qquad \begin{array}{r} 0.76 \\ 0.76 \\ \hline 456 \\ 532 \\ \hline 0.5776 \\ 0.76 \\ \hline 34656 \\ 40432 \\ \hline 0.438976 \end{array}$$

32. Find the value of $(0.125)^2 \times (0.32)^3$.

$$\begin{array}{r} 0.125 \\ 0.125 \\ \hline 625 \\ 250 \\ 125 \\ \hline 0.015625 \end{array} \qquad \begin{array}{r} 0.32 \\ 0.32 \\ \hline 64 \\ 96 \\ \hline 0.1024 \\ 0.32 \\ \hline 2048 \\ 3072 \\ \hline 0.032768 \end{array} \qquad \begin{array}{r} 0.032768 \\ 0.015625 \\ \hline 163840 \\ 65536 \\ 196608 \\ 163840 \\ 32768 \\ \hline 0.000512 \end{array}$$

33. Divide 291.84 by 6; 0.12936 by 12; 7.92801 by 0.9.

6)291.84	12)0.12936	9)7.92801
48.64	0.01078	0.88089

34. Divide 58.383 by 0.39; 0.28744 by 0.08; 491.205 by 0.065.

149.7		7557
39)5838.3	8)28.744	65)491205
39	3.593	455
193		362
156		325
378		370
351		325
273		455
273		455

35. Divide 68.325 by 6.25; 0.732 by 1.6; 1208.88 by 0.438.

10.932	0.4575	2760
625)6832.500	16)7.3200	438)1208880
625	64	876
5825	92	3328
5625	80	3066
2000	120	2628
1875	112	2628
1250	80	
1250	80	

36. Divide 498 by 0.0125; 7 by 0.007; 1000 by 0.0001.

The reciprocal of 0.0125 is 80.

498	7)7000	1)10000000
80	1000	10000000
39840		

37. Divide 0.235 by 10.24; 27 by 12; 0.00507702 by 0.0283.

0.02295
1024)23.50000
2048
3020
2048
9720
9216
5040

12)27.00
2.25

0.1794
283)50.7702
283
2247
1981
2660
2547
1132
1132

38. Divide 89.3 by 0.00752; 74.1 by 0.0256; 1 by 0.128.

11875
752)8930000
752
1410
752
6580
6016
5640
5264
3760
3760

2894.53125
256)741000.00000
512
2290
2048
2420
2304
1160
1024
1360
1280
800
768
320
256
640
512
1280
1280

7.8125
128)1000.0000
896
1040
1024
160
128
320
256
640
640

39. Divide 0.39842 by 3.7164; 281.5 by 13.789; 0.0005 by 0.0028.

```
            0.10720             20.41482           0.17857
37164)3984.20000     13789)281500.00000       28)5.00000
      37164                27578                 28
      ------               -----                 --
       267800               57200                 220
       260148               55156                 196
       ------               -----                 ---
         76520               20440                 240
         74328               13789                 224
         -----               -----                 ---
         21920                66510                 160
                              55156                 140
                              -----                 ---
                             113540                  200
                             110312                  196
                             ------                  ---
                               32280
                               27578
                               -----
```

40. Divide 63.04128 by 912.85; 287.209 by 0.00493; 2000 by 0.0059.

```
           0.06905              58257.40365             338983.05084
91285)6304.12800     493)28720900.00000     59)20000000.00000
      547710             2465                  177
      ------             ----                  ---
       827028             4070                  230
       821565             3944                  177
       ------             ----                  ---
        546300             1269                  530
        456425              986                  472
        ------             ----                  ---
                            2830                  580
                            2465                  531
                            ----                  ---
                             3650                  490
                             3451                  472
                             ----                  ---
                              1990                  180
                              1972                  177
                              ----                  ---
                               1800                  300
                               1479                  295
                               ----                  ---
                                3210                  500
                                2958                  472
                                ----                  ---
                                 2520                  280
                                 2465                  236
                                 ----                  ---
```

EXERCISE II.

1. $1.4 + 2.08 + 3.895 =$

$$\begin{array}{r} 1.4 \\ 2.08 \\ 3.895 \\ \hline 7.375 \end{array}$$

2. $2.8 + 2.08 + 0.28 + 0.028 + 0.812 =$

$$\begin{array}{r} 2.8 \\ 2.08 \\ 0.28 \\ 0.028 \\ 0.812 \\ \hline 6. \end{array}$$

3. $1.667 + 0.4 + 0.286 + 6.08 + 0.636 + 0.931 =$

$$\begin{array}{r} 1.667 \\ 0.4 \\ 0.286 \\ 6.08 \\ 0.636 \\ 0.931 \\ \hline 10. \end{array}$$

4. $6.125 - 0.57 =$

$$\begin{array}{r} 6.125 \\ 0.57 \\ \hline 5.555 \end{array}$$

5. $(4.625 + 1.146) - (1.2 + 3.57)$
$= 5.771 - 4.771$
$= 1.$

6. $6.913 - (2.85 - 0.937)$
$= 6.913 - 1.913$
$= 5.$

7. $24 - 2.4 + (5 - 3.508) - 3.092$
$= 24 - 2.4 + 1.492 - 3.092$
$= 25.492 - 5.492$
$= 20.$

8. $10 - (4.25 - 2.5 + 2 - 0.625 - 0.4 - 2.02) - 0.295$
$= 10 - (6.25 - 5.545) - 0.295$
$= 10 - 0.705 - 0.295$
$= 10 - 1$
$= 9.$

9. $1.5 \times 0.08 \times 0.5 =$

$$\begin{array}{r} 1.5 \\ 0.08 \\ \hline 0.12 \\ 0.5 \\ \hline 0.06 \end{array}$$

10 $0.1204 \times 0.0168 \times 100 =$

$$\begin{array}{r} 0.1204 \\ 0.0168 \\ \hline 9632 \\ 7224 \\ 1204 \\ \hline 0.00202272 \\ 100 \\ \hline 0.202272 \end{array}$$

11. $0.04 \times 3.25 \times 0.06 =$

$$\begin{array}{r} 3.25 \\ \underline{0.04} \\ 0.13 \\ \underline{0.06} \\ 0.0078 \end{array}$$

12. $36 \times 0.002 \times 2.05 \times 0.00765 =$

$$\begin{array}{r} 36 \\ \underline{0.002} \\ 0.072 \\ \underline{2.05} \\ 360 \\ \underline{144} \\ 0.1476 \end{array} \qquad \begin{array}{r} 0.1476 \\ \underline{0.00765} \\ 7380 \\ 8856 \\ \underline{10332} \\ 0.00112904 \end{array}$$

13. $0.139 \times 28 + 42 \times 0.002 + 6 \times 0.004 - 0.05 \times 20$

$= 3.892 + 0.084 + 0.024 - 1$

$= 4 - 1$

$= 3.$

14. $(10 - 1.25) \times 0.2 + 0.02 \times 2.8 + (80.3 \times 0.1 - 5.3) \times 10 - 805.3 \times 0.02$

$= 8.75 \times 0.2 + 0.02 \times 2.8 + (8.03 - 5.3) \times 10 - 805.3 \times 0.02$

$= 1.75 + 0.056 + 27.3 - 16.106$

$= 29.106 - 16.106$

$= 13.$

15. $28.3696 + 1.49 =$

$$\begin{array}{r} 19.04 \\ 149\overline{)2836.96} \\ \underline{149} \\ 1346 \\ \underline{1341} \\ 596 \\ \underline{596} \end{array}$$

16. $0.27 + 0.00225 =$

$$\begin{array}{r} 120 \\ 225\overline{)27000} \\ \underline{225} \\ 450 \\ \underline{450} \end{array}$$

17. $8.8779 + 175.8 =$

$$\begin{array}{r} 0.0505 \\ 1758\overline{)88.7790} \\ \underline{8790} \\ 8790 \\ \underline{8790} \end{array}$$

18. $0.0427 + 92.3 =$

$$\begin{array}{r} 0.00046 \\ 923\overline{)0.42700} \\ \underline{3692} \\ 5780 \\ \underline{5538} \end{array}$$

19. $0.28744 \div 800 =$

$$\begin{array}{r} 8\overline{)0.0028744} \\ 0.0003593 \end{array}$$

20. $491.205 \div 650 =$

$$\begin{array}{r} 0.7557 \\ 65\overline{)49.1205} \\ \underline{455} \\ 362 \\ \underline{325} \\ 370 \\ \underline{325} \\ 455 \\ \underline{455} \end{array}$$

21. $68.325 \div 6250 =$

$$\begin{array}{r} 0.010932 \\ 625\overline{)6.832500} \\ \underline{625} \\ 5825 \\ \underline{5625} \\ 2000 \\ \underline{1875} \\ 1250 \\ \underline{1250} \end{array}$$

22. $0.732 \div 16{,}000 =$

$$\begin{array}{r} 0.00004575 \\ 16\overline{)0.00073200} \\ \underline{64} \\ 92 \\ \underline{80} \\ 120 \\ \underline{112} \\ 80 \\ \underline{80} \end{array}$$

23. $1208.88 \div 0.438 =$

$$\begin{array}{r} 2760 \\ 438\overline{)1208880} \\ \underline{876} \\ 3328 \\ \underline{3066} \\ 2628 \\ \underline{2628} \end{array}$$

24. $2 \div 0.01 - (0.2 \div 0.02 + 0.8 \div 10) + 36.48 \div 8 - (4 \div 0.05 - 2 + 0.6 \div 1.25$
$= 200 - (10 + 0.08) + 4.56 - (80 - 2 + 0.48)$
$= 200 - 10.08 + 4.56 - 78.48$
$= 204.56 - 88.56$
$= 116.$

25. $72.2 \div 10 - 2 \div (0.5 \div 1.60) + 2.125 \div (1.75 - 0.5).$
$= 72.2 \div 10 - 2 \div 0.3125 + 2.125 \div 1.25$
$= 7.22 - 6.4 + 1.7$
$= 8.92 - 6.4$
$= 2.52.$

Exercise III.

1. What number subtracted 88 times from 80,005 will leave 13 as a remainder?

$$\begin{array}{r} 80{,}005 \\ \underline{13} \\ 79{,}992 \end{array} \qquad \begin{array}{r} 909 \\ 88\overline{)79992} \\ \underline{792} \\ 792 \\ \underline{792} \end{array}$$

2. If 7 men can build a wall in 16 days, how many men will it take to build a wall three times as long in half the time?

$$\begin{array}{r} 7 \\ \underline{3} \\ 21 \\ \underline{2} \\ 42 \end{array}$$

3. How many minutes are there between 25 minutes past 8 in the morning and midnight?

$$\begin{array}{r} 35 \\ 180 \\ \underline{720} \\ 935 \end{array}$$

4. The velocity of sound being 1090 feet per second, at what distance is a gun fired, the report of which I hear 11 seconds after seeing the flash? (5280 feet make a mile.)

$$\begin{array}{r} 1090 \\ \underline{11} \\ 1090 \\ \underline{1090} \\ 11990 \end{array} \qquad \begin{array}{r} 2.27083 \\ 5280\overline{)11990.00000} \\ \underline{10560} \\ 14300 \\ \underline{10560} \\ 37400 \\ \underline{36960} \\ 44000 \\ \underline{42240} \\ 17600 \\ \underline{15840} \end{array}$$

5. How long would it take to travel 30.2375 miles at the rate of 8.85 miles per hour ?

```
         3.4167
885)3023.7500
     2655
     ----
      3687
      3540
      ----
       1475
        885
       ----
        5900
        5310
        ----
         5900
         6195
         ----
```

6. The circumference of a circle being 3.1416 times the diameter, find the circumference of a circle whose diameter is 6.8 feet; also, find the diameter of a circle whose circumference is 20 inches.

```
  3.1416
     6.8
  ------
  251328
 188496
 -------
21.36288
= 21.363 ft. Ans.
```

```
                6.366
31416)200000.000
      188496
      ------
       115040
        94248
       ------
        207920
        188496
        ------
         194240
         188496
         ------
```

7. How much wire will be required to make a hoop 30 inches in diameter, allowing two inches for the joining?

```
3.1416
    30
------
94.248
 2
------
96.248
```

8. How many times would such a hoop turn in going half a mile?

```
2)5280
  2640

  2640
    12
  ----
  5280
 2640
 -----
 31680
```

```
             336.
94248)31680000.
      282744
      ------
       340560
       282744
       ------
        578160
        565488
        ------
```

9. Cork, whose weight is 0.24 of that of water, weighs 15 pounds per cubic foot. What is the weight of 6 cubic feet of oak, the weight of oak being 0.934 of that of water?

```
     62.5
24)1500.0
   144
   ---
    60
    48
    --
    120
    120
    ---
```

```
     62.5
        6
    -----
     375.
    0.934
    -----
     1500
    1125
   3375
   ------
   350.25
```

10. From what number can 847 be subtracted 307 times, and leave a remainder of 49?

```
   847
   307
   ---
  5929
2541
------
260029
    49
------
260078
```

11. What is the 235th part of 141,235?

```
       601
235)141235
    1410
    ----
      235
      235
      ---
```

12. What will 343 barrels of flour cost, at \$6.37 a barrel?

$$\begin{array}{r} \$6.37 \\ 343 \\ \hline 1911 \\ 2548 \\ 1911 \\ \hline \$2184.91 \end{array}$$

13. 12 make a dozen, and 12 dozen make a gross. How many steel pens in 28 gross? What will a gross of eggs cost, at 27 cents a dozen?

$$\begin{array}{r} 144 \\ 28 \\ \hline 1152 \\ 288 \\ \hline 4032 \end{array} \qquad \begin{array}{r} \$0.27 \\ 12 \\ \hline 54 \\ 27 \\ \hline \$3.24 \end{array}$$

14. How much must be added to \$4429 in order to make the sum $43 \times \$241$?

$$\begin{array}{r} \$241 \\ 43 \\ \hline 723 \\ 964 \\ \hline 10363 \\ 4429 \\ \hline \$5934 \end{array}$$

15. What number deducted from the 26th part of 2262 will leave the 87th part of the same number?

$$\begin{array}{r} 87 \\ 26\overline{)2262} \\ 208 \\ \hline 182 \\ 182 \\ \hline \end{array} \qquad \begin{array}{r} 26 \\ 87\overline{)2262} \\ 174 \\ \hline 522 \\ 522 \\ \hline \end{array} \qquad \begin{array}{r} 87 \\ 26 \\ \hline 61 \end{array}$$

16. At an ordinary rate, 123 words a minute, how long will it take a man to deliver a speech of 15 pages, each of 28 lines, and each line containing 11 words? How long would it have taken Daniel Webster to deliver the same speech, at the rate of 93 words a minute?

$$\begin{array}{r} 15 \\ 28 \\ \hline 120 \\ 30 \\ \hline 420 \\ 11 \\ \hline 420 \\ 420 \\ \hline 4620 \end{array} \qquad \begin{array}{r} 37.6 \\ 123\overline{)4620.0} \\ 369 \\ \hline 930 \\ 861 \\ \hline 690 \end{array} \qquad \begin{array}{r} 49.7 \\ 93\overline{)4620.0} \\ 372 \\ \hline 900 \\ 837 \\ \hline 630 \end{array}$$

17. How long would it take a railway train to go from New York to San Francisco, 3310 miles, at the rate of 1973 feet a minute?

```
  3310                  8858
  5280          1973)17476800
------               15784
264800               -----
 6620                 16928
16550                 15784
--------              -----
17476800               11440
                        9865
                       -----
                        15750
```

18. How long will it take to count a million, at the rate of 67 a minute?

```
     14925.4
 67)1000000.0
    67
    ---
    330
    268
    ---
     620
     603
     ---
      170
      134
      ---
       360
       335
       ---
        250
```

19. If you put into a box 17 cents a day including Sundays, beginning January 1 and ending July 4, how much money will there be in the box?

```
 31        185
 28        0.17
 31        ----
 30        1295
 31        185
 30        -----
  4        31.45
---        = $31.45.
185
```

20. If a man's income is $3000 a year, and his daily expenses average $7.68, what does he save in a year?

```
 $7.68        $3000.
   365        2803.20
 -----        -------
  3840        $196.80
 4608
2304
--------
$2803.20
```

21. In a question of division the quotient was 87.83, the divisor 759. What was the dividend?

```
   87.83
     759
   -----
   79047
  43915
 61481
--------
66662.97
```

22. It is 3.1416 times as far round a wheel as across it. How many times will a wheel 4.5 feet across turn round in going 23 miles of 5280 feet each?

		8590
5280	3.1416	141372) 1214400000
23	4.5	1130976
15840	157080	834240
10560	125664	706860
121440	14.1372	1273800
		1272348
		14520

23. How many gallons of 231 cubic inches are contained in a cubic foot (1728 cubic inches)? in a bushel of 2150.42 cubic inches? How many cubic feet in a bushel? How many bushels in 31.5 gallons?

(i.)	(ii.)
7.48	9.309
231) 1728.00	231) 2150.420
1617	2079
1110	714
924	693
1860	2120
1848	2079

(iii.)		(iv.)
1.244		3.38
1728) 2150.420	31.5	215042) 727650.00
1728	231	645126
4224	315	825240
3456	945	645126
7682	630	1801140
6912	7276.5	1720336

24. Seven children had left to them $7186 apiece; one died, and his share was divided among the surviving six. How much had each then?

```
6)$7186.00
  $1197.67
   7186
  --------
  $8383.67
```

25. What is the nearest number to 7196 that will contain 372 without a remainder?

```
       19
372)7196        7196
    372          128
    ----        ----
    3476        7068
    3348
    ----
     128
```

26. How long will it take 2 men to do what 1 man can do in 6 days? what 4 men can do in 3 days? what 3 men can do in 4 days?

6 days ÷ 2 = 3 days.
2 × 3 days = 6 days.
(3 × 4 days) ÷ 2 = 6 days.

27. Divide $1.80 among Thomas, Richard, and Henry in such a way that Henry shall receive 3 cents for every 5 cents that Thomas gets, and Richard shall receive 2 cents for every 3 cents that Henry gets.

```
 2        10)$1.80
 3           $0.18
 5               2
--           -----
10           $0.36 = R.'s.

$0.18        $0.18
    3            5
-----        -----
$0.54 = H.'s. $0.90 = T.'s.
```

28. Divide $87.84 between B and C so that C shall get $19 as often as B gets $17.

```
                2.44
19          36)87.84
17             72
--             ---
36             158
               144
               ---
                144
                144
                ---
$2.44         $2.44
   19            17
-----         -----
 2196          1708
 244           244
------        ------
$46.36 = B's.  $41.48 = C's.
```

29. Three partners received for goods: one, $371.63; the second, $285.40; the third, $411.91. They paid for the goods $879.34, and divided the balance equally among them. How much did each receive?

```
 $371.63       $1068.94
  285.40         879.34
  411.91      3)$189.60
 --------      $  63.20
$1068.94
```

30. At 12 inches in a foot, how many inches long is a wall 35 feet in length? A brick and its share of mortar being 8.4 inches long, how many bricks in length is the wall?

```
  35          50
  12     84)4200
  --         420
  70         ---
 35            0
 ---
 420
```

31. A brick and mortar being 2.4 inches in height, how many bricks are required to build the wall 12 feet high, if the wall be two bricks wide?

```
 12        60      60
 12   24)1440      50
 --        144   ----
144        ---   3000
             0      2
                 ----
                 6000
```

32. What is the total weight of the wall, if a brick and its share of the mortar weigh 4.13 pounds? What is the weight after a long rain, when the weight is increased to 4.27 pounds for each brick?

```
 4.13        4.27
  6000        6000
 -----       -----
 24780       25620
```

33. How many pounds does each foot in length of the wall weigh?

```
      708           732
 35)24780      35)25620
    245           245
    ---           ---
     280           112
     280           105
     ---           ---
                    70
                    70
                    --
```

34. If 60.98 cubic inches of brick weigh 4 pounds, how many cubic inches of brick weigh 1 pound? How many pounds would a cubic foot (1728 cubic inches) weigh?

```
 4)60.980
   ------
   15.245

              113.35
 15245)1728000 00
       15245
       -----
        20350
        15245
        -----
         51050
         45735
         -----
          53150
          45735
          -----
          74150
```

35. If a cubic foot of water weigh 62.5 pounds, how many times as heavy as water is brick?

```
        1.8
 625)1133.5
     625
     ----
     5085
     5000
     ----
```

36. Light moves through the air at 186,500 miles in a second. How many times can it go around the earth in a second, the distance round the earth being 24,897.714 miles?

```
                 7.5
24897714)186500000.0
          174283998
          122160020
```

37. Light moves through the air at 300,190 kilometers in a second. How many times can it go around the earth in a second, the distance round the earth being 40,007.5 kilometers?

```
              7.5
400075)3001900.0
        2800525
         2013750
         2000305
```

38. A minute is 60 seconds. How many miles and how many kilometers can light travel through air in a minute?

300190 km.
60
18,011,400 km.

186,500 mi.
60
11,190,000 mi.

39. An hour is 60 minutes. How many miles and how many kilometers can light travel in an hour?

18,011,400 km.
60
1,080,684,000 km.

11,190,000 mi.
60
671,400,000 mi.

40. The distance round the earth, given in Ex. 37, is measured on a north and south line. Around the equator the distance is 40,075.45 kilometers. How many times could light move round the equator in one minute?

```
                 7.49
4007545)30019000.00
         28052815
          19661850
          16030180
           36316700
           36067905
```

7.49
60
449.4

41. Find the reciprocal of the difference between 31.24 and 31.23768.

31.24
31.23768
0.00232

```
         431.034
232)100000.000
     928
      720
      696
       240
       232
        800
        696
        1040
         928
```

42. The Hanoverian mile is 25,400 Hanoverian feet long, each foot being 0.9542 of an English foot. Find to four places of decimals the fraction that an English mile of 5280 English feet is of a Hanoverian mile.

```
      0.9542
       25400
     3816800
    47710
   19084
   24236.6800

                0.2178
2423668)528000.0000
         4847336
          4326640
          2423668
          19029720
          16965676
           21640440
           19389444
```

43. Express in inches the length of a meter, given that a meter is one ten-millionth of a quarter of the earth's circumference, that the circumference is 3.14159 times the diameter, that the diameter is 7911.7 miles, and that a mile is 5280 × 12 inches.

```
              5280
                12
             10560
             5280
             63360
            7911.7
            443520
            63360
           63360
          570240
         443520
         501285312.
           3.14159
        4511567808
       2506426560
       501285312
     2005141248
     501285312
   1503855936
  4)1574832923.32608
    393708230.83152
    0.0000001
    39.370823083152
  = 39.3708 in. Ans.
```

44. How must a number be altered to double its reciprocal? Divided by 2.

45. What effect is produced on the sum of two numbers, if each number is increased by the same number? What effect on the difference?

It is increased by two times the number; not any.

46. What effect is produced on the product of two numbers, if both numbers are multiplied by the same number? What effect on the quotient?

It is multiplied by the square of the number; not any.

47. What effect is produced on the *remainder*, if both divisor and dividend are multiplied by the same number? If both are divided by the same number?

It is multiplied by the number; it is divided by the number.

48. In going from one planet to another light probably moves faster than in air. Suppose it moves at 309,800 kilometers a second, how long would it take light to perform each of the following journeys:

Moon to Earth	375,500	kilometers.
Sun to Earth	147,250,000	"
Sun to Mercury	56,900,000	"
Sun to Venus	106,400,000	"
Sun to Mars	224,100,000	"
Sun to the Asteroids	400,000,000	"
Sun to Jupiter.	765,400,000	"
Sun to Saturn	1,403,000,000	"
Sun to Uranus.	2,817,000,000	"
Sun to Neptune	4,421,000,000	"
Sun to the nearest star . .	24,000,000,000,000	"

1.21	475.3	183.7
3098)3755.00	3098)1472500.0	3098)569000.0
3098	12392	3098
6570	23330	25920
6196	21686	24764
3740	16440	11560
3098	15490	9294
	9500	22660
	9294	21686

343.4
3098)1064000.0
9294
13460
12392
10680
9294
13860
12392

723.4
3090)2241000.0
21686
7240
6196
10440
9294
12460
12392

1291.1
3098)4000000.0
3098
9020
6196
28240
27802
3580
3098
4820
3098

2470.6
3098)7654000.0
6196
14580
12392
21880
21686
19400
18588

4528.7
3098)14030000.0
12392
16380
15490
8900
6196
27040
24784
22560
21686

9092.9
3098)28170000.0
27882
28800
27882
9180
6196
29840
27882

14270.5
3098)44210000.0
3098
13230
12392
8380
6196
21840
21686
15400
15490

77469335
3098)240000000000
21686
23140
21686
14540
12392
21480
18588
28920
27882
10380
9294
10860
9294
15660
15490

49. A kilometer is about 0.6214 ~~of~~ a mile. How many miles is each of the planets from the sun?

14725	5690	10640
6214	6214	6214
58900	22760	42560
14725	5690	10640
29450	11380	21280
88350	34140	63840
Earth, 91,501,150	Mercury, 35,357,660	Venus, 66,116,960

22410		76540
6214		6214
89640		306160
22410		76540
44820	6214	153080
134460	40000	459240
Mars, 139,255,740	Asteroids, 248,560,000	Jupiter, 475,619,560

140300	281700	442100
6214	6214	6214
561200	1126800	1768400
140300	281700	442100
280600	563400	884200
841800	1690200	2652600
Saturn, 871,824,200	Uranus, 1,750,483,800	Neptune, 2,747,209,400

Exercises on Page 73.

1. Convert 5427^{m} into kilometers; into millimeters; into centimeters.

$5427^{m} = 5.427^{km} = 5427000^{mm} = 542700^{cm}$.

2. 6853^{mm} contain how many meters? how many centimeters? what part of a kilometer?

$6853^{mm} = 6.853^{m} = 685.3^{cm} = 0.006853^{km}$.

3. Write 49.7^{m} as centimeters; as millimeters; as part of a kilometer.

$49.7^{m} = 4970^{cm} = 49700^{mm} = 0.0497^{km}$.

4. How many centimeters in 12.4^{km}? how many millimeters?

$12.4^{km} = 1{,}240{,}000^{cm} = 12{,}400{,}000^{mm}$.

5. Change 1230 meters into kilometers; into centimeters.

$1230^{m} = 123^{km} = 123{,}000^{cm}$.

6. Write 1230^{cm} as meters; as millimeters.

$1230^{cm} = 12.3^{m} = 12{,}300^{mm}$.

7. $0.435^{m} + 852^{cm} + 4263^{mm} + 0.1595^{km}$.

$$\begin{array}{r} 0.435^{m} \\ 8.52 \\ 4.26 \\ 159.5 \\ \hline 172.718^{m} \end{array}$$

8. $0.927^{km} - 6495^{cm}$; $4.37^{cm} - 42.87^{mm}$.

$$\begin{array}{r} 927.^{m} \\ 64.95 \\ \hline 862.05^{m} \end{array} \qquad \begin{array}{r} 0.0437^{m} \\ 0.04287 \\ \hline 0.00083^{m} \end{array}$$

9. 8×0.0457^{km}; 3.04×60.93^{cm}; 5.43×67.2^{mm}.

$$\begin{array}{r} 45.7^{m} \\ 8 \\ \hline 365.6^{m} \end{array} \qquad \begin{array}{r} 0.6093^{m} \\ 3.04 \\ \hline 24372 \\ 18279 \\ \hline 1.852272^{m} \end{array} \qquad \begin{array}{r} 0.0672^{m} \\ 5.43 \\ \hline 2016 \\ 2688 \\ 3360 \\ \hline 0.364896^{m} \end{array}$$

10. $38{,}019^{mm} \div 0.097$; $0.41^{km} \div 25.625$.

$$\begin{array}{r} 391.948^{m} \\ 97\overline{)38019.000} \\ 291 \\ \hline 891 \\ 873 \\ \hline 189 \\ 97 \\ \hline 920 \\ 873 \\ \hline 470 \\ 388 \\ \hline 820 \\ 776 \\ \hline \end{array} \qquad \begin{array}{r} 16^{m} \\ 25625\overline{)410000} \\ 25625 \\ \hline 143750 \\ 143750 \\ \hline \end{array}$$

11. At \$1.87 the meter what is the cost of 6.20^{m} of cloth?

$$\begin{array}{r} \$1.87 \\ 6.2 \\ \hline 374 \\ 1122 \\ \hline \$11.594 \end{array}$$

\$11.59. *Ans.*

12. At $0.75 the meter what is the cost of 60^m of cloth?

$$\begin{array}{r} \$0.75 \\ 60 \\ \hline \$45.00 \end{array}$$

13. From a piece of cloth containing 47.60^m a tailor cuts off three pieces: the first of 3.80^m, the second of 1.30^m, and the third of 45^{cm}. How much of the cloth is left?

$$\begin{array}{r} 3.8^m \\ 1.3 \\ 0.45 \\ \hline 5.55^m \end{array} \qquad \begin{array}{r} 47.6^m \\ 5.55 \\ \hline 42.05^m \end{array}$$

14. What is the value of 60^{cm} of cloth, worth $5.20 a meter?

$$\begin{array}{r} \$5.20 \\ 0.6 \\ \hline \$3.12 \end{array}$$

15. If $6.00 are paid for a railroad ticket to travel 440^{km}, what is the fare per kilometer?

$$0.0138 = \$0.014$$

$$\begin{array}{r} 440)\overline{6.000} \\ 440 \\ \hline 1600 \\ 1320 \\ \hline 3800 \\ 3520 \\ \hline 280 \end{array}$$

16. If a train run 288^{km} in 9 hours, how many meters does it run in a minute?

$$\begin{array}{r} 60 \\ 9 \\ \hline 540 \end{array} \qquad \begin{array}{r} 533.33^m \\ 54)\overline{28800.00^m} \\ 270 \\ \hline 180 \\ 162 \\ \hline 180 \\ 162 \\ \hline 180 \\ 162 \\ \hline 180 \\ 162 \\ \hline \end{array}$$

17. If a man walk at the rate of 6^{km} an hour, what part of an hour will it take to walk 420 meters?

$$6^{km} = 6000^m \qquad \begin{array}{r} 0.02 \\ 6000)\overline{420.00} \\ 42000 \\ \hline \end{array}$$

18. A railroad carried 412 passengers 18 kilometers, and received $88.992; at the same rate, what will it receive for carrying 350 passengers 35 kilometers?

$$\begin{array}{r} 412 \\ 18 \\ \hline 3296 \\ 412 \\ \hline 7416 \end{array} \qquad \begin{array}{r} 0.012 \\ 7416)\overline{88.992} \\ 7416 \\ \hline 14832 \\ 14832 \\ \hline \end{array}$$

$$\begin{array}{r} 350 \\ 35 \\ \hline 1750 \\ 1050 \\ \hline 12250 \end{array} \qquad \begin{array}{r} 12250 \\ \$0.012 \\ \hline 24500 \\ 12250 \\ \hline \$147.000 \end{array}$$

Exercises on Pages 75 and 76.

1. Convert $1,854,276^{qm}$ into hektars; into square kilometers.

$1,854,276^{qm} = 185.4276^{ha}$;
$= 1.854276^{qkm}$.

2. How many hektars in 2.7856 square kilometers?

$2.7856^{qkm} = 278.56^{ha}$.

3. Write 1.7431^{qm} as square centimeters; as square millimeters.

$1.7431^{qm} = 17,431^{qcm}$;
$= 1,743,100^{qmm}$.

4. How many square kilometers in 17,467.5 hektars?

$17,467.5^{ha} = 174.675^{qkm}$.

5. How many square meters in 1.3614^{qkm}?

$1.3614^{qkm} = 1,361,400^{qm}$.

6. How many square meters in 2.25 hektars?

$2.25^{ha} = 22,500^{qm}$.

7. How many square centimeters in 0.0137 of a square meter?

$0.0137^{qm} = 137^{qcm}$.

8. Write 3.571^{qcm} as square millimeters.

$3.571^{qcm} = 357.1^{qmm}$.

Exercises on Page 77.

1. How many cubic centimeters in 2.25^{cbm}?

$2.25^{cbm} = 2,250,000^{ccm}$.

2. How many cubic meters in $2,162,875^{ccm}$?

$2,162,875^{ccm} = 2.162875^{cbm}$.

Exercises on Page 78.

1. How many liters in 1.7^{cbm}? in $157,854^{ccm}$?

$1.7^{cbm} = 1700^{l}$;
$157,854^{ccm} = 157.854^{l}$.

2. How many cubic centimeters in 9.5^{l}? in 0.015^{l}?

$9.5^{l} = 9500^{ccm}$;
$0.015^{l} = 15^{ccm}$.

3. Change 1.25^{hl} to cubic centimeters; to the fraction of a cubic meter.

$1.25^{hl} = 125,000^{ccm}$;
$= 0.125^{cbm}$.

4. Convert 431.88^{l} into hektoliters; into the fraction of a cubic meter.

$431.88^{l} = 4.3188^{hl}$;
$= 0.43188^{cbm}$.

5. Write 0.375^{cbm} as liters; as cubic centimeters.

$0.375^{cbm} = 375^{l}$;
$= 375{,}000^{ccm}$.

6. Write $734{,}159.651^{ccm}$ as liters; as hektoliters; as cubic meters.

$734{,}159.651^{ccm} = 734.159651^{l}$;
$= 7.34159651^{hl}$;
$= 0.734159651$

7. How many meters in $8{,}573{,}412.867^{ccm}$?

$8{,}573{,}412.867^{ccm} =$
8.573412867^{cbm}.

8. Change the expression 0.734578912^{cbm} into cubic centimeters; into liters.

$0.734578912^{cbm} = 734{,}578.912^{ccm}$;
$= 734.578912^{l}$.

9. Change 1731.5 liters into cubic meters; into cubic centimeters.

$1731.5^{l} = 1.7315^{cbm}$;
$= 1{,}731{,}500^{ccm}$.

Exercises on Page 79.

1. How many kilos in 1.73^{t}? in 0.341 of a ton?

$1.73^{t} = 1730^{kg}$;
$0.341^{t} = 341^{kg}$.

2. How many kilos will a hektoliter of water weigh?

100^{kg}.

3. Convert $13{,}756^{mg}$ into grams; into the fraction of a kilo.

$13{,}756^{mg} = 13.756^{g}$;
$= 0.013756^{kg}$.

4. What is the weight in grams of 346.1^{ccm} of water?

346.1^{g}.

5. Give the weight in kilograms of 0.37615^{cbm} of water.

376.15^{kg}.

6. Change 0.6778^{kg} into milligrams.

$677{,}800^{mg}$.

7. How many milligrams in the third part of 17.4 grams?

$5{,}800^{mg}$.

Exercise IV.

1. Add 17.3^m, 87.41^m, 271^{cm}, 380^{mm}, and 1.79^m.

$$\begin{array}{r} 17.3^m \\ 87.41 \\ 2.71 \\ 0.38 \\ 1.79 \\ \hline 109.59^m \end{array}$$

2. What is the sum of \$15.87, \$39.46, \$47.52, \$75.38, \$75.89?

$$\begin{array}{r} \$15.87 \\ 39.46 \\ 47.52 \\ 75.38 \\ 75.89 \\ \hline \$254.12 \end{array}$$

3. Add 187^{cm}, 49.3^m, 317^{mm}, and 6.138^m.

$$\begin{array}{r} 1.87^m \\ 49.3 \\ 0.317 \\ 6.138 \\ \hline 57.625^m \end{array}$$

4. The door-sill being 3^{cm} high; the door, 2.34^m; the finish over it, 13.7^{cm}; and the distance from finish to ceiling, 93^{cm}: how far from floor to ceiling?

$$\begin{array}{r} 0.03^m \\ 2.34 \\ 0.137 \\ 0.93 \\ \hline 3.437^m \end{array}$$

5. The distance to the post-office is 3.31^{km}; thence to the mill, 1.711^{km}; thence to the store, 3.718^{km}; thence home, 2.543^{km}. How long is the circuit?

$$\begin{array}{r} 3.31^{km} \\ 1.711 \\ 3.718 \\ 2.543 \\ \hline 11.282^{km} \end{array}$$

6. From Portland, Me., to Boston is about 132^{km}; Boston to Albany, 320^{km}; Albany to Buffalo, 480^{km}; Buffalo to Chicago, 800^{km}; Chicago to Omaha, 800^{km}; Omaha to Cheyenne, 780^{km}; how far from Cheyenne to Portland? to Albany? from Boston to Chicago? from Boston to Cheyenne?

(1)

$$\begin{array}{r} 132^{km} \\ 320 \\ 480 \\ 800 \\ 800 \\ 780 \\ \hline 3312^{km} \end{array}$$

(2)

$$\begin{array}{r} 132^{km} \\ 320 \\ \hline 452^{km} \end{array}$$

$$\begin{array}{r} 3312^{km} \\ 452 \\ \hline 2860^{km} \end{array}$$

(3)

$$\begin{array}{r} 320^{km} \\ 480 \\ 800 \\ \hline 1600^{km} \end{array}$$

(4)

$$\begin{array}{r} 3312^{km} \\ 132 \\ \hline 3180^{km} \end{array}$$

7. If I travel 789.7^{km} a day, how far shall I go in 7 days? in 8.5? in 19.6? in 27.8? in 365 days?

$$\begin{array}{r}
789.7^{km} \\
7 \\ \hline
5527.9^{km}
\end{array}
\qquad
\begin{array}{r}
789.7^{km} \\
8.5 \\ \hline
39485 \\
63176 \\ \hline
6712.45^{km}
\end{array}
\qquad
\begin{array}{r}
789.7^{km} \\
19.6 \\ \hline
47382 \\
71073 \\
7897 \\ \hline
15478.12^{km}
\end{array}
\qquad
\begin{array}{r}
789.7^{km} \\
27.8 \\ \hline
63176 \\
55279 \\
15794 \\ \hline
21{,}953.66^{km}
\end{array}
\qquad
\begin{array}{r}
789.7^{km} \\
365 \\ \hline
39485 \\
47382 \\
23691 \\ \hline
288{,}240.5^{km}
\end{array}$$

8. How much will 3^{m} of cloth cost, at \$1.37 a meter? How much 5.38^{m}, at \$2.63 a meter?

$$\begin{array}{r}
\$1.37 \\
3 \\ \hline
\$4.11
\end{array}
\qquad
\begin{array}{r}
\$2.63 \\
5.38 \\ \hline
2104 \\
789 \\
1315 \\ \hline
\$14.1494 \\
= \$14.15. \ \textit{Ans.}
\end{array}$$

9. How much will 13.4^{kg} of opium be worth, at \$8.48 a kilo? 28.79^{kg}, at \$7.96 a kilo?

$$\begin{array}{r}
\$8.48 \\
13.4 \\ \hline
3392 \\
2544 \\
848 \\ \hline
\$113.63
\end{array}
\qquad
\begin{array}{r}
28.79 \\
\$7.96 \\ \hline
17274 \\
25911 \\
20153 \\ \hline
\$229.1684 \\
= \$229.17. \ \textit{Ans.}
\end{array}$$

10. A man bought 153 barrels of flour, at \$4.875 a bbl. What did the whole cost him?

$$\begin{array}{r}
\$4.875 \\
153 \\ \hline
14625 \\
24375 \\
4875 \\ \hline
\$745.875 \\
= \$745.88. \ \textit{Ans.}
\end{array}$$

11. He gave for it 6 shares of stock, at \$113.50 a share, and the rest in cash. How much money did he pay?

$$\begin{array}{r}
\$113.50 \\
6 \\ \hline
\$681.00
\end{array}$$

$$\begin{array}{r}
\$745.88 \\
681. \\ \hline
\$\ \ 64.88
\end{array}$$

12. He paid \$13.75 for storage; also, 75 cents a barrel for freight. How much do these expenses amount to?

153
$0.75
765
1071
$114.75
13.75
$128.50

13. It was sold, 49 bbls. at $6.50 a bbl.; the rest at $6.25. What were the gross receipts?

$6.50	$6.25
49	104
5850	2500
2600	625
$318.50	$650.00
153	318.50
49	$968.50
104	

14. He paid for commissions, etc., $17.50; and counts his loss of interest at $29.30. What then is his net profit?

$745.88
128.50
17.50
29.30
$921.18

$968.50
921.18
$ 47.32

15. Find the circumference of a circle having a diameter of 1^{m}.

3.1416
1^{m}
3.1416^{m}

16. Find the circumferences of circles of which the diameters are respectively 83^{m}; 3.71^{m}; 32.8^{m}; 10.4^{cm}; 11.8^{cm}; 167.1^{mm}; 39.3^{mm}. Give each to the nearest tenth of a millimeter.

3.1416	3.1416	3.1416
83000^{mm}	3710^{mm}	32800^{mm}
94248000	314160	25132800
251328	219912	62832
260752.8^{mm}	94248	94248
	11655.3^{mm}	103044.4800^{mm}
		$= 103{,}044.5^{mm}$. *Ans.*

3.1416	3.1416	3.1416	3.1416
104^{mm}	118^{mm}	167.1^{mm}	39.3^{mm}
125664	251328	31416	94248
31416	31416	219912	282744
326.7264^{mm}	31416	188496	94248
$= 326.7^{mm}$. *Ans.*	370.7088^{mm}	31416	123.46488^{mm}
	$= 370.7^{mm}$. *Ans.*	524.96136^{mm}	$\doteq 123.5^{mm}$. *Ans.*
		$= 525^{mm}$. *Ans.*	

17. What is the length of the earth's orbit, to the nearest meter, if the diameter of the orbit is 294,481,217^{km}?

294481217^{km}
3.1416
1766887302
294481217
1177924868
294481217
883443651
925,142,191.3272^{km}
= 925,142,191,327^{m}. *Ans.*

18. How far round this world, if its diameter is 12,734^{km}?

12734^{km}
3.1416
76404
12734
50936
12734
38202
40,005.1344^{km}. *Ans.*

19. If a carriage-wheel is 1.31^{m} in diameter, what is its circumference? How far will it go, if it roll without slipping, in turning once? 17 times?

(1)
3.1416
1.31^{m}
31416
94248
31416
4.115496^{m}
= 4.115^{m}. *Ans.*

(2)
= 4.115^{m}. *Ans.*

(3)
4.115^{m}
17
28805
4115
69.955^{m}. *Ans.*

20. How often must that wheel turn in going 69.429^{m}? 73.513^{m}? 17.27^{km}?

(1)
17 nearly
4115) 69429
4115
28279

(2)
18 nearly
4115) 73513
4115
32363

(3)
4197 nearly
4115) 17270000
16460
8100
4115
39850
37035
28150

21. Find the reciprocal of 3.14159 to the 5th place.

```
         0.31831
314159)100000.00000
        942477
        ------
         575230
         314159
         -------
         2610710
         2513272
         -------
           974380
           942477
           ------
            319030
            314159
            ------
```

22. What is the diameter of the circle whose circumference is 314.159^{cm}?

```
           100cm
314159)31415900
       314159
       ------
```

23. What is the diameter of the wheel which revolves 19.5 times in going 107.25^{m}?

```
       5.5m
195)1072.5m
     975
     ---
      975
      975
      ---
```

$$\begin{array}{r} 0.31831 \\ 5.5^{m} \\ \hline 159155 \\ 159155 \\ \hline 1.750705^{m} \end{array}$$

$= 1.75^{m}$. *Ans.*

24. How thick through is a tree which has a girth of 2.97^{m}?

$$\begin{array}{r} 0.31831 \\ 2.97^{m} \\ \hline 222817 \\ 286479 \\ 63662 \\ \hline 0.9453807^{m} \end{array}$$

$= 0.945^{m}$. *Ans.*

25. What is the diameter of a circular field two kilometers in circumference?

$$\begin{array}{r} 0.31831 \\ 2000 \\ \hline 636.62^{m} \end{array}$$

26. What is the diameter of a rope of which the circumference is 20^{cm}?

$$\begin{array}{r} 0.31831 \\ 20^{cm} \\ \hline 6.3662^{cm} \end{array}$$

27. In a park is a fountain whose basin is 75^{m} in circumference. What is the diameter of the basin?

$$\begin{array}{r} 0.31831 \\ 75^{m} \\ \hline 159155 \\ 222817 \\ \hline 23.87325^{m} \end{array}$$

$= 23.87^{m}$. *Ans.*

EXERCISE V.

1. Find the area of a rectangle 17^{cm} by 19^{cm}.

$$\begin{array}{r} 19^{cm} \\ 17^{cm} \\ \hline 133 \\ 19 \\ \hline 323^{qcm} \end{array}$$

2. In a rectangular township 16^{km} by 7^{km}, how many hektars? If there are in it 47.3^{km} of highway, averaging 11.7^{m} wide, how much land is left for other uses?

$$\begin{array}{r} 47300^{m} \\ 11.7 \\ \hline 331100 \\ 473 \\ 473 \\ \hline 553410^{qm} \\ = 55.341^{ha} \end{array} \qquad \begin{array}{r} 16^{km} \\ 7^{km} \\ \hline 112^{qkm} \\ 100 \\ \hline 11200^{ha} \\ 55.341 \\ \hline 11144.659^{ha} \end{array}$$

3. In a rectangular field, 751.3^{m} long and 189.3^{m} wide, is a strawberry bed 31.4^{m} by 17.8^{m}. How many hektars in the field? How many, exclusive of the strawberry bed?

$$\begin{array}{r} 751.3^{m} \\ 189.3^{m} \\ \hline 22539 \\ 67617 \\ 60104 \\ 7513 \\ \hline 142221.09^{qm} \\ = 14.222^{ha}.\ \textit{Ans.} \end{array} \qquad \begin{array}{r} 31.4^{m} \\ 17.8^{m} \\ \hline 2512 \\ 2198 \\ 314 \\ \hline 558.92^{qm} \\ = 0.056^{ha} \end{array}$$

$$\begin{array}{r} 14.222^{ha} \\ 0.056^{ha} \\ \hline 14.166^{ha}.\ \textit{Ans.} \end{array}$$

4. If my garden contain 941.65^{qm}, and my neighbor's 748.37^{qm}, what is the area of both in hektars?

$$\begin{aligned} 941.65^{qm} &= 0.094165^{ha} \\ 748.37^{qm} &= 0.074837^{ha} \\ &\overline{0.169002^{ha}} \\ &= 0.169^{ha}. \textit{ Ans.} \end{aligned}$$

5. If a painter can cover 8.786^{qm} in an hour, how much can he cover in 1.78 hours? in 3.86 hours? in 4.57 hours?

8.786^{qm}	8.786^{qm}	8.786^{qm}
1.78	3.86	4.57
70288	52716	61502
61502	70288	43930
8786	26358	35144
15.639^{qm}	33.91396^{qm}	40.15202^{qm}
	$= 33.914^{qm}$. *Ans.*	$= 40.152^{qm}$. *Ans.*

6. How many hektars in each of three rectangular fields: one measuring 315.71^{m} by 78.91^{m}; a second, 293.6^{m} by 84.84^{m}; the third, 346.8^{m} by 71.82^{m}. How many in the three?

315.71^{m}	293.6^{m}	346.8^{m}	
78.91^{m}	84.84^{m}	71.82^{m}	
31571	11744	6935	
284139	23488	27744	2.4913^{ha}
252568	11744	3468	2.4909^{ha}
220997	23488	24276	2.4907^{ha}
24912.6761^{qm}	24909.024^{qm}	24907.176^{qm}	7.4729^{ha}. *Ans.*
$= 2.4913^{ha}$. *Ans.*	$= 2.4909^{ha}$. *Ans.*	$= 2.4907^{ha}$. *Ans.*	

7. Give the price of each field, and of the whole, at \$67.50 a hektar; at \$384 a hektar; and at \$2.375 a square meter.

2.4913	2.4909	2.4907		2.4913
\$67.50	\$67.50	\$67.50		\$384
1245850	1245450	1245350	\$168.16	99652
174391	174363	174349	168.14	199304
149478	149454	149442	168.12	74739
\$168.162950	\$168.135750	\$168.122250	\$504.42	\$956.6592
=\$168.16. *Ans.*	=\$168.14. *Ans.*	=\$168.12. *Ans.*		=\$956.66 *Ans.*

8. What is the area of a circle 27^{cm} in diameter? of one which is 1^{m} in diameter?

(1)

27^{cm}	0.7854
27^{cm}	729^{qcm}
189	70686
54	15708
729^{qcm}	54978
	572.5566^{qcm}

(2)

$1^{m} \times 1^{m} \times 0.7854 = 0.7854^{qm}$

9. What is the area in hektars of a circular field 784^{m} across?

784^{m}	614656^{qm}
784^{m}	0.7854
3136	2458624
6272	3073280
5488	4917248
614656^{qm}	4302592
	482750.8224^{qm}
	$= 48.275^{ha}$. *Ans.*

10. Give the area of a circle 31^{cm} in diameter.

31^{cm}	0.7854
31^{cm}	961^{qcm}
31	7854
93	47124
961^{qcm}	70686
	754.7694^{qcm}

11. Find the length of a rectangle 17^{cm} wide, and containing 306^{qcm}. What length of carpet 75^{cm} wide is required to make 27^{qm}?

18^{cm}	36^{m}
17)306	75)2700
17	225
136	450
136	450

12. A room is 16^{m} long, 8^{m} wide, and 8^{m} high; another room is 7^{m} long, 7^{m} wide, and 3^{m} high. How many square meters of painting on the walls of both rooms, if no allowance is made for doors and windows? How many more square meters of painting on the walls of the larger room than on those of the smaller?

7^{m}	14^{m}	16^{m}	24^{m}	384^{qm}	384^{qm}
7	2	8	2	84	84
14^{m}	28	24^{m}	48	468^{qm}. *Ans.*	300^{qm}. *Ans.*
	3		8		
	84^{qm}		384^{qm}		

13. How many square centimeters of surface on a ball 7^{cm} in diameter?

7^{cm}
7^{cm}
49^{qcm}

3.1416^{qcm}
49
282744
125664
153.9384^{qcm}

14. How many square centimeters of surface on a ball 18^{cm} in diameter?

18^{cm}
18^{cm}
144
18
324^{qcm}

3.1416
324^{qcm}
125664
62832
94248
1017.8784^{qcm}

15. How many square meters of surface on a hemispherical dome 11.27^{m} in diameter?

11.27^{m}
11.27^{m}
7889
2254
1127
1127
127.0129^{qm}

127.0129^{qm}
3.1416
7620774
1270129
5080516
1270129
3810387
2) 399.02372664^{qm}
199.51186332^{qm}
= 199.5119^{qm}. *Ans.*

16. What is the interior surface of a hemispherical basin 12^{cm} in diameter?

12^{cm}
12^{cm}
144^{qcm}

3.1416
144^{qcm}
125664
125664
31416
2) 452.3904^{qcm}
226.1952^{qcm}

17. What is the interior surface of a hemispherical vase 70^{cm} in diameter?

70^{cm}
70^{cm}
4900^{qcm}

3.1416
4900^{qcm}
27274400
125664
2) 15293.8400^{qcm}
7641.92^{qcm}

18. Find, by this rule, the area of example 9.

2) 784
392
392
784
3528
1176
153664^{qm}

153664^{qm}
3.1416
921984
153664
614656
153664
460992
482750.8224^{qm}
= 48.275^{ha}. *Ans.*

19. How many square centimeters are inclosed in a circle struck with a radius of 7^{cm}?

7^{cm}
7^{cm}
49^{qcm}

3.1416
49^{qcm}
272744
125664
152.9384^{qcm}. *Ans.*

20. In a sheet of zinc 1.76^{m} long and 89^{cm} wide are two circular openings, one of which has a radius 10.5^{cm}, the other a radius 9.2^{cm}. What is the area of the zinc left?

$$\begin{array}{r} 10.5^{cm} \\ 10.5^{cm} \\ \hline 525 \\ 105 \\ \hline 110.25^{qcm} \end{array} \qquad \begin{array}{r} 3.1416 \\ 110.25^{qcm} \\ \hline 157080 \\ 62832 \\ 31416 \\ 31416 \\ \hline 346.361400^{qcm} \\ = 0.03464^{qm} \end{array} \qquad \begin{array}{r} 9.2^{cm} \\ 9.2^{cm} \\ \hline 184 \\ 828 \\ \hline 84.64^{qcm} \end{array}$$

$$\begin{array}{r} 3.1416 \\ 84.64^{qcm} \\ \hline 125664 \\ 188496 \\ 125664 \\ 251328 \\ \hline 265.905024^{qcm} \\ = 0.02659^{qm} \end{array} \qquad \begin{array}{r} 0.02659^{qm} \\ 0.03464^{qm} \\ \hline 0.06123^{qm} \end{array} \qquad \begin{array}{r} 1.76^{m} \\ 0.89^{m} \\ \hline 1584 \\ 1408 \\ \hline 1.5664^{qm} \\ 0.06123^{qm} \\ \hline 1.50517^{qm} \\ = 1.505^{qm}.\ \textit{Ans.} \end{array}$$

21. What is the area of a circle of which the radius is 24^{m}?

$$\begin{array}{r} 24^{m} \\ 24^{m} \\ \hline 96 \\ 48 \\ \hline 576^{qm} \end{array} \qquad \begin{array}{r} 3.1416 \\ 576^{qm} \\ \hline 188496 \\ 219912 \\ 157080 \\ \hline 1809.5616^{qm} \end{array}$$

22. A piece of land in the form of a circle has a radius of 40^{m}; in the middle of it is a pond forming a circle of 15^{m} radius. What is the total surface? the surface of the pond? the surface of the land to cultivate?

$$\begin{array}{r} 3.1416 \\ 225^{qm} \\ \hline 157080 \\ 62832 \\ 62832 \\ \hline 706.86^{qm}, \text{ surface of pond.} \end{array} \qquad \begin{array}{r} 40^{m} \\ 40^{m} \\ \hline 1600^{qm} \end{array}$$

$$\begin{array}{r} 3.1416 \\ 1600^{qm} \\ \hline 18849600 \\ 31416 \\ \hline 5026.56^{qm}, \text{ total surface.} \\ 706.86^{qm} \\ \hline 4219.7^{qm}, \text{ surface of land.} \end{array}$$

24. How many meters of carpet 56^{cm} wide will be required for a room 8.32^{m} long and 6.6^{m} wide, strips running lengthwise?

$$\begin{array}{r} 11.7 = 12 \text{ strips.} \\ 56)\overline{660.0} \\ \underline{56} \\ 100 \\ \underline{56} \\ 440 \\ \underline{392} \\ 48 \end{array} \qquad \begin{array}{r} 8.32^{m} \\ \underline{12} \\ 1664 \\ \underline{832} \\ 99.84^{m} \end{array}$$

25. How many meters of carpet 70^{cm} wide will be required for a room 7^{m} long and 5.4^{m} wide, strips running across the room?

$$\begin{array}{r} 10 \text{ strips.} \\ 70)\overline{700} \\ \underline{70} \\ 0 \end{array} \qquad \begin{array}{r} 5.4^{m} \\ \underline{10} \\ 54^{m} \end{array}$$

26. How many meters of carpet 80^{cm} wide will be required for a room 6^{m} long and 5.47^{m} wide, strips running across the room?

$$\begin{array}{r} 7.5 \text{ strips} = 8 \text{ strips.} \\ 80)\overline{600} \\ \underline{560} \\ 400 \\ \underline{400} \end{array} \qquad \begin{array}{r} 5.47^{m} \\ \underline{8} \\ 43.76^{m} \end{array}$$

27. How many meters of carpet 90^{cm} wide will be required for a room 5^{m} long and 4.5^{m} wide, strips running lengthwise? How much will it cost, at $1.875 a meter?

$$\begin{array}{r} 5 \text{ strips.} \\ 90)\overline{450} \\ \underline{450} \end{array} \qquad \begin{array}{r} 5^{m} \\ \underline{5} \\ 25^{m} \end{array}$$

$$\begin{array}{r} \$1.875 \\ \underline{25} \\ 9375 \\ \underline{3750} \\ \$46.875 \\ = \$46.88 \end{array}$$

28. How many meters of carpet 75^{cm} wide will be required for a room 5.25^{m} long and 4.75^{m} wide, strips running across the room? How much will it cost, at $2.125 a meter?

$$\begin{array}{r} 7 \text{ strips.} \\ 75)\overline{525} \\ \underline{525} \end{array} \qquad \begin{array}{r} 4.75^{m} \\ \underline{7} \\ 33.25^{m} \\ \underline{\$2.125} \\ 16625 \\ 6650 \\ 3325 \\ \underline{6650} \\ \$70.65625 \\ = \$70.66 \end{array}$$

29. How many meters of carpet 75^{cm} wide will be required for a room 5.6^{m} square? How wide a strip will have to be turned

under? How much will the carpet cost, at $1.25 a meter?

$$\begin{array}{l} \ 7.4 = 8 \text{ strips.} \\ 75\overline{)560.0} \\ \ \underline{525} \\ \ \ \ 350 \\ \ \ \ \underline{300} \end{array} \qquad \begin{array}{r} 5.6^{m} \\ \underline{8} \\ 44.8^{m} \\ \underline{\$1.25} \\ 2240 \\ 896 \\ \underline{448} \\ \$56.00 \end{array}$$

$$\begin{array}{l} 75^{cm} \\ \underline{35^{cm}} \\ 40^{cm} \text{ to turn under.} \end{array}$$

31. How many rolls of paper 45^{cm} wide and 8^{m} long, allowing 11.19^{qm} for doors and windows, will be required to paper a room whose length is 6.12^{m}, breadth 5.05^{m}, and height 3.5^{m}?

$$\begin{array}{l} 78.19^{qm} \\ \underline{11.19^{qm}} \\ 67^{qm} \end{array} \qquad \begin{array}{r} 0.45 \\ \underline{8} \\ 3.60^{qm} \end{array}$$

$$\begin{array}{l} 18.6 = 19 \text{ rolls.} \\ 36\overline{)670.0} \\ \underline{36} \\ 310 \\ \underline{288} \\ 220 \\ \underline{216} \end{array}$$

32. Find the cost of papering a room 8^{m} long, 5.5^{m} wide, and 4.5^{m} high, with paper 50^{cm} wide and 7.5^{m} in a roll, at $1.25 a roll, put on? There is a baseboard 25^{cm} wide running round the room, and an allowance of 11^{qm} is made for doors and windows.

$$\begin{array}{r} 8^{m} \\ \underline{5.5} \\ 13.5^{m} \\ \underline{2} \\ 27^{m} \\ \underline{4.5^{m}} \\ 135 \\ \underline{108} \\ 121.5^{qm} \\ \underline{17.75} \\ 103.75^{qm} \end{array} \qquad \begin{array}{r} 27^{m} \\ \underline{0.25^{m}} \\ 135 \\ \underline{54} \\ 6.75^{qm} \\ \underline{11} \\ 17.75^{qm} \end{array} \qquad \begin{array}{r} 7.5^{m} \\ \underline{.5^{m}} \\ 3.75^{qm} \end{array}$$

$$\begin{array}{l} 27.6 = 28 \text{ rolls.} \\ 375\overline{)10375.0} \\ \underline{750} \\ 2875 \\ \underline{2625} \\ 2500 \\ \underline{2250} \end{array} \qquad \begin{array}{r} \$1.25 \\ \underline{28} \\ 1000 \\ \underline{250} \\ \$35.00 \end{array}$$

33. Find the cost of plastering this room, at $0.50 a square meter.

$$\begin{array}{l} 5.5^{m} \\ \underline{8^{m}} \\ 44^{qm}, \text{ ceiling.} \end{array} \qquad \begin{array}{r} 103.75^{qm}, \text{ walls.} \\ \underline{44^{qm}} \\ 147.75^{qm} \\ \underline{\$0.50} \\ \$73.8750 \\ = \$73.88 \end{array}$$

34. Find the cost of papering a room 5.5^{m} long, 4.8^{m} wide, and 3.2^{m} high, with paper 45^{cm} wide, 7.5^{m} in a roll, at \$0.875 a roll, put on, allowing 12^{qm} for base-board, doors, etc.

		15.9 = 16 rolls.	
5.5^{m}	75^{m}	3375)539200	\$0.875
4.8^{m}	0.45^{m}	3375	16
10.3^{m}	375	20170	5250
2	300	16875	875
20.6^{m}	3.375^{qm}	32950	\$14.00
3.2^{m}		30375	
412			
618			
65.92^{qm}			
12 allowance.			
53.92^{qm}			

35. Find the cost of plastering this room, at \$0.45 a square meter.

5.5^{m}	26.40^{qm}, ceiling.	80.32
4.8^{m}	53.92, walls.	\$0.45
440	80.32	40160
220		32128
26.40^{qm}, ceiling.		\$36.14

36. Find the cost of papering a room 6^{m} square and 3.5^{m} high, with paper 45^{cm} wide and 7.5^{m} in a roll, at \$0.75 a roll, put on; and of putting on a border, at 5 cents per running meter.

6^{m}		24.8 = 25 rolls.		
6^{m}	7.5^{m}	3375)84000.0		25
12^{m}	0.45^{m}	6750		\$0.75
2	375	16500		125
24^{m}	300	13500		175
3.5^{m}	3.375^{qm}	30000	24	\$18.75
120		27000	\$0.05	1.20
72			\$1.20	\$19.95
84^{qm}				

37. Find the cost of plastering this room, at \$0.36 a square meter.

6^{m}
6^{m}
36^{qm}, ceiling.
84^{qm}, walls.
120^{qm}

120^{qm}
\$0.36
720
360
\$43.20

38. Find the cost of papering a room 13^{m} long, 12^{m} wide, and 7^{m} high, with paper 45^{cm} wide and 7.5^{m} in a roll, at \$1.50 a roll, put on; and of putting on a border, at \$0.30 a running meter, allowing 15^{qm} for base-boards, doors, etc.

13^{m}
12^{m}
25^{m}
2
50^{m}
7^{m}
350^{qm}
115^{qm}, allowance.
235^{qm}

7.5^{m}
0.45^{m}
375
300
3.375^{qm}

\$0.30
50
\$15.00

69.9 = 70 rolls.
3375)235000.0
20230
33700
30375
33250
30375

\$1.50
70
\$105.00
15.
\$120.00

39. Find the cost of plastering this room, at \$0.60 a square meter.

13^{m}
12^{m}
156^{qm}, ceiling.
235^{qm}, walls.
391^{qm}
\$0.60
\$234.60

40. How many meters, board measure, in a board 8^{m} long and 20^{cm} wide?

8^{m}
0.2^{m}
1.6^{qm}

41. How many meters, board measure, in a joist 5^{m} long, 25^{cm} wide, and 75^{mm} thick?

0.25^{m}
5^{m}
1.25^{qm}
3
3.75^{qm}

42. How many meters, board measure, in a stick of timber 15^{m} long and 40^{cm} square?

15
0.4
6.0

16
25)400
25
150
150

16
6
96

43. How many meters, board measure, in 2 joists 5^{m} long, 27.5^{cm} wide, and 50^{mm} thick?

$$\begin{array}{r} 0.275 \\ 5 \\ \hline 1.375 \\ 2 \\ \hline 2.75 \\ 2 \\ \hline 5.5 \end{array}$$

44. How many meters, board measure, in 10 planks, each 4^{m} long, 45^{cm} wide, and 10^{cm} thick? and what is the value of these planks, at $25 a hundred meters?

$$\begin{array}{r} 0.45 \\ 4 \\ \hline 1.80 \\ 4 \\ \hline 7.2 \\ 10 \\ \hline 72 \end{array} \qquad \begin{array}{r} 4 \\ 25\overline{)100} \\ 100 \\ \\ 72 \\ \$0.25 \\ \hline 360 \\ 144 \\ \hline \$18. \end{array}$$

45. How many meters, board measure, in 25 box boards, each 4^{m} long, 42^{cm} wide, and 20^{mm} thick? and what is their value, at $14 a hundred meters?

$$\begin{array}{r} 0.42^{m} \\ 4^{m} \\ \hline 1.68^{qm} \end{array}$$

$$\begin{array}{r} 1.68^{m} \\ 25 \\ \hline 840 \\ 336 \\ \hline 42 \\ \$0.14 \\ \hline 168 \\ 42 \\ \hline \$5.88 \end{array}$$

46. Find the cost of 10 joists 4.5^{m} long, 10^{cm} wide, and 7.5^{cm} thick, at $11 a hundred meters.

$$\begin{array}{r} 4.5^{m} \\ 1^{m} \\ \hline 0.45^{qm} \\ 3 \\ \hline 1.35^{qm} \end{array} \qquad \begin{array}{r} 3 \\ 25\overline{)75} \\ 75 \\ \hline \end{array} \qquad \begin{array}{r} 1.35^{qm} \\ 10 \\ \hline 13.50^{qm} \\ \$0.11 \\ \hline 135 \\ 135 \\ \hline \$1.485 \\ =\$1.49.\ \textit{Ans.} \end{array}$$

47. Find the cost of 36 planks, each 4^{m} long, 27.8^{cm} wide, and 75^{mm} thick, at $16 a hundred meters.

$$\begin{array}{r} 0.278^{m} \\ 4^{m} \\ \hline 1.112^{qm} \\ 3 \\ \hline 3.336^{qm} \\ 36 \\ \hline 20016 \\ 10008 \\ \hline 120.096^{qm} \end{array} \qquad \begin{array}{r} 3 \\ 25\overline{)75} \\ 75 \\ \hline \end{array} \qquad \begin{array}{r} 120.096^{qm} \\ \$0.16 \\ \hline 720576 \\ 120096 \\ \hline \$19.21536 \\ =\$19.22.\ \textit{Ans.} \end{array}$$

48. Find the cost of 3 sticks of timber, each 8^{m} long, 22.5^{cm} wide, and 20^{cm} thick, at $17.50 a hundred meters.

0.225^{m}	8	43.2^{qm}
8^{m}	25)200	\$0.175
1.8^{qm}	200	2160
8		3024
14.4^{qm}		432
3		7.56
43.2^{qm}		

50. Find the cost of a stick of timber 10^{m} long, 25^{cm} thick, 30^{cm} wide at one end and 25^{cm} wide at the other, at $14 a hundred meters.

0.30^{m}	10	27.5^{qm}
0.25^{m}	25)250	\$0.14
2)0.55^{m}	25	1100
0.275^{m}	0	275
10^{m}		\$3.85
2.75^{qm}		
10		
27.5^{qm}		

49. Find the cost of a board 8.25^{m} long, 28^{cm} wide at one end and 35^{cm} at the other, and 31.25^{mm} thick, at $0.30 a meter.

0.28^{m}	2.59875^{qm}
0.35^{m}	1.25
2)0.63^{m}	1299375
0.315^{m}	519750
8.25	259875
1575	3.2484375
630	\$0.30
2520	\$0.974531250
2.59875^{qm}	= \$0.97.

51. Find the cost of the flooring for a two-story building 16^{m} by 10.5^{m}; the flooring being 32^{mm} thick, and worth $30 a hundred meters.

1.28	10.5^{m}	336^{qm}
25)32.00	16^{m}	1.28
25	630	2688
70	105	672
50	168.0^{qm}	336
200	2	430.08
200	336^{qm}	\$0.30
		\$129.02

52. Find the cost of the flooring timbers for this building, the timbers being 25^{cm} by 50^{mm}, and placed on edge 30^{cm} apart, and worth $11.50 a hundred meters.

30^{cm}	30	16^{m}	4^{qm}	240^{qm}	\$27.60
5^{cm}	35)1050	0.25^{m}	2	\$0.115	2
35^{qcm}	105	80	8^{qm}	1200	\$55.20
		32	30	240	
		4^{qm}	240^{qm}	240	
				\$27.60	

53. Find the cost of the fencing to enclose a field 150^{m} long and 75^{m} wide; the posts are set 2.5^{m} apart, and cost \$0.25 apiece; the fence is 5 boards high; the bottom board is 30^{cm}, the top board 25^{cm}, and the other three each 22.5^{cm} wide, and the boards cost \$13.25 a hundred meters.

30	150^{m}	122.5^{m}
25	75^{m}	450^{m}
67.5	225^{qm}	61250
122.5	2	4900
	450^{qm}	551.25^{qm}
		\$0.1325
		275625
180	180 posts.	110250
\$0.25	25)4500	165375
900	25	55125
360	200	\$73.04
\$45.	200	45
		\$118.04

Exercise VI.

1. How many cubic centimeters in a block 9^{cm} long by 7^{cm} wide, and 6^{cm} deep?

9^{cm}	63^{qcm}
7^{cm}	6^{cm}
63^{qcm}	378^{ccm}

2. If wood is cut into 120^{cm} lengths, and a pile is 43.7^{m} long and 1.4^{m} high, how many sters of wood are there in it?

43.7^{m}	52.44^{qm}
1.2^{m}	1.4^{m}
874	20976
437	5244
52.44^{qm}	73.416^{s}

3. In a grain elevator is a bin 11.2^{m} long, 4.34^{m} wide, and 2.83^{m} deep. How many hektoliters of grain will it hold?

11.2^{m}	48.608^{qm}
4.34^{m}	2.83^{m}
448	145824
336	388864
448	97216
48.608^{qm}	137.56064^{cbm}
	$= 1375.6064^{hl}$.

4. If a liter of grain weigh 0.81 of the weight of a liter of water, how much will the grain in that bin weigh?

$$\begin{array}{r} 1375.6064^{hl} \\ = 137560.64^{kg} \\ 0.81 \\ \hline 13756064 \\ 110048512 \\ \hline 111,424.1184^{kg} \end{array}$$

5. A bin measuring 16^m by 9.7^m, and 2.8^m deep, is full of oats, worth $0.98 a hektoliter. What is the whole worth?

$$\begin{array}{r} 16^m \\ 9.7^m \\ \hline 112 \\ 144 \\ \hline 155.2^{qm} \\ 2.8^m \\ \hline 12416 \\ 3104 \\ \hline 434.56^{cbm} \\ = 4345.6^{hl}. \end{array} \qquad \begin{array}{r} 4345.6^{hl} \\ \$0.98 \\ \hline 347648 \\ 391104 \\ \hline \$4258.688 \\ = \$4258.69.\ \textit{Ans.} \end{array}$$

6. A vat 197^{cm} long, 87^{cm} wide, and 63^{cm} deep, holds how many liters? What would be the weight of water required to fill it?

$$\begin{array}{r} 197^{cm} \\ 87^{cm} \\ \hline 1379 \\ 1576 \\ \hline 17139^{qcm} \end{array} \qquad \begin{array}{r} 17139^{qcm} \\ 63^{cm} \\ \hline 51417 \\ 102834 \\ \hline 1079757^{ccm} \\ = 1079.757^{l} \\ = 1079.757^{kg}. \end{array}$$

7. Add 1341^{ccm}, 231^{l}, and 2.13^{hl}, and give the sum in terms of each of the three units.

$$\begin{array}{r} 1341^{ccm} \\ 231^{l} = 231000^{ccm} \\ 2.13^{hl} = 213000^{ccm} \\ \hline 445341^{ccm} \\ = 445.341^{l} \\ = 4.45341^{hl}. \end{array}$$

8. If a spring pours out 467.8^{l} each minute, how many hektoliters will it deliver in 60 minutes? in 37 minutes? in 78 minutes?

$$\begin{array}{r} 467.8^{l} \\ = 4.678^{hl} \\ 60 \\ \hline 280.68^{hl} \end{array} \qquad \begin{array}{r} 4.678^{hl} \\ 37 \\ \hline 32746 \\ 14034 \\ \hline 173.086^{hl} \end{array}$$

$$\begin{array}{r} 4.678^{hl} \\ 78 \\ \hline 37424 \\ 32746 \\ \hline 364.884^{hl} \end{array}$$

9. If 67.3^{l} of oil in a vat with perpendicular sides fill it to a depth of 173^{mm}, how deep will 13.7 times that quantity fill it? and how many hektoliters will there be?

$$\begin{array}{r} 173^{mm} \\ 13.7 \\ \hline 1211 \\ 519 \\ 173 \\ \hline 2370.1^{mm} \\ = 2.3701^{m}. \end{array} \qquad \begin{array}{r} 67.3^{l} \\ = 0.673^{hl} \\ 13.7 \\ \hline 4711 \\ 2019 \\ 673 \\ \hline 9.2201^{hl} \end{array}$$

10. Into a round cup 10^{cm} across, with perpendicular sides, pour oil until it is 1^{cm} deep; then there are 78.54^{ccm} of oil in the cup. How many cubic centimeters will there be when the oil is 38^{mm} deep?

$38^{mm} = 3.8^{cm}$

$$\begin{array}{r} 78.54^{ccm} \\ 3.8^{cm} \\ \hline 62832 \\ 23562 \\ \hline 298.452^{ccm} \end{array}$$

11. What is the capacity of a tin cup 95^{mm} across and 11.08^{cm} deep?

$95^{mm} = 9.5^{cm}$

$$\begin{array}{r} 9.5^{cm} \\ 9.5^{cm} \\ \hline 475 \\ 855 \\ \hline 90.25^{qcm} \end{array}$$

$$\begin{array}{r} 0.7854 \\ 90.25^{qcm} \\ \hline 39270 \\ 15708 \\ 70686 \\ \hline 70.882350^{qcm} \\ 11.08^{cm} \\ \hline 56705880 \\ 7088235 \\ 7088235 \\ \hline 785.3764380^{ccm} \end{array}$$

$= 0.785^{l}$. *Ans.*

12. What are the capacities of two cylindrical vessels, one being 16.24^{cm} across and 19.95^{cm} deep, the other 75.4^{mm} across and 87.9^{mm} deep?

$$\begin{array}{r} 16.24^{cm} \\ 16.24^{cm} \\ \hline 6496 \\ 3248 \\ 9744 \\ 1624 \\ \hline 263.7376^{qcm} \\ 0.7854 \\ \hline 10549504 \\ 13186880 \\ 21099008 \\ 18461632 \\ \hline 207.1395 \\ 19.95^{cm} \\ \hline 10356975 \\ 18642555 \\ 18642555 \\ 2071395 \\ \hline 4132.433025^{ccm} \end{array}$$

$4132.433^{ccm} = 4.132^{l}$

$$\begin{array}{r} 75.4^{mm} \\ 75.4^{mm} \\ \hline 3016 \\ 3770 \\ 5278 \\ \hline 5685.16^{qmm} \\ 0.7854 \\ \hline 2274064 \\ 2842580 \\ 4548128 \\ 3979612 \\ \hline 4465.124664^{qmm} \\ = 4465.125^{qmm} \\ 87.9^{mm} \\ \hline 40196125 \\ 31255875 \\ 35721000 \\ \hline 392485.4875^{cmm} \end{array}$$

$= 0.392^{l}$.

13. How many cubic centimeters in a ball 10^{cm} in diameter? How much less if you take the more exact multiplier?

10^{cm}
10^{cm}
100^{qcm}
10^{cm}
1000^{ccm}

0.5236
1000^{ccm}
523.6^{ccm}

$6\overline{)3.1415927}$
0.5235988
1000^{ccm}
523.5988^{ccm}

523.6^{ccm}
523.5988^{ccm}
0.0012^{ccm}

14. Into a cubical box 20^{cm} on a side, and full of water, an iron ball 20^{cm} in diameter is gently lowered until it touches bottom. How much water is left in the box? Answer in liters and in cubic centimeters.

20^{cm}
20^{cm}
400^{qcm}
20
8000^{ccm}
4188.8^{ccm}
3811.2^{ccm}
$= 3.8112^{l}.$

0.5236
8000^{ccm}
4188.8^{ccm}

15. One cask contains 171.4^{l} of oil; another, 209.3^{l}; a third, 73.8^{l}; while a square vat, 137^{cm} each way, is filled to a depth of 69^{cm}. How much oil in all the vessels? in liters and in hektoliters.

137^{cm}
137^{cm}
959
411
137
18769^{qcm}

18769^{qcm}
69^{cm}
168921
112614
1295061^{ccm}
$= 1295.061^{l}.$

171.4^{l}
209.3^{l}
73.8^{l}
1295.061^{l}
1749.561^{l}
$= 17.49561^{hl}.$

16. How many liters of air in a room 7.8^{m} long, 6.23^{m} wide, and 3^{m} high?

6.23^{m}
7.8^{m}
4984
4361
48.594^{qm}
3^{m}
145.782^{cbm}
$= 145782^{l}.$

17. If a person's breathing spoils the air at the rate of 0.2175^{cbm} a minute, how long will it take 3 persons sitting

in the room, closed, to spoil the air?

$$\begin{array}{r} 0.2175 \\ 3 \\ \hline 0.6525 \end{array}$$

$$\begin{array}{r} 223.42 \\ 6525\overline{)1457820.00} \\ 13050 \\ \hline 15282 \\ 13050 \\ \hline 22320 \\ 19575 \\ \hline 27450 \\ 26100 \\ \hline 13500 \end{array}$$

18. How long, at the same rate, would the air in a hall 22^{m} long, 16^{m} wide, and 7^{m} high, last an audience of 280 persons?

$$\begin{array}{r} 22^{m} \\ 16^{m} \\ \hline 132 \\ 22 \\ \hline 352^{qm} \\ 7^{m} \\ \hline 2464^{cbm} \end{array} \qquad \begin{array}{r} 2175 \\ 280 \\ \hline 174000 \\ 4350 \\ \hline 60.9000 \end{array}$$

$$\begin{array}{r} 40.5 \\ 609\overline{)24640.0} \\ 2436 \\ \hline 2800 \end{array}$$

19. How many cubic meters of wood in a round stick of equal size throughout, 37^{cm} in diameter and 8.4^{m} long?

$$\begin{array}{r} 37^{cm} \\ 37^{cm} \\ \hline 259 \\ 111 \\ \hline 1369^{qcm} \end{array} \qquad \begin{array}{r} 0.7854 \\ 1369^{qcm} \\ \hline 70686 \\ 47124 \\ 23562 \\ 7854 \\ \hline 1075.2126^{qcm} \\ = 0.10752126^{qm}. \end{array} \qquad \begin{array}{r} 0.10752126^{qm} \\ 8.4^{m} \\ \hline 43008504 \\ 86017008 \\ \hline 0.903178584^{cbm} \\ = 0.9032^{cbm}. \end{array}$$

Exercise VII.

1. What is the weight, in kilograms, of a hektoliter of water? of 73.8^{l} of water? of a cubic meter of water? of a cubic centimeter?

$$1^{hl} = 100^{kg}$$
$$73.8^{l} = 73.8^{kg}$$
$$1^{cbm} = 1000^{kg}$$
$$1^{ccm} = 0.001^{kg}$$

2. If a man buys half a ton of potatoes for \$20, and retails them all, without waste, at 5 cents a kilogram, what profit does he make on the whole?

$$\begin{array}{r} \$0.05 \\ 500 \\ \hline \$25.00 \\ 20. \\ \hline \$5. \end{array}$$

3. What is the weight of water required to fill a vat 98^{cm} long, 71^{cm} wide, and 38^{cm} deep?

$$\begin{array}{r} 98^{cm} \\ 71^{cm} \\ \hline 98 \\ 686 \\ \hline 6958^{qcm} \end{array} \qquad \begin{array}{r} 6958^{qcm} \\ 38 \\ \hline 55664 \\ 20874 \\ \hline 264.404^{ccm} \\ = 264.404^{kg}. \end{array}$$

4. If the vat of the last example were filled with brine weighing 1.04^{kg} to the liter, what would be the weight of the brine?

$$\begin{array}{r} 264.404^{kg} \\ 1.04 \\ \hline 1057616 \\ 264404 \\ \hline 274.98^{kg} \end{array}$$

5. If the vat of Example 3 were filled with wine weighing 0.981^{kg} to the liter, what would be its weight?

$$\begin{array}{r} 264.404^{kg} \\ 0.981 \\ \hline 264404 \\ 2115232 \\ 2379636 \\ \hline 259.38^{kg} \end{array}$$

6. What is the total weight of 13 men averaging 73.48^{kg} each?

$$\begin{array}{r} 73.48^{kg} \\ 13 \\ \hline 22044 \\ 7348 \\ \hline 955.24^{kg} \end{array}$$

7. How many kilograms, and how many tons, would 3.6175^{cbm} of brick weigh, at 2 tons to a cubic meter? at 2.34 tons?

$$\begin{array}{r} 3.6175 \\ 2^{t} \\ \hline 7.235^{t} \\ = 7235^{kg} \end{array} \qquad \begin{array}{r} 3.6175 \\ 2.34^{t} \\ \hline 144700 \\ 108525 \\ 72350 \\ \hline 8.46495^{t} \\ = 8464.95^{kg} \end{array}$$

8. From a barrel containing 67^{kg} of granulated sugar were taken three parcels of 2.75^{kg} each, and four parcels of 7.50^{kg} each. How much is left in the barrel?

$$\begin{array}{r} 2.75^{kg} \\ 3 \\ \hline 8.25^{kg} \end{array} \qquad \begin{array}{r} 7.5^{kg} \\ 4 \\ \hline 30.0^{kg} \end{array}$$

$$\begin{array}{r} 30.0^{kg} \\ 8.25 \\ \hline 38.25^{kg} \end{array}$$

$$\begin{array}{r} 67^{kg} \\ 38.25 \\ \hline 28.75^{kg} \end{array}$$

$$\begin{array}{r} 24 \\ 325\overline{)7800} \\ 650 \\ \hline 1300 \\ 1300 \\ \hline \end{array}$$

9. Into how many pills of 325^{mg} each can a mass of 7.8^{g} be divided?

10. A mass of 21.8^{g} is divided into 60 pills. What is the weight of each pill?

$$\begin{array}{r} 6\underline{)2180.000^{mg}} \\ 363.333^{mg} \end{array}$$

Exercise VIII.

1. If a stone weighs 1.3^{kg} in air and 0.68^{kg} in water, and the stone and a block of wood together weigh 1.55^{kg} in air and 0.63^{kg} in water, what is the specific gravity of the block of wood?

$1.55^{kg} - 1.3^{kg} = 0.25^{kg}$, the weight of the wood in the air.

$1.55^{kg} - 0.63^{kg} = 0.92^{kg}$, the weight of water displaced by the stone and the wood.

$1.3^{kg} - 0.68^{kg} = 0.62^{kg}$, the weight of water displaced by the stone alone.

Therefore $0.92^{kg} - 0.62^{kg} = 0.3^{kg}$, the weight of water displaced by the wood.

And $0.25 \div 0.3 = 0.833$, the specific gravity of the wood.

2. What is the weight of 8.17^{hl} of alcohol, specific gravity 0.83?

$$\begin{array}{r} 8.17^{hl} = 817^{kg} \\ 0.83 \\ \hline 2451 \\ 6536 \\ \hline 678.11^{kg} \end{array}$$

3. What will 97^{l} alcohol weigh, of specific gravity 0.817? of specific gravity 0.819? of specific gravity 0.823? 0.838? 0.847?

$$\begin{array}{r} 0.817 \\ 97^{kg} \\ \hline 5719 \\ 7353 \\ \hline 79.249^{kg} \end{array} \quad \begin{array}{r} 0.819 \\ 97^{kg} \\ \hline 5733 \\ 7371 \\ \hline 79.443^{kg} \end{array} \quad \begin{array}{r} 0.823 \\ 97^{kg} \\ \hline 5761 \\ 7407 \\ \hline 79.831^{kg} \end{array} \quad \begin{array}{r} 0.838 \\ 97^{kg} \\ \hline 5866 \\ 7542 \\ \hline 81.286^{kg} \end{array} \quad \begin{array}{r} 0.847 \\ 97^{kg} \\ \hline 5929 \\ 7623 \\ \hline 82.159^{kg} \end{array}$$

4. A bar of aluminum 113^{mm} long, 17^{mm} wide, and 13^{mm} thick, is said to be of specific gravity 2.57. What does it weigh? If it really is of specific gravity 2.67, what does it weigh?

113^{mm}	1921^{qmm}	24.973^{g}	24.973^{g}
17^{mm}	13^{mm}	2.57	2.67
791	5763	174811	174811
113	1921	124865	149838
1921^{qmm}	24973^{cmm}	49946	49946
	$= 24.973^{ccm}$	64.18^{g}	66.67791^{g}
	$= 24.973^{g}$		$= 66.68^{g}$

5. What would be the specific gravity of the bar of the last example if it weighed 65.137^{g}?

```
           2.608
24973)65137.000
      49946
      151910
      149838
        207200
        199784
```

6. What is the weight of a bar of aluminum 371^{mm} by 63^{mm} by 84^{mm}, specific gravity being 2.63?

371^{mm}	1963332^{cmm}
63^{mm}	weigh
1113	1.963332^{kg}
2226	2.63
23373^{qmm}	5889996
84	11779992
93492	3926664
186984	5.16356316^{kg}
1963332^{cmm}	$= 5.1636^{kg}$

7. An irregular mass of copper, gently lowered into a pail brimful of water, caused 1.374^{l} to run over. What did it weigh if of specific gravity 8.91? if 8.89?

1.374^{kg}	1.374^{kg}
8.91	8.89
1374	12366
12366	10992
10992	10992
12.242^{kg}	12.21486^{kg}
	$= 12.215^{kg}$

8. What was the specific gravity of that copper if the mass weighed 12.3016^{kg}?

```
         8.953
1374)12301.600
     10992
      13096
      12366
        7300
        6870
         4300
         4122
```

9. A plate of iron 137^{cm} long, 64.3^{cm} wide, and 4.31^{cm} thick, weighs 277.54^{kg}. What is its specific gravity? What would the same mass weigh at specific gravity 7.47? at 7.79?

137^{cm}
64.3^{cm}
411
548
822
8809.1

8809.1^{qcm}
4.31
88091
264273
352364
37967.221^{ccm}
$= 37.97^{l}$.

7.309
3797)27754.000
26579
11750
11391
35900
34173

37.967221^{kg}
7.47
265770547
151868884
265770547
283.615^{kg}

37.967221^{kg}
7.79
341704989
265770547
265770547
295.76465159^{kg}
$= 295.765^{kg}$.

10. What is the specific gravity of sea-water when a hektoliter weighs 102.58^{kg}? what when 3^{l} weigh 3077^{g}?

100)102.58
1.0258

3)3.077
1.0257

11. What is the specific gravity of a substance of which 7.3^{ccm} weighs 31.5^{g}?

4.315
73)315.000
292
230
219
110
73
370
265

12. If a cubic meter of sand weighs 1723^{kg}, what is its specific gravity? If 3.4^{cbm} of gravel weigh 7.134 tons, what is the specific gravity?

1000)1723.
1.723

2.098
34)71.340
68
334
306
280
272

13. If a cubic centimeter of metal weighs 7.3^{g}, what is its specific gravity?

$$1\overline{)7.3}$$
$$7.3$$

14. What is the specific gravity of a fluid weighing 2.317^{kg} to a liter?

$$1\overline{)2.317}$$
$$2.317$$

15. If a body weigh 3.71^{kg} in air and 2.38^{kg} in water, what is its specific gravity?

```
3.71             2.789
2.38       133)371.000
----           266
1.33           ---
               1050
                931
               ----
                1190
                1064
                ----
                 1260
                 1197
                 ----
```

16. A piece of ore weighing 3.77^{kg} weighs in water only 2.53^{kg}. What is its specific gravity?

```
3.77            3.04
2.53      124)377.00
----            372
1.24            ---
                 500
                 496
                 ---
```

17. How many cubic centimeters in a stone which loses 17.8^{g} of its weight when weighed in water? What is its specific gravity if weighed in air it weighs 33.7^{g}?

```
      1)17.8
        17.8ccm

        1.893
   178)337.000
       178
       ---
       1590
       1424
       ----
        1660
        1602
        ----
          580
          534
          ---
```

18. In a wrought-iron bottle I find 2.63^{l} of quicksilver, weighing 35.81^{kg}; in another 2.59^{l}, weighing 35.193^{kg}; in a third, 2.617^{l}, weighing 35.571^{kg}. What is the specific gravity of each? What would be the specific gravity if the three were emptied into one vessel and mixed?

```
         13.616
   263)3581.000
       263
       ---
        951
        789
        ----
        1620
        1578
        ----
          420
          263
          ----
          1570
          1578
          ----
```

$$\begin{array}{r}
13.588 \\
259\overline{)3519.300} \\
259 \\
\hline
929 \\
777 \\
\hline
1520 \\
1295 \\
\hline
2250 \\
2054 \\
\hline
1960 \\
2054 \\
\hline
\end{array}$$

$$\begin{array}{r}
13.592 \\
2617\overline{)35571.000} \\
2617 \\
\hline
9401 \\
7851 \\
\hline
15500 \\
13085 \\
\hline
24150 \\
23553 \\
\hline
5970 \\
5234 \\
\hline
\end{array}$$

$$\begin{array}{r}
13.616 \\
13.588 \\
13.592 \\
\hline
3\overline{)40.796} \\
\hline
13.59867
\end{array}$$

$= 13.599$. *Ans.*

19. A plate of iron 89^{cm} by 17^{cm} by 7^{cm} weighs 79.43^{kg}. What is its specific gravity?

$$\begin{array}{r}
89^{cm} \\
17^{cm} \\
\hline
623 \\
89 \\
\hline
1513^{qcm} \\
7^{cm} \\
\hline
10591^{ccm}
\end{array}$$

$$\begin{array}{r}
7.5 \\
10591\overline{)79430.0} \\
74137 \\
\hline
52930
\end{array}$$

Exercise IX.

1. If three men eat 8^{kg} a week, how much would one man eat at the same rate? How much would seven men? At the same rate, how much do the three men eat in one day? and how much each man? At the same rate, how much would seven men eat each day? each week? in five weeks?

$3\overline{)8.00}$	2.67	$7\overline{)8.00}$	$3\overline{)1.14}$	0.38	2.67	18.67
2.67^{kg}	7	1.14^{kg}	0.38^{kg}	8	7	5
	18.67^{kg}			2.67^{kg}	18.67^{kg}	93.33^{kg}

2. At the same rate, how much would 17 men eat in 3 weeks and 4 days?

$$\begin{array}{r} 7 \\ \underline{3} \\ 21 \\ \underline{4} \\ 25 \\ \underline{17} \\ 175 \\ \underline{25} \\ 425 \\ \underline{0.38^{kg}} \\ 3400 \\ \underline{1275} \\ 161.50^{kg} \end{array}$$

3. If one hektoliter of oats is enough for 5 horses 1 week, how much is enough for 1 horse 1 week? for 1 horse 7 weeks? for 11 horses 17 weeks?

$5)\underline{1.0}$
$0.2^{hl} = 20^{l}$

$$\begin{array}{r} 20^{l} \\ \underline{7} \\ 140^{l} \end{array}$$

$$\begin{array}{r} 17 \\ \underline{11} \\ 17 \\ \underline{17} \\ 187 \end{array} \qquad \begin{array}{r} 187 \\ \underline{20^{l}} \\ 3740^{l} \end{array}$$

4. If two hektoliters of grain are enough for 3 horses 5 days, how much is enough for 3 horses 1 day? for 1 horse 1 day? for 7 horses 6 days?

$$\begin{array}{r} 3 \\ \underline{5} \\ 3)\underline{15} \\ 5 \end{array}$$

$5)\underline{2.0}$
$0.4^{hl} = 40^{l}$

$$\begin{array}{r} 3)\underline{40} \\ 13.33 \\ \underline{42} \\ 2666 \\ \underline{5332} \\ 559.86^{l} = 560^{l} \end{array}$$

5. Mix 17 liters of vinegar, costing 6 cents a liter, with 39^{l} at 5 cents, 21^{l} at 7 cents, and 13^{l} of water costing nothing. Find the amount of the mixture, and its cost.

$$\begin{array}{r} 17^{l} \\ \underline{\$0.06} \\ \$1.02 \end{array} \qquad \begin{array}{r} 39^{l} \\ \underline{\$0.05} \\ \$1.95 \end{array} \qquad \begin{array}{r} 21^{l} \\ \underline{\$0.07} \\ \$1.47 \end{array}$$

$$\begin{array}{r} 17 \\ 39 \\ 21 \\ \underline{13} \\ 90^{l} \end{array} \qquad \begin{array}{r} \$1.02 \\ 1.95 \\ \underline{1.47} \\ \$4.44 \end{array}$$

6. For how much a liter must I sell that mixture, in order to gain 96 cents? for how much to clear \$1.41?

$4.44
0.96
$5.40

$0.06
90)$5.40
540

$4.44
1.41
$5.85

$0.065
90)5.850
540
450
450

7. A grocer sold 421 kegs of butter for $4995.25; 56 kegs brought $12.50 a keg; 91 brought $11.75 a keg; and 100 kegs brought $12.25 a keg. For how much a keg were the other kegs sold?

$12.50
56
7500
6250
$700.00

$11.75
91
1175
10575
$1069.25
700.00
1225.00
$2994.25

$12.25
100
$1225.00

$4995.25
2994.25
$2001.00

56
91
100
247

421
247
174

$11.50
174)$2001.00
174
261
174
870
870

8. If 3 tons of coal cost $15.75, how many tons will $36.75 buy?

3)$15.75
$5.25

7 tons.
525)3675
3675

9. If 5^m of cloth cost $18.75, what should 7^m cost?

5)$18.75
$3.75
7
$26.25

10. If a tap running 3.5^l a minute fills a tub in 16 minutes, how long should a tap delivering 5^l a minute be in filling the same tub?

16
3.5
80
48
5)56.0 min.
11.2 min.

11. If both taps of the last example be opened at once, how soon will they fill the tub?

$$\begin{array}{r} 3.5 \\ \underline{5.} \\ 8.5 \end{array} \qquad \begin{array}{r} 6.6 \\ 85\overline{)560.0} \\ \underline{510} \\ 500 \end{array}$$

12. If 3 men can dig 378^m of ditch in 2 days, how long will it take 5 men, at the same rate, to dig 787^m?

$$\begin{array}{r|l} 2 & 378^m \\ 3 & 189^m \\ & 63^m \\ & \underline{5} \\ & 315^m \end{array} \qquad \begin{array}{r} 2.5 \\ 315\overline{)787.0} \\ \underline{630} \\ 1570 \end{array}$$

13. Into a tub which will hold 48^l, one tap is delivering water at the rate of 3.7^l a minute; while out of it, by another tap, the water is running at 2.5^l a minute. How long will it take to fill the tub, beginning with it empty?

$$\begin{array}{r} 3.7^l \\ \underline{2.5^l} \\ 1.2^l \end{array} \qquad \begin{array}{r} 40 \text{ min.} \\ 12\overline{)480} \end{array}$$

14. A tap discharges into a tub 4.2^l a minute; from the tub water is also running, by a second tap; the water in the tub gains 30^l in 18 minutes. How fast is the second tap discharging?

$$\begin{array}{r} 4.2 \\ \underline{18} \\ 336 \\ \underline{42} \\ 75.6 \\ \underline{30} \\ 45.6 \end{array} \qquad \begin{array}{r} 2.5 \\ 18\overline{)45.6} \\ \underline{36} \\ 96 \\ \underline{90} \end{array}$$

15. If a wheel is 1.2^m across, how often will it turn in going one kilometer?

$$\begin{array}{r} 3.1416 \\ \underline{1.2^m} \\ 62832 \\ \underline{31416} \\ 3.76992^m \\ = 3.77^m \\ = 0.00377^{km} \end{array} \qquad \begin{array}{r} 265 \\ 377\overline{)100000} \\ \underline{754} \\ 2460 \\ \underline{2262} \\ 1980 \\ \underline{1885} \end{array}$$

16. How many times in a minute does the wheel of the last example turn, when the carriage is driven 14^{km} an hour?

$$14 \div 60 = 0.23$$

$$\begin{array}{r} 265 \\ \underline{0.23} \\ 795 \\ \underline{530} \\ 60.95 \\ = 61 \text{ times.} \end{array}$$

17. What is the weight of the water in a tank if it would take a flow of 8.7^l a minute 1 hour and 38 minutes to empty it?

$$
\begin{array}{r}
60 \text{ min.} \\
38 \text{ min.} \\
\hline
98 \text{ min.} \\
8.7^l \\
\hline
686 \\
784 \\
\hline
852.6^l \\
= 852.6^{kg}
\end{array}
$$

18. Replace the bulk of water with oil worth \$18.75 a hektoliter, and what will the contents of the tank be worth?

$$
\begin{array}{r}
852.6^{kg} = 8.526^{hl} \\
\$18.75 \\
\hline
42630 \\
59682 \\
68208 \\
8526 \\
\hline
\$159.86
\end{array}
$$

Exercise X.

1. A train leaves Paris at 11 o'clock A.M., and reaches Lyons at 10 o'clock P.M. How many meters does it travel in a hour, the distance from Paris to Lyons being 512.7^{km}?

There are 11 hours between 11 A.M. and 10 P.M. Therefore the train runs $512.7^{km} \div 11 = 46.609^{km} = 46,609^m$.

2. A railroad has a single track 11.450^{km} long. How many rails 4.569^m in length did it require to lay the track?

There are two lines of rails. Therefore length of rails is

$$2 \times 11.450^{km} = 22.900^{km} = 22,900^m.$$

Therefore the number of rails is $22,900 \div 4.569$.

$$
\begin{array}{r}
5012 \\
4569\overline{)22900000} \\
22845 \\
\hline
5500 \\
4569 \\
\hline
9310 \\
9138 \\
\hline
\end{array}
$$

∴ number of rails required was 5013. *Ans.*

3. A book is 2.1^{cm} in thickness; each leaf is 0.05^{mm} thick. Find the number of pages in the book.

The number of leaves is $21 \div 0.05 = 420$.
The number of pages is $2 \times 420 = 840$.

4. The cost of opening a canal amounts to \$25,400 a kilometer. How much would a canal cost which was 113.253^{km} long?

If it cost \$25,400 to open 1^{km}, to open 113.253 it will cost $113.253 \times \$25,400$.

$$\begin{array}{r} 113.253 \\ \$25400 \\ \hline 45301200 \\ 566265 \\ 226506 \\ \hline \$2,876,626.200 \end{array}$$

5. The expense of laying out a paved road is \$12,500 a kilometer. How much would a road cost which was 72.053^{km} long?

If it cost \$12,500 to lay out 1^{km}, to lay out 72.053^{km} it will cost $72.053 \times \$12,500$.

$$\begin{array}{r} 72.053 \\ \$12500 \\ \hline 36026500 \\ 144106 \\ 72053 \\ \hline \$900,662.50 \end{array}$$

6. The cost of building a railroad is about \$78,000 a kilometer in France, and only \$25,000 in the United States. How much would it cost in each country to make a road 295.671^{km} long?

If it cost \$78,000 to build 1^{km}, to build 295.671^{km} it will cost $295.671 \times \$78,000 = \$23,062,338$. If it cost \$25,000 to build 1^{km}, to build 295.671^{km} it will cost $295.671 \times \$25,000 = \$7,391,775$.

$$\begin{array}{r} 295.671 \\ \$78000 \\ \hline 2365368000 \\ 2069697 \\ \hline \$23,062,338.000 \end{array} \qquad \begin{array}{r} 4\overline{)29567100} \\ \hline \$7,391,775 \end{array}$$

7. If you must go up 211 steps to reach the top of a tower, and each step is 195^{mm} high, what is the height of the tower?

$195^{mm} = 0.195^{m}$.

If one step is 0.195^{m} high, 211 steps are 211×0.195^{m} high.

$$\begin{array}{r} 0.195^{m} \\ 211 \\ \hline 195 \\ 195 \\ 390 \\ \hline 41.145^{m} \end{array}$$

8. A house has 5 stories, each story has 19 stairs, each stair is 16^{cm} in height. Calculate how high the floor of the fifth story is from the ground.

$16^{cm} = 0.16^{m}$.

If one step is 0.16^{m} high, 19 steps are $19 \times 0.16^{m} = 3.04^{m}$, and 4 flights of 19 steps are $4 \times 3.04^{m} = 12.16^{m}$.

$$\begin{array}{r} 0.16^{m} \\ 19 \\ \hline 144 \\ 16 \\ \hline 3.04 \\ 4 \\ \hline 12.16 \end{array}$$

9. A ream of paper contains 20 quires, each quire has 24 sheets, the ream is 13.5^{cm} in thickness. Find the thickness of each sheet.

In one ream there are $20 \times 24 = 480$ sheets. If 480 sheets are 13.5^{cm} thick, $13.5^{cm} \div 480 = 0.028^{cm}$, thickness of one sheet.

$$\begin{array}{r} 0.028 \\ 48\overline{)1.350} \\ 96 \\ \hline 390 \\ 384 \\ \hline \end{array}$$

10. The equator on a terrestrial globe measures 0.80^{m} in circumference. By the aid of a tape-measure we find that the distance between two cities on this globe is 0.046^{m}. What is really the distance in kilometers between the two cities? (The earth's equator is $40{,}075.45^{km}$.)

The ratio of distance on globe between the two cities to the equator is $0.046^{m} \div 0.80^{m} = 0.0575$. Therefore the actual distance between the two cities is $0.0575 \times 40{,}075.45^{km} = 2304.338^{km}$.

```
8)0.4600            40075.45km
  0.0575               0.0575
                    20037725
                   28052815
                  20037725
                  2304.338km
```

11. Upon a military map we find that the distance from Paris to St. Denis is 78^{mm}. What is the distance in kilometers from Paris to St. Denis? The map is made on the scale of 1 to $80{,}000^{m}$; that is, 1^{m} on the map represents $80{,}000^{m}$ of actual measurement upon the ground.

The actual distance is 80,000 times the distance on the map; that is, $80{,}000 \times 78^{mm} = 6{,}240{,}000^{mm} = 6.24^{km}$.

12. Give the number of revolutions made by the wheels of a carriage in travelling 82^{km}. The wheels are 1354^{mm} in diameter.

$82^{km} = 82{,}000{,}000^{mm}$.

The circumference of the wheels is $3.1416 \times 1354^{mm} = 4253.7264^{mm}$. The number of revolutions is the total distance divided by the circumference of the wheel, or $82{,}000{,}000^{mm} \div 4253.7264^{mm} = 19{,}277$ times.

```
  3.1416                            19277
    1354mm          42537264)820000000000
  125664                     42537264
 157080                      394627360
 94248                       382835376
31416                         117919840
4253.7264mm                    85074528
                               328453120
                               297760848
                                306922720
                                297760848
```

13. How many hektars in a square kilometer? how many ars? how many square meters?

$$1^{qkm} = 100^{ha},$$
$$= 10000^{a},$$
$$= 1000000^{qm}.$$

14. France has about $542{,}000^{qkm}$. How many hektars does it measure?

$$1^{qkm} = 100^{ha}.$$
$$\therefore\ 542000^{qkm} = 542000 \times 100^{ha},$$
$$= 54200000^{ha}.$$

15. A piece of land 1224.5^{m} square is sold at \$140 a hektar. How much does the land bring?

$$\begin{array}{r} 1224.5 \\ 1224.5 \\ \hline 61225 \\ 48980 \\ 24490 \\ 24490 \\ 12245 \\ \hline 1499400.25^{qm} \end{array}$$
$$= 149.94^{ha}.$$

$$\begin{array}{r} 149.94 \\ \$140 \\ \hline 599760 \\ 14994 \\ \hline \$20{,}991.60 \end{array}$$

16. The total surface measurement of the glass in the windows of a house is 182^{qm}. How many panes of 53^{cm} by 48^{cm} will it take to supply the windows?

$182^{qm} = 1{,}820{,}000^{qcm}$.

$$\begin{array}{r} 53^{cm} \\ 48^{cm} \\ \hline 424 \\ 212 \\ \hline 2544^{qcm} \end{array}$$, area of one pane.

$$\begin{array}{r} 715.4 \\ 2544\overline{)1820000.0} \\ 17808 \\ \hline 3920 \\ 2544 \\ \hline 13760 \\ 12720 \\ \hline 10400 \\ 10176 \\ \hline \end{array}$$

$\therefore$ it will take 716 panes.

17. How many square slabs of marble 150^{qcm} on the surface will it require to pave a court whose area is 25.35^{qm}?

$25.35^{qm} = 253{,}500^{qcm}$.

The number of slabs required is $253{,}500^{qcm} \div 150^{qcm} = 1690$.

```
     1690
 15)25350
    15
    103
     90
     135
     135
```

18. A speculator bought 31.0728^{ha} of land for \$1296 a hektar. For how much a square meter must he sell it to realize a profit of \$1937?

```
      31.0728
       $1296
      1864368
     2796552
     621456
    310728
 $40,270.3488 cost.
     1937      profit.
 $42,207.3488 selling price.
```

```
                  $0.136
  310728)$42207.349
           310728
          1113454
           932184
           1812709
```

19. A man is offered \$6000 for 2.5 ars of land. He declines to sell; and soon after, the town gives him \$25.20 a square meter. How much did he make by refusing the first offer?

$2.5^{a} = 250^{qm}$.

```
  $25.20
     250
  126000
  5040
 $6300.00
  6000
  $300
```

20. A man surveys a piece of land and finds that it measures 14.0715^{ha}. He afterwards discovers that his chain was too short by 0.03^{m}. How can he calculate the real superficial measurement of his land without surveying it again? (A surveyor's chain is 10^{m} long.)

$$10.00 - 0.03 = 9.97.$$
$$9.97 \div 10 = 0.997.$$

$$\begin{array}{r}
0.997 \\
0.997 \\
\hline
6979 \\
8973 \\
8973 \\
\hline
0.994009
\end{array}
\qquad\qquad
\begin{array}{r}
14.0715^{ha} \\
0.994009 \\
\hline
1266435 \\
562860 \\
1266435 \\
1266435 \\
\hline
13.987^{ha}
\end{array}$$

21. The railroad from Paris to Orleans has a double track; each rail is 4^{m} long, and the distance from Paris to Orleans is 121^{km}. What is the number of rails used in laying the track? The width of the road is 15^{m}; how many hektars of land does the road include?

There are four lines of rails. $4 \times 121^{km} = 484^{km} = 484{,}000^{m}$ of rails. If one rail is 4^{m} long, in $484{,}000^{m}$ there are $484{,}000 \div 4 = 121{,}000$ rails. $15^{m} = 0.015^{km}$. The area of road is

$$121^{km} \times 0.015^{km} = 1.815^{qkm} = 181.5^{ha}. \quad \textit{Ans.}$$

$$\begin{array}{r}
121^{km} \\
4\phantom{^{km}} \\
\hline
484^{km}
\end{array}
\qquad\qquad
\begin{array}{l}
4\underline{)484000} \\
121000 \text{ rails.}
\end{array}
\qquad\qquad
\begin{array}{r}
121^{km} \\
0.015\phantom{^{km}} \\
\hline
605\phantom{^{km}} \\
121\phantom{0^{km}} \\
\hline
1.815^{qkm}
\end{array}$$

22. Calculate the number of ars in a surface which a ream of paper (480 sheets) will cover. The sheets are 30.3^{cm} long and 195^{mm} wide.

$195^{mm} = 19.5^{cm}$.

The area of one sheet is $30.3^{cm} \times 19.5^{cm} = 590.85^{qcm}$.

The area of 480 sheets is $480 \times 590.85^{qcm} = 283{,}608^{qcm} = 0.283608^{a}$.

$$\begin{array}{r}
19.5 \\
30.3 \\
\hline
585 \\
585 \\
\hline
590.85^{qcm}
\end{array}
\qquad\qquad
\begin{array}{r}
590.85 \\
480 \\
\hline
4726800 \\
236340 \\
\hline
283608.00^{qcm}
\end{array}$$

23. A pile of wood is 4.25^{m} long, 1.33^{m} thick, and 2.60^{m} high. How many sters are there in it?

$1^{s} = 1^{cbm}$. In the pile of wood there are

$4.25 \times 1.33 \times 2.60 = 14.6965^{cbm} = 14.696^{s}$. *Ans.*

$$\begin{array}{r} 4.25 \\ \underline{1.33} \\ 1275 \\ 1275 \\ \underline{425} \\ 5.6525 \end{array} \qquad \begin{array}{r} 5.6525 \\ \underline{2.6} \\ 339150 \\ \underline{113050} \\ 14.69650^{cbm} \end{array}$$

24. A beam is 7.070^{m} long; its two other dimensions are 0.258^{m} and 87^{mm}. Find its volume.

$87^{mm} = 0.087^{m}$. Its volume is $0.258^{m} \times 0.087^{m} \times 7.070^{m} = 0.15869^{cbm}$.

$$\begin{array}{r} 0.258 \\ \underline{0.087} \\ 1806 \\ \underline{2064} \\ 0.022446 \end{array} \qquad \begin{array}{r} 0.022446 \\ \underline{7.07} \\ 157122 \\ \underline{157122} \\ 0.15869322^{cbm} \end{array}$$

25. A bar of iron 3^{m} long measures 45^{mm} square on the end where it has been evenly cut. The bar is heated and drawn out to a greater length by being passed through an orifice 24^{mm} square. What is the length of the bar after the operation?

$45^{mm} = 0.045^{m}$. $24^{mm} = 0.024^{m}$.

The volume of the bar is $0.045^{m} \times 0.045^{m} \times 3^{m} = 0.006075^{cbm}$. The area of the end, after the bar has been heated is

$$0.024^{m} \times 0.024^{m} = 0.000576^{qm}.$$

Therefore the length of the bar is $0.006075 \div 0.000576 = 10.547^{m}$.

$$\begin{array}{r} 0.045^{m} \\ \underline{0.045} \\ 225 \\ \underline{180} \\ 0.002025^{qm} \\ \underline{3} \\ 0.006075^{cbm} \end{array} \qquad \begin{array}{r} 0.024^{m} \\ \underline{0.024} \\ 96 \\ \underline{48} \\ 0.000576^{qm} \end{array} \qquad \begin{array}{r} 10.547^{m} \\ 576\overline{)6075.000} \\ \underline{576} \\ 3150 \\ \underline{2880} \\ 2700 \\ \underline{2304} \\ 3960 \\ \underline{3892} \end{array}$$

26. A reservoir is 1.50^{m} wide, 2.80^{m} long, and 1.25^{m} deep. Find how many liters it contains when full, and to what height it would be necessary to raise it that it might contain 10^{cbm}.

The volume of the reservoir is $1.5 \times 2.8 \times 1.25 = 5.25^{cbm} = 5250^{l}$. The area of the bottom is $1.5 \times 2.8 = 4.2^{qm}$; therefore, in order to contain 10^{cbm}, the height must be $10^{cbm} \div 4.2^{qm} = 2.38^{m}$.

$$\begin{array}{r} 1.5^{m} \\ \underline{2.8} \\ 120 \\ \underline{30} \\ 4.20^{qm} \\ \underline{1.25} \\ 2100 \\ 840 \\ \underline{420} \\ 5.2500 \end{array} \qquad\qquad \begin{array}{r} 2.38^{m} \\ 42\overline{)100.00} \\ \underline{84} \\ 160 \\ \underline{126} \\ 340 \\ \underline{336} \end{array}$$

27. Suppose a box to be 3.75^{m} long, 3.50^{m} wide, and 0.50^{m} high. How much lime would it take to fill it with mortar, reckoning that 1^{cbm} of lime after being slaked becomes $1\ 80^{cbm}$ of mortar?

Volume of the box is $3.75 \times 3.50 \times 0.50 = 6.5625^{cbm}$. Since 1^{cbm} of mortar when slaked becomes 1.8^{cbm}, the box will hold 6.5625^{cbm} of slaked mortar, which is the same as $6.5625^{cbm} \div 1.8 = 3.646^{cbm}$ of dry mortar.

$$\begin{array}{r} 3.75 \\ \underline{3.5} \\ 1875 \\ \underline{1125} \\ 13.125 \\ \underline{0.5} \\ 6.5625^{cbm} \end{array} \qquad\qquad \begin{array}{r} 3.646^{cbm} \\ 18\overline{)65.625}^{cbm} \\ \underline{54} \\ 116 \\ \underline{108} \\ 82 \\ \underline{72} \\ 105 \end{array}$$

28. A chest has the following dimensions: 1.17^{m}, 90^{m}, 1.04^{m}. How many cakes of soap 13^{cm} square on the bottom and 29^{cm} thick could be put in it? 0.12 of the volume of the chest must be deducted for packing.

The volume of a cake of soap is $13 \times 13 \times 29 = 4901^{ccm}$. The volume of the chest, deducting waste of room in packing, is

$$1.17 \times 0.90 \times 1.04 \times 0.88 = 0.9637056^{cbm} = 963705.6^{ccm}.$$

Therefore the chest will hold $(963,705.6 \div 4901)$ cakes of soap.

$$\begin{array}{r} 13 \\ \underline{13} \\ 39 \\ \underline{13} \\ 169 \\ \underline{29} \\ 1521 \\ \underline{338} \\ 4901^{ccm} \end{array} \qquad \begin{array}{r} 1.17 \\ \underline{1.04} \\ 468 \\ \underline{117} \\ 12168 \\ \underline{0.9} \\ 1.09512 \\ \underline{0.88} \\ 876096 \\ \underline{876096} \\ 0.9637056^{cbm} \end{array} \qquad \begin{array}{r} 196 \\ 4901\overline{)963705.6} \\ \underline{4901} \\ 47360 \\ \underline{44109} \\ 32515 \\ \underline{29406} \end{array}$$

29. A cubic meter of dry plaster makes 1.18^{cbm} when tempered; tempered plaster increases 1 in every 100, twenty-four hours after it is mixed. What volume of tempered plaster would be obtained from 55 sacks of 25^{l} each of dry plaster?

$25^{l} = 0.025^{cbm}$. The volume of the plaster is $55 \times 0.025^{cbm} = 1.375^{cbm}$. As 1^{cbm} makes 1.18^{cbm} when tempered, 1.375^{cbm} will make $1.375 \times 1.18^{cbm} = 1.6225^{cbm}$. In twenty-four hours its volume will be $1.01 \times 1.6225^{cbm} = 1.6387^{cbm}$.

$$\begin{array}{r} 0.025^{cbm} \\ \underline{55} \\ 125 \\ \underline{125} \\ 1.375^{cbm} \end{array} \qquad \begin{array}{r} 1.375^{cbm} \\ \underline{1.18} \\ 11000 \\ 1375 \\ \underline{1375} \\ 1.62250^{cbm} \end{array} \qquad \begin{array}{r} 1.6225 \\ \underline{1.01} \\ 16225 \\ \underline{16225} \\ 1.6387^{cbm} \end{array}$$

30. A reservoir is 2.80^{m} long, 1.50^{m} wide, and 1.25^{m} deep. How many liters will be required to fill 0.80 of it?

$$\begin{array}{r} 1.5^{m} \\ \underline{2.8} \\ 120 \\ \underline{30} \\ 4.20^{qm} \end{array} \qquad \begin{array}{r} 4.20^{qm} \\ \underline{1.25} \\ 2100 \\ 840 \\ \underline{420} \\ 5.2500^{cbm} \end{array} \qquad \begin{array}{r} 5250^{l} \\ \underline{0.8} \\ 4200.0 \end{array}$$

$= 5250^{l}$, volume.

31. A man buys 1415^{hl} of wheat for \$3.50 a hektoliter; but the measure used proves too small, the mistake amounting to 3^{l} in every hektoliter. What was the quantity of wheat delivered to the purchaser, the cost, and the reduction which ought to be made to him on account of the error?

The mistake was 3^{l} in 100^{l}, or he received only 0.97 of 1415^{hl} $= 1372.55^{hl}$. If 1^{hl} of wheat cost \$3.50, 1415^{hl} cost $1415 \times \$3.50 = \$4952.50$. A reduction of 0.03 of $\$4952.50 = \$148.58$ ought to be made.

$$\begin{array}{r} 1415^{hl} \\ \underline{0.97} \\ 9905 \\ \underline{12735} \\ 1372.55^{hl} \end{array} \qquad \begin{array}{r} 1415 \\ \underline{\$3.50} \\ 70750 \\ \underline{4245} \\ \$4952.50 \end{array} \qquad \begin{array}{r} \$4952.50 \\ \underline{0.03} \\ \$148.5750 \end{array}$$

32. The dimensions of a tile are as follows: length 22^{cm}, width 11^{cm}, thickness 55^{mm}. Find the volume of the tile, and the number of tiles in a pile of 25^{cbm}.

$55^{mm} = 5.5^{cm}$. The volume of a tile is $22 \times 11 \times 5.5 = 1331^{ccm}$. $25^{cbm} = 25{,}000{,}000^{ccm}$. In the pile there will be $25{,}000{,}000 \div 1331 = 18{,}732$ tiles.

$$\begin{array}{r} 22^{cm} \\ \underline{11} \\ 22 \\ \underline{22} \\ 242 \\ \underline{5.5} \\ 1210 \\ \underline{1210} \\ 1331.0^{ccm} \end{array} \qquad \begin{array}{r} 18732 \\ 1331\overline{)25000000} \\ \underline{1331} \\ 11690 \\ \underline{10648} \\ 10420 \\ \underline{9317} \\ 11030 \\ \underline{10648} \\ 3820 \\ \underline{2662} \end{array}$$

33. The measurement of a pile of wood shows that a ster could be filled from it 25.68 times. Give the volume of the pile in cubic meters, reckoning the length of the logs to be 1.15^{m}.

The volume of a pile is $1 \times 1 \times 1.15 \times 25.68 = 29.532^{cbm}$. *Ans.*

$$
\begin{array}{r}
25.68 \\
1.15^{cbm} \\
\hline
12840 \\
2568 \\
2568 \\
\hline
29.5320^{cbm}
\end{array}
$$

34. A liter of air weighs 1.273^{g}. How much does a cubic meter of air weigh?

$1^{cbm} = 1000^{l}$. Therefore 1^{cbm} of air weighs $1000 \times 1.273^{g} = 1273^{g} = 1.273^{kg}$. *Ans.*

35. A package of candles which weighs 465^{g} is sold at 28 cents. What is the price of a kilogram of candles?

1^{g} of candles costs $\$0.28 \div 465 = \$0.000602$. Therefore 1^{kg} costs $1000 \times \$0.000602 = \$0.602$.

36. How many times would 3.243^{t} of water fill a liter?

As 1^{t} of water will fill a cubic meter, 3.243^{t} will fill $3.243^{cbm} = 3243^{l}$. 3243 times. *Ans.*

37. Give the weight in kilograms of 43.4578^{ccm} of pure water.

43.4578^{ccm} of water weigh $43.4578^{g} = 0.0434578^{kg}$.

38. The volume of an engine's axletree is 0.245^{cbm}. Find its weight, the specific gravity of the iron being 7.8.

0.245^{cbm} of water weigh 0.245^{t}, and 0.245^{cbm} of iron weigh

$$7.8 \times 0.245_{t} = 1.911^{t}.$$

$$
\begin{array}{r}
0.245 \\
7.8 \\
\hline
1960 \\
1715 \\
\hline
1.9110^{t}
\end{array}
$$

39. Calculate the volume of a gram of the following substances: proof spirit, specific gravity 0.865; tin, specific gravity 7.291; lead, specific gravity 11.35; copper, specific gravity 8.85; silver, specific gravity 10.47; cork, specific gravity 0.240.

The volume equals 1^{ccm}, which, filled with water, weighs 1^{g}, divided by the specific gravity.

(i.)

$$\begin{array}{r} 1.16^{ccm} \\ 865\overline{)\,1000.00} \\ 865 \\ \hline 1350 \\ 865 \\ \hline 4850 \end{array}$$

(ii.)

$$\begin{array}{r} 0.14^{ccm} \\ 7291\overline{)\,1000.00} \\ 7291 \\ \hline 27090 \end{array}$$

(iii.)

$$\begin{array}{r} 0.088^{ccm} \\ 1135\overline{)\,100.000} \\ 9080 \\ \hline 9200 \\ 9080 \\ \hline \end{array}$$

(iv.)

$$\begin{array}{r} 0.113^{ccm} \\ 885\overline{)\,100.000} \\ 885 \\ \hline 1150 \\ 885 \\ \hline 2650 \end{array}$$

(v.)

$$\begin{array}{r} 0.095^{ccm} \\ 1047\overline{)\,100.000} \\ 9423 \\ \hline 5770 \\ 5235 \\ \hline \end{array}$$

(vi.)

$$\begin{array}{r} 4.167^{ccm} \\ 24\overline{)\,100.000} \\ 96 \\ \hline 40 \\ 24 \\ \hline 160 \\ 144 \\ \hline 160 \end{array}$$

40. Olive oil costs 60 cents a kilogram. What is the price of a liter? The specific gravity of olive oil is 0.914.

1^{l} of olive oil weighs 0.914^{kg}. As 1^{kg} costs \$0.60, 1^{l} costs $0.914 \times \$0.60 = \$0.548$.

$$\begin{array}{r} 0.914 \\ \$0.60 \\ \hline \$0.54840 \end{array}$$

41. Pure alcohol costs \$1.87 a kilogram. What is the price of a liter? The specific gravity of alcohol is 0.792.

1^{l} of alcohol weighs 0.792^{kg}. As 1^{kg} costs \$1.87, 1^{l} costs $0.792 \times \$1.87 = \$1.48$.

$$\begin{array}{r} \$1.87 \\ 0.792 \\ \hline 374 \\ 1683 \\ 1309 \\ \hline \$1.48 \end{array}$$

42. A man wants to build a shed large enough to hold 135^{st} of wood; if the shed is to be 3^{m} high and 5^{m} wide, how long must it be?

$135^{st} = 135^{cbm}$. The ground area is $3 \times 5 = 15^{qm}$. Therefore the height must be $135 \div 15 = 7^{m}$.

43. In a country where fire-wood is cut 1.16^{m} long what height must the sides of the ster be to hold a cubic meter?

The height must be

$1^{cbm} \div 1.16^{qm} = 0.86207^{m}$.

$$\begin{array}{r} 0.86207^{m} \\ 116)\overline{100.00000} \\ 928 \\ \hline 720 \\ 696 \\ \hline 240 \\ 232 \\ \hline 800 \end{array}$$

44. If a ster of cork cost \$20.00, how much would 100^{kg} cost, the cork weighing one quarter as much as water?

1^{st} of cork weighs 250^{kg}, and costs \$20.00. 100^{kg} will cost $\frac{100}{250}$ of \$20.00 $= \frac{2}{5}$ of \$20.00 $=$ \$8.00.

45. A liter of powder weighs 825^{g}. What would be the volume of a charge for a gun if the charge weighed 5^{g}? Calculate the volume in cubic centimeters.

The specific gravity of powder is 0.825. It takes $(1 \div 0.825)^{ccm}$ of powder to weigh 1^{g}; therefore to weigh 5^{g} it takes $5^{ccm} \div 0.825 = 6.06^{ccm}$.

$$\begin{array}{r} 6.06 \\ 825)\overline{5000.00} \\ 4950 \\ \hline 5000 \\ 4950 \\ \hline \end{array}$$

46. Out of gold which weighs 19.362 times as much as water, sheets of gold foil are made which are 0.010^{mm} in thickness. What surface would 3^{g} of gold cover?

$0.010^{mm} = 0.001^{cm}$. The volume of the gold is $3^{ccm} \div 19.362 = 0.154943^{ccm}$. Therefore, the surface is $0.154943^{ccm} \div 0.001^{cm} = 154.943^{qcm}$.

$$\begin{array}{r} 0.154943 \\ 19362)\overline{3000.000000} \\ 19362 \\ \hline 106380 \\ 96810 \\ \hline 95700 \\ 77448 \\ \hline 182520 \\ 174258 \\ \hline 82620 \\ 77448 \\ \hline 51720 \end{array}$$

47. Find the weight of an oak board 3.25^{m} long, 0.31^{m} wide, and 0.04^{m} thick; the specific gravity of the oak being 0.808.

The volume of the board is $3.25 \times 0.31 \times 0.04 = 0.0403^{cbm}$. 1^{cbm} of oak weighs 0.808^{t}; therefore 0.0403^{cbm} weigh $00.403 \times 0.808^{t} = 0.0325624^{t} = 32.5624^{kg}$.

```
   3.25
   0.31
   ----
    325
   975
 ------
 1.0075
   0.04
 ------
0.040300cbm, volume.

  0.0403
   0.808
  ------
    3224
  3224
---------
0.0325624
```

48. Find the weight of a bar of iron having the following dimensions: length 3.6^{m}, width 6^{cm}, thickness 2^{cm}; the specific gravity of the iron being 7.8.

$3.6^{m} = 360^{cm}$.

```
  360
    6
 ----
 2160
    2
 ----
 4320ccm, volume.

  4320
   7.8
 -----
 34560
 3024
 -------
 33696.0g
 = 33.696kg
```

49. How many lead balls each weighing 27^{g} could be obtained by melting a mass of lead, cubic in form, the edge measuring 0.356^{m}, the specific gravity of the lead being 11.35?

$0.356^{m} = 35.6^{cm}$.

```
   35.6cm
   35.6
   ----
   2136
  1780
 1068
 -------
 1267.36
    35.6
 -------
  760416
 633680
380208
----------
45118.016ccm, volume.

    45118.016
        11.35
    ---------
    225590080
   135354048
   45118016
  45118016
 ---------------
 512089.48160g

     18966
 27)512089
    27
    ---
    242
    216
    ---
     260
     243
     ---
      178
      162
      ---
       169
       162
       ---
```

50. Marble costs \$30.95 a cubic meter, and the specific gravity of marble is 2.73. If a block of marble weighs 1260^{kg}, what is its volume, and what is it worth?

1^{cbm} of marble weighs 2.73^{t}. $1260^{kg} = 1.26^{t}$.

$$\begin{array}{r} 0.4515^{cbm} \\ 273)\overline{126.0000} \\ \underline{1092} \\ 1680 \\ \underline{1638} \\ 420 \\ \underline{273} \\ 1470 \\ \underline{1365} \end{array} \qquad \begin{array}{r} 0.4615 \\ \underline{\$30.95} \\ 23075 \\ 41535 \\ \underline{13845} \\ \$14.28 \end{array}$$

51. Sea-water contains 28 parts, by weight, of salt in 1000. A liter of sea-water weighs 1.025^{kg}. How many kilograms of salt could be obtained from 126.276842^{cbm} of sea-water?

1^{kg} of sea-water contains 0.028^{kg} of salt.

$$\begin{array}{r} 126.276842 \\ \underline{1.025^{kg}} \\ 631384210 \\ 252553684 \\ \underline{126276842} \\ 129.433753050^{kg} \end{array} \qquad \begin{array}{r} 129433.753 \\ \underline{0.028^{kg}} \\ 1035470024 \\ \underline{258867506} \\ 3624.145084^{kg} \end{array}$$

52. An empty cask weighs 17.06^{kg}; when filled with water it weighs 275.8^{kg}. How many liters does it hold? How many casks of this size would it require to receive the wine from a vat containing 3.008^{cbm}?

The cask will hold $275.8^{kg} - 17.06^{kg} = 258.74^{kg}$ of water. It takes 258.74^{l} of water to weigh 258.74^{kg}. Therefore the cask will hold 258.74^{l}.

$3.008^{cbm} = 3008^{l}$. If one barrel holds 258.74^{l}, to hold 3008^{l}, it will take $3008 \div 258.74 = 12$ barrels.

$$\begin{array}{r} 275.80^{kg} \\ \underline{17.06} \\ 258.74^{kg} \end{array} \qquad \begin{array}{r} 12 \\ 25874)\overline{300800} \\ \underline{25874} \\ 42060 \end{array}$$

53. It takes about 204.8^{l} of wheat to sow a hektar. How many cubic meters would it take to sow a square kilometer?

$1^{qkm} = 100^{ha}$. 1^{ha} will require $100 \times 204.8^{l} = 20{,}480^{l} = 20.48^{cbm}$. *Ans.*

54. A piece of road 1^{km} long and 7^{m} wide is to be macadamized; the macadamizing is to be 33^{cm} deep; it costs 43 cents a cubic meter to prepare the stones. What will enough for the road cost?

$1^{km} = 1000^{m}$; $33^{cm} = 0.33^{m}$.

$$\begin{array}{r} 0.33 \\ \underline{7} \\ 2.31 \\ \underline{1000} \\ 2310.00 \end{array} \qquad\qquad \begin{array}{r} 2310 \\ \underline{\$0.43} \\ 6930 \\ \underline{9240} \\ \$993.30 \end{array}$$

55. A gasometer holds $28{,}000^{cbm}$ of gas. How many jets would this gasometer feed, when each jet burns 125^{l} an hour, and is used 4 hours every evening?

Each jet will burn $4 \times 125^{l} = 500^{l}$ each evening. $28{,}000^{cbm} = 28{,}000{,}000^{l}$. The gasometer will feed $28{,}000{,}000 \div 500 = 56{,}000$ jets.

56. The city of Venice is situated in the midst of a great lake of salt water, communicating with the sea, and all the rain-water is caught for the cisterns. Ordinary years the fall of rain in Venice is 82^{cm}; the surface of the city, after the canals have been deducted, is 520^{ha}; reckoning the population at 115,330, how many liters a day of rain-water could each inhabitant have?

$520^{ha} = 5{,}200{,}000^{qm}$; $82^{cm} = 0.82^{m}$.

The average amount of rain-water is $5{,}200{,}000 \times 0.82 = 4{,}264{,}000^{cbm} = 4{,}264{,}000{,}000^{l}$. Each person can use per year $4{,}264{,}000{,}000 \div 115{,}530$, or, per day, $4{,}264{,}000{,}000 \div (115{,}530 \times 365) = 101.118^{l}$.

$$\begin{array}{r} 0.82 \\ \underline{5200000} \\ 16400000 \\ \underline{410} \\ 4264000.00 \end{array} \qquad \begin{array}{r} 115{,}530 \\ \underline{365} \\ 577650 \\ 693180 \\ \underline{346590} \\ 42168450 \end{array} \qquad \begin{array}{r} 101.118^{l} \\ 4216845)\overline{426400000.000} \\ \underline{4216845} \\ 4715500 \\ \underline{4216845} \\ 4986550 \\ \underline{4216845} \\ 7697050 \\ \underline{4216845} \\ 34802050 \\ \underline{33734760} \end{array}$$

57. Find the weight of a bar of iron 5.35^{m} long, 4.56^{cm} thick, and 3.54^{cm} wide. Find, also, the width of an oak beam 4.30^{m} long, 9.12^{cm} thick, which has the same weight. The specific gravity of the oak to be reckoned at 1.026, that of the iron 7.788.

$5.35^{m} = 535^{cm}$. $4.30^{m} = 430^{cm}$. The volume of the oak beam is $67{,}258.596992^{g} \div 1.026^{g} = 65{,}554.188^{ccm}$. The area of one side of the oak beam is $430 \times 9.12 = 3921.6^{qcm}$; therefore thickness is $65{,}554.188^{ccm} \div 3921.6^{qcm} = 16.72^{cm}$.

4.56^{cm}	16.1424	8636.184
3.54	535	7.788^{g}
1824	807120	69089472
2280	484272	69089472
1368	807120	60453288
16.1424^{qcm}	8636.1840^{ccm}, volume.	60453288
		67258.596992^{g} $= 67.259^{kg}$.

```
            65554.188
1026)67258596.992
     6156
      5698
      5130
       5685
       5130
        5559
        5130
         4296
         4104
          1929
          1026
           9039
           8208
            8212
```

```
          16.72cm
39216)655541.88
      39216
      263381
      235296
       280858
       274512
        63468
```

58. Give the specific gravity and volume of a body weighing 35^{kg} in air and 30^{kg} in water.

The weight of water displaced by the body is 5^{kg}.

The weight of body in air is 35^{kg}.

Therefore specific gravity is $35 \div 5 = 7$. 35^{l} of water weigh 35^{kg}; $35 \div 7 = 5^{l}$, volume.

59. A ster of piled oak wood weighs 425^{kg}; the specific gravity of the wood is 0.74. What is the volume occupied by the spaces between the logs? For how much must 100^{kg} of separate sticks be sold in order to bring the same amount as when sold by the ster; a ster selling for \$2.20?

If there were no spaces between the logs, the ster of wood would weigh 740^{kg}. Therefore the spaces, if filled with wood, would weigh $740^{kg} - 425^{kg} = 315^{kg}$. Therefore volume of spaces is $315 \div 740 = 0.42568^{cbm}$. 100^{kg} ought to be sold for $\frac{100}{425}$ of \$2.20 = \$220 ÷ 425 = \$0.518.

```
     0.42568cbm
74)31.50000
   296
    190
    148
     420
     370
      500
      444
       560
```

```
       $0.518
425)$220.000
     2125
       750
       425
       3250
```

60. Wrought iron sells for \$7.00 per 100^{kg}. A bar of iron 4.5^{cm} wide, 3.3^{cm} thick costs \$5.08; what is its length, reckoning the specific gravity of the iron at 7.4?

\$7.00 per 100^{kg} is the same as \$0.07 per kilogram. An iron bar that costs \$5.08 must weigh $5.08 \div 0.07 = 72.57143^{kg}$, and its volume is $72.57143 \div 7.4 = 9.8066^{l} = 9806.5^{ccm}$. The area of an end of the bar is $4.5^{cm} \times 3.3^{cm} = 14.85^{qcm}$. Therefore the length is $9806.6 \div 14.85 = 660.4^{cm} = 6.604^{m}$.

```
     9.8066l
74)725.7143
   666
    597
    592
     514
     444
      703
      666
```

```
         660.4cm
1485)980650.0
     8910
      8965
      8910
       5500
```

61. Experiment shows that water weighs 770 times as much as air; and the specific gravity of mercury, in comparison with water, is 13.6. How many liters of air will it take to weigh as much as a liter of mercury?

Water is 770 times as heavy as air, and mercury is 13.6 times as heavy as water. Therefore mercury is 13.6×770 times as heavy as air.

```
   13.6
    770
   9520
  9520
10472.0
```

62. A mass of lead weighing 753^{kg} is made into sheets 0.1^{mm} thick. Calculate, in square meters, the surface which can be covered by the sheets thus obtained. The specific gravity of the lead is 11.3.

The volume of the lead is $753 \div 11.3 = 66.637^{l} = 0.066637^{cbm}$.

$0.1^{mm} = 0.0001^{m}$. The surface of the lead is

$0.066637^{cbm} \div 0.0001^{m} = 666.37^{qm}$. *Ans.*

```
      66.637l
113)7530.000
    678
     750
     678
      720
      678
       420
       339
        810
        797
```

63. A rectangular sheet of tin of uniform thickness is 85^{cm} wide, 1.35^{m} long; it weighs 268^{g}. What is its thickness, reckoning the specific gravity of tin at 7.3?

The volume of the tin is $268 \div 7.3 = 36.7109^{ccm}$; $1.35^{m} = 135^{cm}$. The area of the tin is $135^{cm} \times 85^{cm} = 11,475^{qcm}$; therefore its thickness is $36.7109^{ccm} \div 11,475^{qcm} = 0.0032^{cm}$.

```
    36.7109ccm
73)2680.0000
   219
    490
    438
     520
     511
      90
      73
      700
      657
```

```
  135
   85
  675
1080
11475qcm
```

```
          0.0032cm
11475)36.7109
       34325
        23859
        22950
```

64. The fine coal which collects about the shafts of the mines and in the coal-yards, was for a long time wasted, because it could not be burned in stoves and grates. Now, this dust is mixed with tar in proportion of 92^{kg} of dust and 8^{kg} of tar; the mixture is heated, and afterwards pressed in rectangular moulds of 14.75^{cm}, 18.5^{cm}, and 29^{cm}; each one of these blocks weighs 10^{kg}; they are sold at \$3.00 a ton, and make excellent fuel for heating steam boilers. Give the specific gravity of this fuel; also, the sum which would be realized in thus utilizing $800{,}000^{t}$ of coal dust, the cost of tar, mixing, etc., being \$0.50 a ton.

Volume of a block is $14.75 \times 18.5 \times 29 = 7913.305^{ccm} = 7.913305^{l}$. Specific gravity is $10 \div 7.913305 = 1.264$. $800{,}000^{t}$ of coal dust will make $800{,}000^{t} \div 0.92 = 869{,}565.217^{t}$ of the mixture. $869{,}565.217^{t}$ at \$2.50 per ton $= 869{,}565.217 \times \$2.50 = \$2{,}173{,}913.04$.

14.75^{cm}
18.5
7375
11800
1475
272.875
29
2455805
545750
7913.305^{ccm}

1.264
79133) 100000.000
79133
208670
158266
504040
474798
292420

869565.217
92) 80000000.00
736
640
552
880
828
520
460
600
552
480
460
200
184
160
92
680
644

869565.217
\$2.50
43478260850
1739130434
\$2,173,913.04

65. A bar of iron a millimeter square on the end will break under a tension of 30^{kg}. Find the length at which a suspended bar of iron will break from its own weight, the specific gravity of the iron being 7.8.

$30^{kg} = 0.03^{t}$.

The volume of the iron bar is $0.03 \div 7.8 = 0.00384615^{cbm}$. The area of an end of the bar is $1^{mm} \times 1^{mm} = 1^{qmm} = 0.000001^{qm}$. Therefore the length of the bar is $0.00384615^{cbm} \div 0.000001^{qm} = 3846.15^{m}$.

0.00384615^{cbm}
78)0.30000000
234
660
624
360
312
480
468
120
78
420
390

66. Fifty-three kilograms of starch are obtained from 100^{kg} of wheat. A hektar of land produces 1363 of wheat; a hektoliter of wheat weighs 78^{kg}. If the wheat harvested from a field measuring 2^{ha} and 33^{qm} is taken to a starch factory, how much starch will be made from it?

0.53^{kg} of starch are obtained from 1^{kg} of wheat. 1^{l} of wheat weighs 0.78^{kg}. 1^{ha} produces 1363×0.78^{kg} of wheat $= 1063.14^{kg}$, $2^{ha}\ 33^{qm} = 2.0033^{ha}$. 2.0033^{ha} produce $2.0033 \times 1063.14^{kg} = 2129.788362^{kg}$ of wheat. The amount of starch is 1128.7878^{kg}.

1363
0.78^{kg}
10904
9541
1063.14^{kg}
2.0033
318942
318942
212628
2129.788362^{kg}

2129.788
0.53
6389364
10648940
1128.78764^{kg}

67. A gardener wishes to provide glass for his hot-beds. The beds cover 2.65^{a}; the panes will cover 0.75 of the whole surface, the rest being taken up by the frames and alleys. First, find how many panes measuring 45^{cm} by 37^{cm} it will take to cover the beds; then find the price of the glass, at a cost of 95 cents a square meter.

$45^{cm} = 0.45^{m}$; $37^{cm} = 0.37^{m}$; $2.65^{a} = 265^{qm}$.

Total area of the glass is 0.75 of $265^{qm} = 198.75^{qm}$. The area of one pane is $0.45 \times 0.37 = 0.1665^{qm}$. Therefore the number of panes needed is $198.75 \div 0.1665 = 1194$. At \$0.95 per square meter, 198.75^{qm} will cost $198.75 \times \$0.95 = \$188.81$.

$$\begin{array}{r} 0.45^{m} \\ \underline{0.37^{m}} \\ 315 \\ \underline{135} \\ 0.1665^{qm} \end{array} \qquad \begin{array}{r} 1194 \\ 1665\overline{)1987500} \\ \underline{1665} \\ 8225 \\ \underline{1665} \\ 15600 \\ \underline{14985} \\ .6150 \end{array} \qquad \begin{array}{r} 198.75 \\ \underline{\$0.95} \\ 99375 \\ \underline{178875} \\ \$188.81 \end{array}$$

68. A jar full of water weighs 1.325^{kg}; filled with mercury it weighs 12.540^{kg}. What is the capacity of the jar, and its weight? The specific gravity of the mercury is 13.59.

The weight of the jar and the jar full of mercury is 12.540^{kg}. The weight of the jar and the jar full of water is 1.325^{kg}. Therefore the difference in weight between the mercury and the water is $12.540^{kg} - 1.325^{kg} = 11.215^{kg}$. $13.59 - 1 = 12.59$, the specific gravity of a liquid of which the jar full without the jar weighs 11.215^{kg}. Hence the capacity of the jar is $11.215 \div 12.59 = 0.89079^{l}$. 0.89079^{l} of water weigh 0.89079^{kg}. Therefore weight of jar is $1.325 - 0.89079 = 0.43421^{kg} = 434.21^{g}$.

$$\begin{array}{r} 12.540^{kg} \\ \underline{1.325\phantom{^{kg}}} \\ 11.215^{kg} \end{array} \qquad \begin{array}{r} 0.89079^{l} \\ 1259\overline{)1121.50000} \\ \underline{10072} \\ 11430 \\ \underline{11331} \\ 9900 \\ \underline{8813} \\ 10870 \\ \underline{10072} \end{array} \qquad \begin{array}{r} 1.32500^{kg} \\ \underline{0.89079\phantom{^{kg}}} \\ 0.43421^{kg} \end{array}$$

69. A hektoliter of rape-seed weighs 63^{kg}, and 32^{l} of oil can be extracted from it. How many kilograms of the seed will it take to make a hektoliter of oil?

$1^{hl} = 100^{l}$. If 32^{l} of oil can be extracted from 63^{kg} of seed, 1^{l} of oil can be extracted from $63 \div 32 = 1.96875^{kg}$ of seed, and 100^{l} of oil can be extracted from $100 \times 1.96875^{kg} = 196.875^{kg}$ of seed.

$$
\begin{array}{r|l}
 & 1.96875 \\
32 & 63.00000 \\
 & \underline{32} \\
 & 310 \\
 & \underline{288} \\
 & 220 \\
 & \underline{192} \\
 & 280 \\
 & \underline{256} \\
 & 240 \\
 & \underline{224} \\
 & 160 \\
 & \underline{160}
\end{array}
$$

70. Common burning gas is 0.97 of the weight of air, and a liter of air weighs 1.293^{g}. In a shop there are 65 jets, each one of which burns 123^{l} an hour, and is used 5 hours in the winter evenings. Calculate the weight of the gas used in a month, and the expense of lighting the shop, when gas costs 6 cents a cubic meter.

1^{l} of gas weighs $0.97 \times 1.293^{g} = 1.25421^{g}$. 65 jets, each burning 123^{l} an hour, and used 5 hours an evening for 30 days, will use $65 \times 5 \times 30 \times 123^{l} = 1{,}199{,}250^{l}$, the weight of which is $1{,}199{,}250 \times 1.25421^{g} = 1{,}504{,}111.34^{g} = 1504.11134^{kg}$. $1{,}199{,}250^{l} = 1199.25^{cbm}$. The expense at \$0.06 per cubic meter is $1199.25 \times \$0.06 = \$71.96$.

$$
\begin{array}{r}
1.293^{g} \\
\underline{0.97} \\
9051 \\
\underline{11637} \\
1.25421^{g}
\end{array}
\qquad
\begin{array}{r}
123^{l} \\
\underline{65} \\
615 \\
\underline{738} \\
7995^{l} \\
\underline{5} \\
39975^{l} \\
\underline{30} \\
1199250
\end{array}
\qquad
\begin{array}{r}
1199250 \\
\underline{1.25421^{g}} \\
1199250 \\
239850 \\
479700 \\
599625 \\
239850 \\
\underline{119925} \\
1504111.34250^{g}
\end{array}
\qquad
\begin{array}{r}
1199.25 \\
\underline{\$0.06} \\
\$71.9550 \\
= \$71.96
\end{array}
$$

71. A merchant buys one kind of wine at 30 cents a liter, another kind at 21 cents a liter; he mixes the two kinds by putting 5^l of the first with 8^l of the second. For how much a liter must he sell the mixture in order to gain $3.75 a hektoliter?

5^l at $0.30 per liter cost $1.50.

8^l at $0.21 per liter cost $1.68.

Therefore 13^l of the mixture cost $1.50 + $1.68 = $3.18, and 1^l costs $3.18 ÷ 13 = $0.2446. Again, if $3.75 per hektoliter is equivalent to a gain of $0.0375 per liter, to make $3.75 per hektoliter the merchant must sell the wine for $0.0375 + $0.2446 = $0.2821 per liter.

$$\begin{array}{r} \$0.30 \\ 5 \\ \hline \$1.50 \end{array} \qquad \begin{array}{r} \$0.21 \\ 8 \\ \hline \$1.68 \\ 1.50 \\ \hline \$3.18 \end{array} \qquad \begin{array}{r} 0.2446 \\ 13\overline{)3.1800} \\ 26 \\ \hline 58 \\ 52 \\ \hline 60 \\ 52 \\ \hline 80 \\ 78 \\ \hline \end{array}$$

72. If it requires 360 tiles to drain an ar of land, what will it cost to drain 17.784^{ha}, when the tiles cost $20 a thousand, and the expense of laying is the same as the cost of the tiles?

The expense of laying the tiles and their cost is $40 per thousand. $17.784^{ha} = 1778.4^{a}$. To drain 1778.4^{a} of land 1778.4×360 tiles = 640,224 tiles = 640.224 thousand are needed. 640.224 thousand at $40 per thousand cost 640.224 × $40 = $25,608.96. *Ans.*

$$\begin{array}{r} 1778.4 \\ 360 \\ \hline 1067040 \\ 53352 \\ \hline 640224.0 \end{array} \qquad \begin{array}{r} 640.224 \\ \$40 \\ \hline \$25608.960 \end{array}$$

73. It is found in building that hewn stone of medium durability ought not to support, as a permanent weight, more than 0.07 of the

weight that it would require to crush it. A certain kind of stone used for building will be crushed under a weight of 250^{kg} a square centimeter. What is the greatest height to which a wall constructed of this material can be safely carried, the specific gravity of the stone being 2.1?

250^{kg} per square centimeter is equivalent to 250,000^{g} per square centimeter. 0.07 of 250,000^{g} = 17,500^{g} ought to be the pressure on a square centimeter. Therefore volume of imaginary prism ought to be 17,500 ÷ 2.1 = 8333.33^{ccm}, or the height ought to be 8333.33^{cm} = 83.333^{m}.

$$\begin{array}{r} 8333.33^{ccm} \\ 21\overline{)175000.00} \\ \underline{168} \\ 70 \\ \underline{63} \\ 70 \end{array}$$

74. Several kinds of wines are mixed as follows: 245^{l} at 20 cents a liter, 547^{l} at 15 cents a liter, 344^{l} at 25 cents a liter. How much does the mixture cost a liter?

245^{l}	at \$0.20 per liter cost	\$49.00
547^{l}	at \$0.15 per liter cost	\$82.05
344^{l}	at \$0.25 per liter cost	\$86.00
1136^{l}	of the mixture cost	\$217.05

Therefore 1^{l} costs \$217.05 ÷ 1136 = \$0.191. *Ans.*

245	547	344
\$0.20	\$0.15	\$0.25
\$49.00	2735	1720
	547	688
	\$82.05	\$86.00

$$\begin{array}{r} \$0.191 \\ 1136\overline{)217.050} \\ \underline{1136} \\ 10345 \\ \underline{10224} \\ 1210 \\ \underline{1136} \end{array}$$

75. A farmer wishes to drain a field of 8.75^{ha}. Each hektar requires 750^{m} of ditches. The opening of these ditches costs 10 cents a running meter; the tiles are 30^{cm} long, and cost \$15 a thousand. He pays 2 cents a meter for laying the tiles, and 4 cents a meter for filling the ditches. What is the cost of draining the field?

There are required $8.75 \times 750^{m} = 6562.5^{m}$ of ditches. The expense of opening the ditches, laying the tiles, and filling the ditches is $\$0.10 + \$0.02 + \$0.04 = \$0.16$ per meter. 6562.5^{m} will cost $6562.5 \times \$0.16 = \$1050.00$. $30^{cm} = 0.3^{m}$. For 6562.5^{m}, $6562.5 \div 0.3 = 21,875$ tiles are necessary. The tiles cost \$15 per thousand. Therefore 21.875 thousand cost $21.875 \times \$15 = \$328.13$. Hence cost of draining the field is $\$1050.00 + \$328.13 = \$1378.13$.

$$\begin{array}{r} 8.75 \\ \underline{750^{m}} \\ 43750 \\ \underline{6125} \\ 6562.50^{m} \end{array} \qquad \begin{array}{r} 6562.5 \\ \underline{\$0.16} \\ 393750 \\ \underline{65625} \\ \$1050.000 \end{array} \qquad \begin{array}{r} 21.875 \\ \underline{\$15} \\ 109375 \\ \underline{21875} \\ \$328.125 = \$328.13. \end{array} \qquad \begin{array}{r} \$1050.00 \\ \underline{328.13} \\ \$1378.13 \end{array}$$

Exercise XI.

1. Find the prime factors of 148; 264; 178; 183; 173; 187; 346; 343.

$$\begin{array}{r|l} 2^2 & 148 \\ \hline & 37 \end{array} \qquad \begin{array}{r|l} 2^3 & 264 \\ \hline 3 & 33 \\ \hline & 11 \end{array} \qquad \begin{array}{r|l} 2 & 178 \\ \hline & 89 \end{array} \qquad \begin{array}{r|l} 3 & 183 \\ \hline & 61 \end{array}$$

$2^2 \times 37$. *Ans.* $2^3 \times 3 \times 11$. *Ans.* 2×89. *Ans.* 3×61. *Ans.*

$$\begin{array}{r|l} 1 & 173 \\ \hline & 173 \end{array} \qquad \begin{array}{r|l} 11 & 187 \\ \hline & 17 \end{array} \qquad \begin{array}{r|l} 2 & 346 \\ \hline & 173 \end{array} \qquad \begin{array}{r|l} 7^3 & 343 \\ \hline & 1 \end{array}$$

1×173. *Ans.* 11×17. *Ans.* 2×173. *Ans.* 7^3. *Ans.*

2. Find the prime factors of 210; 353; 5280; 231; 31,416; 1369; 1368.

$$\begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 1 & 353 \\ \hline & 353 \end{array} \qquad \begin{array}{r|l} 2^5 & 5280 \\ \hline 3 & 165 \\ \hline 5 & 55 \\ \hline & 11 \end{array} \qquad \begin{array}{r|l} 3 & 231 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

$2 \times 3 \times 5 \times 7$. *Ans.* 1×353. *Ans.* $2^5 \times 3 \times 5 \times 11$. *Ans.* $3 \times 7 \times 11$. *Ans.*

$$\begin{array}{r|l} 2^3 & 31416 \\ \hline 3 & 3927 \\ \hline 7 & 1309 \\ \hline 11 & 187 \\ \hline & 17 \end{array} \qquad \begin{array}{r|l} 37 & 1369 \\ \hline & 37 \end{array} \qquad \begin{array}{r|l} 2^3 & 1368 \\ \hline 3^2 & 171 \\ \hline & 19 \end{array}$$

$2^3 \times 3 \times 7 \times 11 \times 17$. *Ans.* 37×37. *Ans.* $2^3 \times 3^2 \times 19$. *Ans.*

3. Find the prime factors of 247; 327; 179; 83; 2125; 2353; 2333.

$\begin{array}{r|l} 13 & 247 \\ \hline & 19 \end{array}$ 13×19. *Ans.*

$\begin{array}{r|l} 3 & 327 \\ \hline & 109 \end{array}$ 3×109. *Ans.*

$\begin{array}{r|l} 1 & 179 \\ \hline & 179 \end{array}$ 1×179. *Ans.*

$\begin{array}{r|l} 1 & 83 \\ \hline & 83 \end{array}$ 1×83. *Ans.*

$\begin{array}{r|l} 5^3 & 2125 \\ \hline & 17 \end{array}$ $5^3 \times 17$. *Ans.*

$\begin{array}{r|l} 13 & 2353 \\ \hline & 181 \end{array}$ 13×181. *Ans.*

$\begin{array}{r|l} 1 & 2333 \\ \hline & 2333 \end{array}$ 1×2333. *Ans.*

4. Find the prime factors of 165; 168; 2148; 16,662; 321; 1551; 38.

$\begin{array}{r|l} 3 & 165 \\ \hline 5 & 55 \\ \hline & 11 \end{array}$ $3 \times 5 \times 11$. *Ans.*

$\begin{array}{r|l} 2^3 & 168 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$ $2^3 \times 3 \times 7$. *Ans.*

$\begin{array}{r|l} 2^2 & 2148 \\ \hline 3 & 537 \\ \hline & 179 \end{array}$ $2^2 \times 3 \times 179$. *Ans.*

$\begin{array}{r|l} 2 & 16662 \\ \hline 3 & 8331 \\ \hline & 2777 \end{array}$ $2 \times 3 \times 2777$. *Ans.*

$\begin{array}{r|l} 3 & 321 \\ \hline & 107 \end{array}$ 3×107. *Ans.*

$\begin{array}{r|l} 3 & 1551 \\ \hline 11 & 517 \\ \hline & 47 \end{array}$ $3 \times 11 \times 47$. *Ans.*

$\begin{array}{r|l} 2 & 38 \\ \hline & 19 \end{array}$ 2×19. *Ans.*

5. Find the prime factors of 82; 129; 72; 66; 68; 65; 76; 86; 88; 142.

$\begin{array}{r|l} 2 & 82 \\ \hline & 41 \end{array}$ 2×41. *Ans.*

$\begin{array}{r|l} 3 & 129 \\ \hline & 43 \end{array}$ 3×43. *Ans.*

$\begin{array}{r|l} 2^3 & 72 \\ \hline 3^2 & 9 \\ \hline & 1 \end{array}$ $2^3 \times 3^2$. *Ans.*

$\begin{array}{r|l} 2 & 66 \\ \hline 3 & 33 \\ \hline & 11 \end{array}$ $2 \times 3 \times 11$. *Ans.*

$\begin{array}{r|l} 2^2 & 68 \\ \hline & 17 \end{array}$ $2^2 \times 17$. *Ans.*

$\begin{array}{r|l} 5 & 65 \\ \hline & 13 \end{array}$ 5×13. *Ans.*

$\begin{array}{r|l} 2^2 & 76 \\ \hline & 19 \end{array}$ $2^2 \times 19$. *Ans.*

$\begin{array}{r|l} 2 & 86 \\ \hline & 43 \end{array}$ 2×43. *Ans.*

$\begin{array}{r|l} 2^3 & 88 \\ \hline & 11 \end{array}$ $2^3 \times 11$. *Ans.*

$\begin{array}{r|l} 2 & 142 \\ \hline & 71 \end{array}$ 2×71. *Ans.*

6. Find the prime factors of 326; 368; 464; 292; 362; 365; 730; 42.

$\begin{array}{r|l} 2 & 326 \\ \hline & 163 \end{array}$ 2×163. *Ans.*

$\begin{array}{r|l} 2^4 & 368 \\ \hline & 23 \end{array}$ $2^4 \times 23$. *Ans.*

$\begin{array}{r|l} 2^4 & 464 \\ \hline & 29 \end{array}$ $2^4 \times 29$. *Ans.*

$\begin{array}{r|l} 2^2 & 292 \\ \hline & 73 \end{array}$ $2^2 \times 73$. *Ans.*

$\begin{array}{r|l} 2 & 362 \\ \hline & 181 \end{array}$ 2×181. *Ans.*

$\begin{array}{r|l} 5 & 365 \\ \hline & 73 \end{array}$ 5×73. *Ans.*

$\begin{array}{r|l} 2 & 730 \\ \hline 5 & 365 \\ \hline & 73 \end{array}$ $2 \times 5 \times 73$. *Ans.*

$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$ $2 \times 3 \times 7$. *Ans.*

7. Find the prime factors of 868; 999; 822; 1346; 7641; 6234; 234.

$$\begin{array}{r|l} 2^2 & 868 \\ \hline 7 & 217 \\ \hline & 31 \end{array}$$

$2^2 \times 7 \times 31$. *Ans.*

$$\begin{array}{r|l} 3^3 & 999 \\ \hline & 37 \end{array}$$

$3^3 \times 37$. *Ans.*

$$\begin{array}{r|l} 2 & 822 \\ \hline 3 & 411 \\ \hline & 137 \end{array}$$

$2 \times 3 \times 137$. *Ans.*

$$\begin{array}{r|l} 2 & 1346 \\ \hline & 673 \end{array}$$

2×673. *Ans.*

$$\begin{array}{r|l} 3^3 & 7641 \\ \hline & 283 \end{array}$$

$3^3 \times 283$. *Ans.*

$$\begin{array}{r|l} 2 & 6234 \\ \hline 3 & 3117 \\ \hline & 1039 \end{array}$$

$2 \times 3 \times 1039$. *Ans.*

$$\begin{array}{r|l} 2 & 234 \\ \hline 3^2 & 117 \\ \hline & 13 \end{array}$$

$2 \times 3^2 \times 13$. *Ans.*

8. Find the prime factors of 579; 577; 212; 126; 128; 8192; 8190.

$$\begin{array}{r|l} 3 & 579 \\ \hline & 193 \end{array}$$

3×193. *Ans.*

$$\begin{array}{r|l} 1 & 577 \\ \hline & 577 \end{array}$$

1×577. *Ans.*

$$\begin{array}{r|l} 2^2 & 212 \\ \hline & 53 \end{array}$$

$2^2 \times 53$. *Ans.*

$$\begin{array}{r|r} 2 & 126 \\ \hline 3^2 & 63 \\ \hline & 7 \end{array}$$

$2 \times 3^2 \times 7$. *Ans.*

$$\begin{array}{r|l} 2^7 & 128 \\ \hline & 1 \end{array}$$

2^7. *Ans.*

$$\begin{array}{r|l} 2^{13} & 8192 \\ \hline & 1 \end{array}$$

2^{13}. *Ans.*

$$\begin{array}{r|r} 2 & 8190 \\ \hline 3^2 & 4095 \\ \hline 5 & 455 \\ \hline 7 & 91 \\ \hline & 13 \end{array}$$

$2 \times 3^2 \times 5 \times 7 \times 13$. *Ans.*

9. Find the prime factors of 8197; 3125; 2401; 1331; 78,309; 25,179.

$$\begin{array}{r|l} 7 & 8197 \\ \hline & 1171 \end{array}$$

7×1171. *Ans.*

$$\begin{array}{r|l} 5^5 & 3125 \\ \hline & 1 \end{array}$$

5^5. *Ans.*

$$\begin{array}{r|l} 7^4 & 2401 \\ \hline & 1 \end{array}$$

7^4. *Ans.*

$$\begin{array}{r|l} 11^3 & 1331 \\ \hline & 1 \end{array}$$

11^3. *Ans.*

$$\begin{array}{r|r} 3^2 & 78309 \\ \hline 7 & 8701 \\ \hline 11 & 1243 \\ \hline & 113 \end{array}$$

$3^2 \times 7 \times 11 \times 113$. *Ans.*

$$\begin{array}{r|r} 3 & 25179 \\ \hline 7 & 8393 \\ \hline 11 & 1199 \\ \hline & 109 \end{array}$$

$3 \times 7 \times 11 \times 109$. *Ans.*

EXERCISE XII.

1. Find the prime factors of 8.4; 7.6; 1.08; 0.144; 0.036; 0.037; 21.45.

$8.4 = 84 \times 0.1.$ $7.6 = 76 \times 0.1.$ $1.08 = 108 \times 0.01.$ $0.144 = 144 \times 0.001.$

$$\begin{array}{r|l} 2^2 & 84 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 2^2 & 76 \\ & 19 \end{array} \qquad \begin{array}{r|l} 2^2 & 108 \\ 3^3 & 27 \\ & 1 \end{array} \qquad \begin{array}{r|l} 2^4 & 144 \\ 3^2 & 9 \\ & 1 \end{array}$$

$2^2 \times 3 \times 7 \times 0.1.$ *Ans.* $2^2 \times 19 \times 0.1.$ *Ans.* $2^2 \times 3^3 \times 0.01.$ *Ans.* $2^4 \times 3^2 \times 0.001.$ *Ans.*

$0.036 = 36 \times 0.001.$ $0.037 = 37 \times 0.001.$ $21.45 = 2145 \times 0.01.$

$$\begin{array}{r|l} 2^2 & 36 \\ 3^2 & 9 \\ & 1 \end{array} \qquad \begin{array}{r|l} 1 & 37 \\ & 37 \end{array} \qquad \begin{array}{r|l} 3 & 2145 \\ 5 & 715 \\ 11 & 143 \\ & 13 \end{array}$$

$1 \times 37 \times 0.001.$ *Ans.*

$2^2 \times 3^2 \times 0.001.$ *Ans.*

$3 \times 5 \times 11 \times 13 \times 0.01.$ *Ans.*

2. Find the prime factors of 14.6; 2.61; 21.2; 78.54; 0.5236; 0.00052.

$14.6 = 146 \times 0.1$ $2.61 = 261 \times 0.01.$ $21.2 = 212 \times 0.1.$

$$\begin{array}{r|l} 2 & 146 \\ & 73 \end{array} \qquad \begin{array}{r|l} 3^2 & 261 \\ & 29 \end{array} \qquad \begin{array}{r|l} 2^2 & 212 \\ & 53 \end{array}$$

$2 \times 73 \times 0.1.$ *Ans.* $3^2 \times 29 \times 0.01.$ *Ans.* $2^2 \times 53 \times 0.1.$ *Ans.*

$78.54 = 7854 \times 0.01.$ $0.5236 = 5236 \times 0.0001.$

$$\begin{array}{r|l} 2 & 7854 \\ 3 & 3927 \\ 7 & 1309 \\ 11 & 187 \\ & 17 \end{array} \qquad \begin{array}{r|l} 2^2 & 5236 \\ 7 & 1309 \\ 11 & 187 \\ & 17 \end{array}$$

$2^2 \times 7 \times 11 \times 17 \times 0.0001.$ *Ans.*

$2 \times 3 \times 7 \times 11 \times 17 \times 0.01.$ *Ans.*

$0.00052 = 52 \times 0.00001$

$$\begin{array}{r|l} 2^2 & 52 \\ & 13 \end{array}$$

$2^2 \times 13 \times 0.00001.$ *Ans.*

3. Find the prime factors of 86.7; 48.3; 99.99; 5.04; 1.485; 0.216.

$86.7 = 867 \times 0.1.$ $48.3 = 483 \times 0.1.$ $99.99 = 9999 \times 0.01.$

$$\begin{array}{r|l} 3 & 867 \\ \hline 17^2 & 289 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 483 \\ \hline 7 & 161 \\ \hline & 23 \end{array} \qquad \begin{array}{r|l} 3^2 & 9999 \\ \hline 11 & 1111 \\ \hline & 101 \end{array}$$

$3 \times 17^2 \times 0.1$. *Ans.* $3 \times 7 \times 23 \times 0.1$. *Ans.* $3^2 \times 11 \times 101 \times 0.01$. *Ans.*

$5.04 = 504 \times 0.01.$ $1.485 = 1485 \times 0.001.$ $0.216 = 216 \times 0.001.$

$$\begin{array}{r|l} 2^3 & 504 \\ \hline 3^2 & 63 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3^3 & 1485 \\ \hline 5 & 55 \\ \hline & 11 \end{array} \qquad \begin{array}{r|l} 2^3 & 216 \\ \hline 3^3 & 27 \\ \hline & 1 \end{array}$$

$2^3 \times 3^2 \times 7 \times 0.01$. *Ans.* $3^3 \times 5 \times 11 \times 0.001$. *Ans.* $2^3 \times 3^3 \times 0.001$. *Ans.*

4. Find the prime factors of 34.87; 32.4; 5.115; 71.2; 2.993.

$34.87 = 3487 \times 0.01.$ $32.4 = 324 \times 0.1.$ $5.115 = 5115 \times 0.001.$

$$\begin{array}{r|l} 11 & 3487 \\ \hline & 317 \end{array} \qquad \begin{array}{r|l} 2^2 & 324 \\ \hline 3^4 & 81 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 5115 \\ \hline 5 & 1705 \\ \hline 11 & 341 \\ \hline & 31 \end{array}$$

$11 \times 317 \times 0.01$. *Ans.*

$2^2 \times 3^4 \times 0.1$. *Ans.*

$3 \times 5 \times 11 \times 31 \times 0.001$. *Ans.*

$71.2 = 712 \times 0.1.$ $2.993 = 2993 \times 0.001.$

$$\begin{array}{r|l} 2^3 & 712 \\ \hline & 89 \end{array} \qquad \begin{array}{r|l} 41 & 2993 \\ \hline & 73 \end{array}$$

$2^3 \times 89 \times 0.1$. *Ans.* $41 \times 73 \times 0.001$. *Ans.*

Exercise XIII.

1. Find the G. C. M. of 27 and 33.

$$\begin{array}{r|ll} 3 & 27 & 33 \\ \hline & 9 & 11 \end{array}$$

3. *Ans.*

2. Find the G. C. M. of 13 and 39.

$$\begin{array}{r|ll} 13 & 13 & 39 \\ \hline & 1 & 3 \end{array}$$

13. *Ans.*

3. Find the G. C. M. of 8 and 28.

$$\begin{array}{r|ll} 2^2 & 8 & 28 \\ \hline & 2 & 7 \end{array}$$

$2^2 = 4$. *Ans.*

4. Find the G. C. M. of 27 and 45.

$$\begin{array}{r|ll} 3^2 & 27 & 45 \\ \hline & 3 & 5 \end{array}$$

$3^2 = 9$. *Ans.*

5. Find the G.C.M. of 81 and 108.

$$\begin{array}{r|rr} 3^3 & 81 & 108 \\ \hline & 3 & 4 \end{array}$$

$3^3 = 27$. *Ans.*

6. Find the G.C.M. of 4, 10, 12.

$$\begin{array}{r|rrr} 2 & 4 & 10 & 12 \\ \hline & 2 & 5 & 6 \end{array}$$

2. *Ans.*

7. Find the G.C.M. of 4, 6, 10.

$$\begin{array}{r|rrr} 2 & 4 & 6 & 10 \\ \hline & 2 & 3 & 5 \end{array}$$

2. *Ans.*

8. Find the G.C.M. of 9, 12, 21.

$$\begin{array}{r|rrr} 3 & 9 & 12 & 21 \\ \hline & 3 & 4 & 7 \end{array}$$

3. *Ans.*

9. Find the G.C.M. of 10, 15, 25.

$$\begin{array}{r|rrr} 5 & 10 & 15 & 25 \\ \hline & 2 & 3 & 5 \end{array}$$

5. *Ans.*

10. Find the G.C.M. of 14, 98, 42.

$$\begin{array}{r|rrr} 2 & 14 & 98 & 42 \\ \hline 7 & 7 & 49 & 21 \\ \hline & 1 & 7 & 3 \end{array}$$

$2 \times 7 = 14$. *Ans.*

11. Find the G.C.M. of 30, 18, 54.

$$\begin{array}{r|rrr} 2 & 30 & 18 & 54 \\ \hline 3 & 15 & 9 & 27 \\ \hline & 5 & 3 & 9 \end{array}$$

$2 \times 3 = 6$. *Ans.*

12. Find the G.C.M. of 14, 56, 42.

$$\begin{array}{r|rrr} 2 & 14 & 56 & 42 \\ \hline 7 & 7 & 28 & 21 \\ \hline & 1 & 4 & 3 \end{array}$$

$2 \times 7 = 14$. *Ans.*

13. Find the G.C.M. of 96, 36, 48.

$$\begin{array}{r|rrr} 2^2 & 96 & 36 & 48 \\ \hline 3 & 24 & 9 & 12 \\ \hline & 8 & 3 & 4 \end{array}$$

$2^2 \times 3 = 12$. *Ans.*

14. Find the G.C.M. of 84, 105, 63.

$$\begin{array}{r|rrr} 3 & 84 & 105 & 63 \\ \hline 7 & 28 & 35 & 21 \\ \hline & 4 & 5 & 3 \end{array}$$

$3 \times 7 = 21$. *Ans.*

15. Find the G.C.M. of 24, 60, 84, 128.

$$\begin{array}{r|rrrr} 2^2 & 24 & 60 & 84 & 128 \\ \hline & 6 & 15 & 21 & 32 \end{array}$$

$2^2 = 4$. *Ans.*

16. Find the G.C.M. of 45, 81, 27, 90.

$$\begin{array}{r|rrrr} 3^2 & 45 & 81 & 27 & 90 \\ \hline & 5 & 9 & 3 & 10 \end{array}$$

$3^2 = 9$. *Ans.*

17. Find the G.C.M. of 78, 18, 54, 42.

$$\begin{array}{r|rrrr} 2 & 78 & 18 & 54 & 42 \\ \hline 3 & 39 & 9 & 27 & 21 \\ \hline & 13 & 3 & 9 & 7 \end{array}$$

$2 \times 3 = 6$. *Ans.*

18. Find the G. C. M. of 98, 28, 70, 42.

$$\begin{array}{r|rrrr} 2 & 98 & 28 & 70 & 42 \\ \hline 7 & 49 & 14 & 35 & 21 \\ \hline & 7 & 2 & 5 & 3 \end{array}$$

$2 \times 7 = 14$. *Ans.*

19. Find the G. C. M. of 96, 112, 80, 32.

$$\begin{array}{r|rrrr} 2^4 & 96 & 112 & 80 & 32 \\ \hline & 6 & 7 & 5 & 2 \end{array}$$

$2^4 = 16$. *Ans.*

20. Find the G. C. M. of 24, 96, 48, 120.

$$\begin{array}{r|rrrr} 2^3 & 24 & 96 & 48 & 120 \\ \hline 3 & 3 & 12 & 6 & 15 \\ \hline & 1 & 4 & 2 & 5 \end{array}$$

$2^3 \times 3 = 24$. *Ans.*

21. Find the G. C. M. of 84, 252, 168, 210.

$$\begin{array}{r|rrrr} 2 & 84 & 252 & 168 & 210 \\ \hline 3 & 42 & 126 & 84 & 105 \\ \hline 7 & 14 & 42 & 28 & 35 \\ \hline & 2 & 6 & 4 & 5 \end{array}$$

$2 \times 3 \times 7 = 42$. *Ans.*

22. Find the G. C. M. of 33, 88, 77, 55.

$$\begin{array}{r|rrrr} 11 & 33 & 88 & 77 & 55 \\ \hline & 3 & 8 & 7 & 5 \end{array}$$

11. *Ans.*

23. Find the G. C. M. of 252, 315, 420, 504.

$$\begin{array}{r|rrrr} 3 & 252 & 315 & 420 & 504 \\ \hline 7 & 84 & 105 & 140 & 168 \\ \hline & 12 & 15 & 20 & 24 \end{array}$$

$3 \times 7 = 21$. *Ans.*

24. Find the G. C. M. of 128, 192, 320, 368, 432.

$$\begin{array}{r|rrrrr} 2^4 & 128 & 192 & 320 & 368 & 432 \\ \hline & 8 & 12 & 20 & 23 & 27 \end{array}$$

$2^4 = 16$. *Ans.*

25. Find the G. C. M. of 136, 204, 357, 459.

$$\begin{array}{r|rrrr} 17 & 136 & 204 & 357 & 459 \\ \hline & 8 & 12 & 21 & 27 \end{array}$$

17. *Ans.*

26. Find the G. C. M. of 909, 1414, 2323, 4242.

$$\begin{array}{r|rrrr} 101 & 909 & 1414 & 2323 & 4242 \\ \hline & 9 & 14 & 23 & 42 \end{array}$$

101. *Ans.*

Exercise XIV.

1. Find the G.C.M. of 2479 and 3589.

$$\begin{array}{l} 2479)3589(1 \\ \quad\;\; \underline{2479} \\ 10\,|\,\overline{1110} \\ \;\;3\,|\,\underline{111} \\ \qquad\quad 37)2479(67 \\ \qquad\qquad\;\; \underline{222} \\ \qquad\qquad\;\; 259 \\ \qquad\qquad\;\; \underline{259} \end{array}$$

37. *Ans.*

2. Find the G.C.M. of 3045 and 6195.

$$\begin{array}{r|rr} 5 & 3045 & 6195 \\ \hline 3 & 609 & 1239 \\ \hline & 203 & \end{array}$$

$$\begin{array}{l} 203)413(2 \\ \qquad \underline{406} \\ \qquad\;\; 7)203(29 \\ \qquad\qquad \underline{14} \\ \qquad\qquad\; 63 \\ \qquad\qquad\; \underline{63} \end{array}$$

$5 \times 3 \times 7 = 105$. *Ans.*

3. Find the G. C. M. of 568 and 712.

8 | 568   712
71) 89(1
71
2 | 18
3^2 | 9
1

8. *Ans.*

4. Find the G. C. M. of 11,023 and 6493.

6493) 11023 (1
6493
10 | 4530
3 | 453
151) 6493 (43
604
453
453

151. *Ans.*

5. Find the G. C. M. of 1485 and 2160.

5 | 1485   2160
3^3 | 297   432
11   16

$5 \times 3^3 = 135$. *Ans.*

6. Find the G. C. M. of 7040 and 7392.

32 | 7040   7392
10 | 220   231
2 | 22
11) 231 (21
22
11
11

$32 \times 11 = 352$. *Ans.*

7. Find the G. C. M. of 2760 and 4485.

3 | 2760   4485
5 | 920   1495
8 | 184   299
23) 299 (13
23
69
69

$3 \times 5 \times 23 = 345$. *Ans.*

8. Find the G. C. M. of 1177 and 2675.

11 | 1177
107) 2675 (25
214
535
535

107. *Ans.*

9. Find the G. C. M. of 78,473 and 94,653.

78473) 94653 (1
78473
10 | 16180
2 | 1618
809) 78473 (97
7281
5663
5663

809. *Ans.*

10. Find the G.C.M. of 35,143 and 10,283.

```
10283)35143(3
      30849
    2| 4294
   19| 2147
        113)10283(91
             1017
              113
              113
```

113. *Ans.*

11. Find the G.C.M. of 44,323 and 61,087.

```
44323)61087(1
      44323
    4|16764
    3| 4191
   11| 1397
        127)44323(349
             381
             622
             508
             1143
             1143
```

127. *Ans.*

12. Find the G.C.M. of 232,353 and 39,699.

```
11|39699   232353
 9| 3609    21123
    401  )  2347(5
            2005
          6| 342
          3|  57
              19)401(21
                  38
                   21
                   19
                    2)19(9
                      18
                       1
```

$11 \times 9 = 99$. *Ans.*

13. Find the G.C.M. of 33,853 and 35,017.

```
33853)35017(1
      33853
    4| 1164
    3|  291
         97)33853(349
            291
             475
             388
              873
              873
```

97. *Ans.*

14. Find the G.C.M. of 5115 and 7254.

```
 3|5115   7254
 5|1705   2418
11| 341
     31)2418(78
        217
         248
         248
```

$3 \times 31 = 93$. *Ans.*

15. Find the G.C.M. of 2268 and 3348.

```
4|2268   3348
9| 567    837
3|  63     93
    21     31
```

$4 \times 9 \times 3 = 108$. *Ans.*

16. Find the G. C. M. of 1003 and 2419.

```
1003)2419(2
     2006
   7| 413
       59)1003(17
            59
           413
59. Ans.   413
```

17. Find the G. C. M. of 419 and 52,301.

```
419)52301(124
    419
    1040
     838
     2021
     1676
   3| 345
   5| 115
   3|  21
   7|   7
        1|419
1. Ans.   419
```

18. Find the G. C. M. of 30,072 and 133,784.

```
8|30072    133784
7| 3759     16723
3|  537      2389
   179)2389(13
       179
       599
       537
     2| 42
     3| 21
     7|  7
        1|179
1. Ans.   179
```

19. Find the G. C. M. of 4527 and 10,836.

```
 9|4527     10836
11| 473   7|1204
     43  )   172(4
             172
9 × 43 = 387. Ans.
```

20. Find the G. C. M. of 17,104 and 27,794.

```
2|17104   27794
8| 8552   13897
  1069)13897(13
       1069
       3207
       3207
2 × 1069 = 2168. Ans.
```

Exercise XV.

1. Find the G. C. M. of 855, 1197, 1596.

3 | 855   1197   1596
3 | 285   399   4 | 532
5 | 95   7 | 133   7 | 133
19   19   19

$3 \times 19 = 57$. *Ans.*

2. Find the G. C. M. of 3864, 3404, 3657.

4 | 3864   3404   3)3657
6 | 966   851   1219
7 | 161
23)   851   1219
37   53

23. *Ans.*

3. Find the G. C. M. of 15,561, 11,115, 13,585.

13 | 11115   13585   15561
9 | 855   5 | 1045   7 | 1197
5 | 95   11 | 209   9 | 171
19   19   19

$13 \times 19 = 247$. *Ans.*

4. Find the G. C. M. of 2943, 2616, 4578.

3 | 2943   2616   4578
9 | 981   8 | 872   2 | 1526
109   109   7 | 763
109

$3 \times 109 = 327$. *Ans.*

5. Find the G. C. M. of 1177, 1391, 1819.

11 | 1177
107) 1819 (17
107
749
749

107) 1391 (13
107
321
321

107. *Ans.*

6. Find the G. C. M. of 4939, 1347, 3143.

11 | 4939
449) 1347 (3
1347

449) 3143 (7
3143

449. *Ans.*

7. Find the G. C. M. of 740, 333, 296.

2 | 740   9 | 333   8 | 296
10 | 370   37   37
37

37. *Ans.*

8. Find the G. C. M. of 833, 1785, 1309.

7 | 833   3 | 1785   11 | 1309
119   5 | 595   119
119

119. *Ans.*

9. Find the G. C. M. of 7326, 8547, 9768, 22,755.

$$\begin{array}{r|l} 6 & 7326 \\ \hline 11 & 1221 \\ \hline & 111 \end{array} \quad \begin{array}{r|l} 7 & 8547 \\ \hline 11 & 1221 \\ \hline & 111 \end{array} \quad \begin{array}{r|l} 8 & 9768 \\ \hline 11 & 1221 \\ \hline & 111 \end{array} \quad \begin{array}{r|l} 5 & 22755 \\ \hline 41 & 4551 \\ \hline & 111 \end{array}$$

111. *Ans.*

10. Find the G. C. M. of 4994, 7491, 9988, 12,485, 16,571.

$$\begin{array}{r|l} 2 & 4994 \\ \hline 11 & 2497 \\ \hline & 227 \end{array} \quad \begin{array}{r|l} 3 & 7491 \\ \hline 11 & 2497 \\ \hline & 227 \end{array} \quad \begin{array}{r|l} 4 & 9988 \\ \hline 11 & 2497 \\ \hline & 227 \end{array} \quad \begin{array}{r|l} 5 & 12485 \\ \hline 11 & 2497 \\ \hline & 227 \end{array} \quad \begin{array}{r} 73 \\ 227\overline{)16571} \\ \underline{1589} \\ 681 \\ \underline{681} \end{array}$$

227. *Ans.*

Exercise XVI.

1. Find the L. C. M. of 6, 14, 21.

$$\begin{array}{r|lll} 2 & 6 & 14 & 21 \\ \hline & \cancel{3} & \cancel{7} & 21 \end{array}$$

$2 \times 3 \times 7 = 42$. *Ans.*

2. Find the L. C. M. of 8, 12, 3, 24.

$\cancel{8}$ $\cancel{12}$ $\cancel{3}$ 24.

24. *Ans.*

3. Find the L. C. M. of 6, 10, 15.

$$\begin{array}{r|lll} 2 & 6 & 10 & 15 \\ \hline & \cancel{3} & \cancel{5} & 15 \end{array}$$

$2 \times 3 \times 5 = 30$. *Ans.*

4. Find the L. C. M. of 9, 12, 18, 4.

$$\begin{array}{r|llll} 2 & \cancel{9} & 12 & 18 & \cancel{4} \\ \hline 3 & & 6 & 9 & \\ \hline & & 2 & 3 & \end{array}$$

$2^2 \times 3^2 = 36$. *Ans.*

5. Find the L. C. M. of 15, 21, 35.

$$\begin{array}{r|lll} 3 & 15 & 21 & 35 \\ \hline & \cancel{5} & \cancel{7} & 35 \end{array}$$

$3 \times 5 \times 7 = 105$. *Ans.*

6. Find the L. C. M. of 12, 20, 24.

$$\begin{array}{r|lll} 2 & \cancel{12} & 20 & 24 \\ \hline 2 & & 10 & 12 \\ \hline 2 & & 5 & 6 \\ \hline & & 5 & 3 \end{array}$$

$2^3 \times 3 \times 5 = 120$. *Ans.*

7. Find the L. C. M. of 14, 24, 28.

$$\begin{array}{r|lll} 2^2 & \cancel{14} & 24 & 28 \\ \hline & & 6 & 7 \end{array}$$

$2^3 \times 3 \times 7 = 168$. *Ans.*

8. Find the L. C. M. of 12, 15, 20.

$$\begin{array}{r|lll} 3 & 12 & 15 & 20 \\ \hline & \cancel{4} & \cancel{5} & 20 \end{array}$$

$2^2 \times 3 \times 5 = 60$. *Ans.*

9. Find the L. C. M. of 16, 24, 32.

$$\begin{array}{r|rrr} 2^3 & \cancel{16} & 24 & 32 \\ \hline & & 3 & 4 \end{array}$$

$2^5 \times 3 = 96$. *Ans.*

10. Find the L. C. M. of 21, 33, 77.

$$\begin{array}{r|rrr} 3 & 21 & 33 & 77 \\ \hline & \cancel{7} & \cancel{11} & 77 \end{array}$$

$3 \times 7 \times 11 = 231$. *Ans.*

11. Find the L. C. M. of 27, 33, 99.

$$\begin{array}{r|rrr} 3^2 & 27 & \cancel{33} & 99 \\ \hline & 3 & & 11 \end{array}$$

$3^3 \times 11 = 297$. *Ans.*

12. Find the L. C. M. of 7, 11, 13.

$$\begin{array}{r|rrr} & 7 & 11 & 13 \\ \hline \end{array}$$

$7 \times 11 \times 13 = 1001$. *Ans.*

13. Find the L. C. M. of 77, 55, 35.

$$\begin{array}{r|rrr} 5 & 77 & 55 & 35 \\ \hline & 77 & \cancel{11} & \cancel{7} \end{array}$$

$5 \times 7 \times 11 = 385$. *Ans.*

14. Find the L. C. M. of 16, 18, 27, 72.

$$\begin{array}{r|rrrr} 2^3 & 16 & \cancel{18} & 27 & 72 \\ \hline & 2 & & 27 & \cancel{9} \end{array}$$

$2^4 \times 3^3 = 432$. *Ans.*

15. Find the L. C. M. of 10, 12, 22, 33, 60.

$$\begin{array}{r|rrrrr} 2 & \cancel{10} & \cancel{12} & 22 & 33 & 60 \\ 3 & & & \cancel{11} & 33 & 30 \\ \hline & & & & 11 & 10 \end{array}$$

$2^2 \times 3 \times 5 \times 11 = 660$. *Ans.*

16. Find the L. C. M. of 15, 16, 18, 20, 22, 24.

$$\begin{array}{r|rrrrrr} 2 & 15 & 16 & 18 & 20 & 22 & 24 \\ \hline 2 & 15 & 8 & 9 & 10 & 11 & 12 \\ \hline 2 & 15 & 4 & 9 & \cancel{5} & 11 & 6 \\ \hline 3 & 15 & 2 & 9 & & 11 & \cancel{3} \\ \hline & 5 & 2 & 3 & & 11 & \end{array}$$

$2^4 \times 3^2 \times 5 \times 11 = 7920$. *Ans.*

17. Find the L. C. M. of 56, 64, 70, 84, 112.

$$\begin{array}{r|rrrrr} 2 & \cancel{56} & 64 & 70 & 84 & 112 \\ \hline 2 & & 32 & 35 & 42 & 56 \\ \hline 2^2 & & 16 & 35 & 21 & 28 \\ \hline 7 & & 4 & 35 & 21 & \cancel{7} \\ \hline & & 4 & 5 & 3 & \end{array}$$

$2^6 \times 3 \times 5 \times 7 = 6720$. *Ans.*

18. Find the L. C. M. of 48, 54, 81, 144, 162.

$$\begin{array}{r|rrrrr} 2 & \cancel{48} & \cancel{54} & \cancel{81} & 144 & 162 \\ \hline 3^2 & & & & 72 & 81 \\ \hline & & & & 8 & 9 \end{array}$$

$2^4 \times 3^4 = 1296$. *Ans.*

19. Find the L. C. M. of 75, 100, 120, 150, 180.

$$\begin{array}{r|rrrrr} 10 & \cancel{75} & 100 & 120 & 150 & 180 \\ \hline 2 & & 10 & 12 & 15 & 18 \\ \hline 3 & & \cancel{5} & 6 & 15 & 9 \\ \hline & & & 2 & 5 & 3 \end{array}$$

$2^3 \times 3^2 \times 5^2 = 1800$. *Ans.*

20. Find the L. C. M. of 112, 168, 196, 224.

$$\begin{array}{r|rrrr}
2^2 & \cancel{112} & 168 & 196 & 224 \\ \hline
2 & & 42 & 49 & 56 \\ \hline
7 & & 21 & 49 & 28 \\ \hline
 & & 3 & 7 & 4
\end{array}$$

$2^5 \times 3 \times 7^2 = 4704$. *Ans.*

21. Find the L. C. M. of 7, 14, 15, 21, 45.

$$\begin{array}{r|rrrrr}
3 & \cancel{7} & 14 & \cancel{15} & 21 & 45 \\ \hline
 & & 14 & & \cancel{7} & 15
\end{array}$$

$2 \times 3^2 \times 5 \times 7 = 630$. *Ans.*

22. Find the L. C. M. of 16, 25, 81.

$$\begin{array}{|rrr}
16 & 25 & 81 \\ \hline
\end{array}$$

$16 \times 25 \times 81 = 32{,}400$. *Ans.*

23. Find the L. C. M. of 26, 39, 52, 65.

$$\begin{array}{r|rrrr}
13 & \cancel{26} & 39 & 52 & 65 \\ \hline
 & & 3 & 4 & 5
\end{array}$$

$2^2 \times 3 \times 5 \times 13 = 780$. *Ans.*

24. Find the L. C. M. of 80, 72, 225, 48.

$$\begin{array}{r|rrrr}
2^3 & 80 & 72 & 225 & 48 \\ \hline
2 & 10 & 9 & 225 & 6 \\ \hline
 & \cancel{5} & \cancel{9} & 255 & \cancel{3}
\end{array}$$

$2^4 \times 3^2 \times 5^2 = 3600$. *Ans.*

25. Find the L. C. M. of 10, 20, 30, 40, 50, 60.

$$\begin{array}{r|rrrrrr}
2 & \cancel{10} & \cancel{20} & \cancel{30} & 40 & 50 & 60 \\ \hline
2 & & & & 20 & 25 & 30 \\ \hline
5 & & & & 10 & 25 & 15 \\ \hline
 & & & & 2 & 5 & 3
\end{array}$$

$2^3 \times 3 \times 5^2 = 600$. *Ans.*

26. Find the L. C. M. of 30, 42, 105, 70.

$$\begin{array}{r|rrrr}
2 & 30 & 42 & 105 & 70 \\ \hline
 & \cancel{15} & \cancel{21} & 105 & \cancel{35}
\end{array}$$

$2 \times 3 \times 5 \times 7 = 210$. *Ans.*

27. Find the L. C. M. of 36, 24, 35, 20.

$$\begin{array}{r|rrrr}
2^2 & 36 & 24 & 35 & 20 \\ \hline
3 & 9 & 6 & 35 & \cancel{5} \\ \hline
 & 3 & 2 & 35 &
\end{array}$$

$2^3 \times 3^2 \times 5 \times 7 = 2520$. *Ans.*

28. Find the L. C. M. of 7, 11, 14, 15.

$$\begin{array}{|rrrr}
\cancel{7} & 11 & 14 & 15 \\ \hline
\end{array}$$

$2 \times 7 \times 11 \times 3 \times 5 = 2310$. *Ans.*

29. Find the L. C. M. of 12, 18, 27, 63, 28.

$$\begin{array}{r|rrrrr}
2 & 12 & 18 & 27 & 63 & 28 \\ \hline
2 & 6 & \cancel{9} & 27 & 63 & 14 \\ \hline
3^2 & \cancel{3} & & 27 & 63 & \cancel{7} \\ \hline
 & & & 3 & 7 &
\end{array}$$

$2^2 \times 3^3 \times 7 = 756$. *Ans.*

30. Find the L. C. M. of 34, 26, 65, 85, 51, 39.

$$\begin{array}{r|rrrrrr}
2 & 34 & 26 & 65 & 85 & 51 & 39 \\ \hline
5 & \cancel{17} & \cancel{13} & 65 & 85 & 51 & 39 \\ \hline
3 & & & \cancel{13} & \cancel{17} & 51 & 39 \\ \hline
 & & & & & 17 & 13
\end{array}$$

$2 \times 3 \times 5 \times 13 \times 17 = 6630$. *Ans.*

31. Find the L. C. M. of 12, 18, 96, 144.

$$\begin{array}{r|rrrr}
2^3 & \cancel{12} & \cancel{18} & 96 & 144 \\ \hline
2 & & & 12 & 18 \\ \hline
3 & & & 6 & 9 \\ \hline
 & & & 2 & 3
\end{array}$$

$2^5 \times 3^2 = 288$. *Ans.*

32. Find the L. C. M. of 84, 156, 63, 99.

2^2	84	156	63	99
3	~~21~~	39	63	99
3		13	21	33
		13	7	11

$2^2 \times 3^2 \times 7 \times 11 \times 13 = 36,036$. *Ans.*

33. Find the L. C. M. of 17, 51, 119, 210.

17	~~17~~	51	119	210
		~~3~~	~~7~~	210

$2 \times 3 \times 7 \times 5 \times 17 = 3570$. *Ans.*

34. Find the L. C. M. of 16, 30, 48, 56, 72.

2	~~16~~	30	48	56	72
2^2		15	24	28	36
3		15	6	7	9
		5	2	7	3

$2^4 \times 3^2 \times 5 \times 7 = 5040$. *Ans.*

35. Find the L. C. M. of 27, 33, 54, 69, 132.

2	~~27~~	~~33~~	54	69	132
3			27	69	66
			9	23	22

$2^2 \times 3^3 \times 11 \times 33 = 27,324$. *Ans.*

36. Find the L. C. M. of 15, 26, 39, 65, 180.

2	~~15~~	26	39	65	180
3		~~13~~	39	65	90
5			~~13~~	65	30
				13	6

$2^2 \times 3^2 \times 5 \times 13 = 2340$. *Ans.*

37. Find the L. C. M. of 44, 126, 198, 280, 330.

2	44	126	198	280	330
2	22	63	99	140	165
3	~~11~~	63	99	70	165
7		21	33	70	55
5		~~3~~	33	10	55
			33	2	~~11~~

$2^3 \times 3^2 \times 5 \times 7 \times 11 = 27,720$. *Ans.*

38. Find the L. C. M. of 50, 338, 675, 975.

5	50	338	675	975
5	10	338	135	195
3	~~2~~	338	27	39
		338	9	~~13~~

$2 \times 3^3 \times 5^2 \times 13^2 = 228,150$. *Ans.*

39. Find the L. C. M. of 552, 575, 920.

2^3	552	575	920
5	69	575	115
2^3	69	115	~~23~~
	3	5	

$2^3 \times 3 \times 5^2 \times 23 = 13,800$. *Ans.*

40. Find the L. C. M. of 228, 304, 342.

2	228	304	342
2	114	152	171
19	~~57~~	76	171
		4	9

$2^4 \times 3^2 \times 19 = 2736$. *Ans.*

41. Find the L. C. M. of 1080 and 1260.

10	1080	1260
2	108	126
3^2	54	63
	6	7

$2^3 \times 3^3 \times 5 \times 7 = 7560$. *Ans.*

42. Find the L. C. M. of 600 and 480.

$$\begin{array}{r|rr} 2^3 & 600 & 480 \\ \hline 3 & 75 & 60 \\ \hline 5 & 25 & 20 \\ \hline & 5 & 4 \end{array}$$

$2^5 \times 3 \times 5^2 = 2400$. *Ans.*

43. Find the L. C. M. of 1564 and 1932.

$$\begin{array}{r|rr} 2^2 & 1564 & 1932 \\ \hline 23 & 391 & 483 \\ \hline & 17 & 21 \end{array}$$

$2^2 \times 17 \times 3 \times 7 \times 23 = 32{,}844$. *Ans.*

44. Find the L. C. M. of 2530 and 1760.

$$\begin{array}{r|rr} 2 & 2530 & 1760 \\ \hline 5 & 1265 & 880 \\ \hline 11 & 253 & 176 \\ \hline & 23 & 16 \end{array}$$

$2^5 \times 5 \times 11 \times 23 = 40{,}480$. *Ans.*

45. Find the L. C. M. of 936 and 2925.

$$\begin{array}{r|rr} 3^2 & 936 & 2925 \\ \hline 13 & 104 & 325 \\ \hline & 8 & 25 \end{array}$$

$2^3 \times 3^2 \times 5^2 \times 13 = 23{,}400$. *Ans.*

46. Find the L. C. M. of 3432 and 4032.

$$\begin{array}{r|rr} 2^3 & 3432 & 4032 \\ \hline 3 & 429 & 504 \\ \hline & 143 & 168 \end{array}$$

$2^4 \times 3^5 \times 11 \times 13 = 576{,}576$. *Ans.*

47. Find the L. C. M. of 1875 and 2425.

$$\begin{array}{r|rr} 5^2 & 1875 & 2425 \\ \hline & 75 & 97 \end{array}$$

$3 \times 5^4 \times 97 = 181{,}875$. *Ans.*

48. Find the L. C. M. of 1632 and 2976.

$$\begin{array}{r|rr} 2^3 & 1632 & 2976 \\ \hline 2^2 & 204 & 372 \\ \hline 3 & 51 & 93 \\ \hline & 17 & 31 \end{array}$$

$2^5 \times 3 \times 17 \times 31 = 50{,}592$. *Ans.*

49. Find the L. C. M. of 1001 and 2233.

$$\begin{array}{r|rr} 11 & 1001 & 2233 \\ \hline 7 & 91 & 203 \\ \hline & 13 & 29 \end{array}$$

$7 \times 11 \times 13 \times 29 = 29{,}029$. *Ans.*

50. Find the L. C. M. of 539 and 1463.

$$\begin{array}{r|rr} 7 & 539 & 1463 \\ \hline 11 & 77 & 209 \\ \hline & 7 & 19 \end{array}$$

$7^2 \times 11 \times 19 = 10{,}241$. *Ans.*

EXERCISE XVII.

1. Find the L. C. M. of 424 and 583.

8|424
53)583(11
583

G. C. M. = 53.
L. C. M. = 11 × 424 = 4664. *Ans.*

2. Find the L. C. M. of 319 and 407.

11|319 407
29 37

G. C. M. = 11.
L. C. M. = 29 × 407 = 11,803. *Ans.*

3. Find the L. C. M. of 1679 and 1932.

4|1932
3|483
7|161
23)1679(73
161
69
69

G. C. M. = 23.
L. C. M. = 73 × 1932 = 141,036. *Ans.*

4. Find the L. C. M. of 1003 and 2419.

1003)2419(2
2006
7|413
59)1003(17
59
413
413

L. C. M. = 17 × 2419 = 41,123. *Ans.*

5. Find the L. C. M. of 1003 and 1357.

1003)1357(1
1003
6|354
59)1003(17
59
413
413

L. C. M. = 17 × 1357 = 23,069. *Ans.*

6. Find the L. C. M. of 899 and 961.

899)961(1
899
2|62
31)899(29
62
279
279

L. C. M. = 29 × 961 = 27,869. *Ans.*

7. Find the L. C. M. of 407, 703, 444.

11|407
37)703(19
37
333
333

L. C. M. = 11 × 19 × 444 = 92,796. *Ans.*

8. Find the L. C. M. of 411, 959, 2055.

$\cancel{411}$ 959 2055

959)2055(2
1918
137)959(7
959

L. C. M. = 7 × 2055 = 14,385. *Ans.*

9. Find the L. C. M. of 221 and 351.

221)351(1
221
10 | 130
13)221(17
221

L. C. M. = 17 × 351 = 5967. *Ans.*

10. Find the L. C. M. of 1426 and 989.

2 | 1426
713)989(1
713
4 | 276
3 | 69
23)713(31
69
23
23

L.C.M. = 2 × 31 × 989 = 61,318. *Ans.*

11. Find the L. C. M. of 3864, 3404, 3657.

2^2	3864	3404	3657
3	966	851	3657
23	322	851	1219
	14	37	53

2 | 322
7 | 161
23)851(37
69
161
161

$2^3 \times 3 \times 7 \times 23 \times 37 \times 53$
= 7,577,304. *Ans.*

12. Find the L. C. M. of 539 and 253.

11	253	539
	23	49

L. C. M. = 23 × 539 = 12,397. *Ans.*

13. Find the L. C. M. of 2943, 2616, 4578.

8 | 2616
327)2943(9
2943

2	2616	2943	4578
327	1308	2943	2289
	4	9	7

2 × 4 × 7 × 9 × 327 = 164,808. *Ans.*

14. Find the L. C. M. of 2863 and 1151.

L. C. M. = 1151 × 2863
= 3,295,313. *Ans.*

15. Find the L. C. M. of 1177, 1391, 1819.

107	1177	1391	1819
	11	13	17

```
11|1177
   107)1391(13
       107
        321
        321
```

L. C. M. $=13\times17\times1177=260,117$. *Ans.*

16. Find the L. C. M. of 5317 and 2863.

```
7|2863
   409)5317(13
       409
       1227
       1227
```

L. C. M. $=13\times2863=37,219$. *Ans.*

17. Find the L. C. M. of 12,703 and 12,879.

L. C. M. $= 12,703 \times 12,879$
$= 163,601,937$. *Ans.*

18. Find the L. C. M. of 23,309 and 10,753.

L. C. M. $= 10,753 \times 23,309$
$= 240,631,677$. *Ans.*

19. Find the L. C. M. of 4939 and 3143.

```
7|3143
   449)4939(11
       449
        449
        449
```

L. C. M. $=11\times3143=34,573$. *Ans.*

20. Find the L. C. M. of 4199 and 6137.

```
13|4199
    323)6137(19
        323
        2907
        2907
```

L. C. M. $=19\times4199=79,891$. *Ans.*

Exercise XVIII.

1. Reduce to whole or mixed numbers $\frac{13}{7}$.

$\frac{13}{7} = 1\frac{6}{7}$. *Ans.*

2. Reduce to whole or mixed numbers $\frac{21}{8}$.

$\frac{21}{8} = 2\frac{5}{8}$. *Ans.*

3. Reduce to whole or mixed numbers $\frac{25}{4}$.

$\frac{25}{4} = 6\frac{1}{4}$. *Ans.*

4. Reduce to whole or mixed numbers $\frac{107}{11}$.

$\frac{107}{11} = 9\frac{8}{11}$. *Ans.*

5. Reduce to whole or mixed numbers $\frac{213}{16}$.

$\frac{213}{16} = 13\frac{5}{16}$. *Ans.*

6. Reduce to whole or mixed numbers $\frac{242}{22}$.

$\frac{242}{22} = 11$. *Ans.*

7. Reduce to whole or mixed numbers $\frac{374}{25}$.

$\frac{374}{25} = 14\frac{24}{25}$. *Ans.*

8. Reduce to whole or mixed numbers $\frac{481}{13}$.

$\frac{481}{13} = 37$. *Ans.*

9. Reduce to whole or mixed numbers $\frac{3828}{75}$.

$\frac{3828}{75} = 50\frac{78}{75}$. *Ans.*

10. Reduce to whole or mixed numbers $\frac{4684}{177}$.

$\frac{4684}{177} = 26\frac{82}{177}$. *Ans.*

11. Reduce to whole or mixed numbers $\frac{2829}{753}$.

$\frac{2829}{753} = 3\frac{570}{753}$. *Ans.*

12. Reduce to whole or mixed numbers $\frac{299}{23}$.

$\frac{299}{23} = 13$. *Ans.*

13. Reduce to whole or mixed numbers $\frac{529}{29}$.

$\frac{529}{29} = 18\frac{7}{29}$. *Ans.*

14. Reduce to whole or mixed numbers $\frac{786}{42}$.

$\frac{786}{42} = 18\frac{30}{42}$. *Ans.*

15. Reduce to whole or mixed numbers $\frac{8975}{25}$.

$\frac{8975}{25} = 359$. *Ans.*

Exercise XIX.

1. Reduce to improper fractions $3\frac{4}{9}$.

$3\frac{4}{9} = \frac{31}{9}$. *Ans.*

2. Reduce to improper fractions $5\frac{9}{10}$.

$5\frac{9}{10} = \frac{59}{10}$. *Ans.*

3. Reduce to improper fractions $12\frac{4}{11}$.

$12\frac{4}{11} = \frac{136}{11}$. *Ans.*

4. Reduce to improper fractions $10\frac{7}{19}$.

$10\frac{7}{19} = \frac{197}{19}$. *Ans.*

5. Reduce to improper fractions $8\frac{3}{7}$.

$8\frac{3}{7} = \frac{59}{7}$. *Ans.*

6. Reduce to improper fractions $12\frac{13}{15}$.

$12\frac{13}{15} = \frac{193}{15}$. *Ans.*

7. Reduce to improper fractions $84\frac{17}{20}$.

$84\frac{17}{20} = \frac{1697}{20}$. *Ans.*

8. Reduce to improper fractions $864\frac{13}{97}$.

$864\frac{13}{97} = \frac{83821}{97}$. *Ans.*

9. Reduce to improper fractions $41\frac{83}{1000}$.

$41\frac{83}{1000} = \frac{41083}{1000}$. *Ans.*

10. Reduce to improper fractions $41\frac{3}{10000}$.

$41\frac{3}{10000} = \frac{410003}{10000}$. *Ans.*

11. Reduce to improper fractions $400\frac{127}{400}$.

$400\frac{127}{400} = \frac{160127}{400}$. *Ans.*

12. Reduce to improper fractions $5000\frac{83}{5000}$.

$5000\frac{83}{5000} = \frac{25000083}{5000}$. *Ans.*

13. Reduce to improper fractions $10000\frac{14}{15}$.

$10000\frac{14}{15} = \frac{150014}{15}$. *Ans.*

14. Reduce to improper fractions $3001\frac{83}{700}$.

$3001\frac{83}{700} = \frac{2100783}{700}$. *Ans.*

15. Reduce to improper fractions $73\frac{73}{99}$.

$73\frac{73}{99} = \frac{7300}{99}$. *Ans.*

16. Express 8, 7, 3, 5, 12, 13, 18, 29, 25 in the form of fractions, each having 5 for a denominator.

8	7	3	5	12	13	18	20	25
$=\frac{40}{5}$.	$=\frac{35}{5}$.	$=\frac{15}{5}$.	$=\frac{25}{5}$.	$=\frac{60}{5}$.	$=\frac{65}{5}$.	$=\frac{90}{5}$.	$=\frac{100}{5}$.	$=\frac{125}{5}$.

17. Express 21 in the form of fractions, having for denominators 3, 5, 7, 8, 12, 13, 20, 25, 30, 37.

21	21	21	21	21
$=\frac{63}{3}$.	$=\frac{105}{5}$.	$=\frac{147}{7}$.	$=\frac{168}{8}$.	$=\frac{252}{12}$.
21	21	21	21	21
$=\frac{273}{13}$.	$=\frac{420}{20}$.	$=\frac{525}{25}$.	$=\frac{630}{30}$.	$=\frac{777}{37}$.

18. Express 12, 15, 23 in the form of fractions, each having for denominators 12, 15, 23, respectively.

12	12	12	15	15
$=\frac{144}{12}$.	$=\frac{180}{15}$.	$=\frac{276}{23}$.	$=\frac{180}{12}$.	$=\frac{225}{15}$.
15 .	23	23	23	
$=\frac{345}{23}$.	$=\frac{276}{12}$.	$=\frac{345}{15}$.	$=\frac{529}{23}$.	

Exercise XX.

1. Reduce to lowest terms $\frac{120}{192}$.
$\frac{120}{192} = \frac{15}{24} = \frac{5}{8}$. *Ans.*

2. Reduce to lowest terms $\frac{105}{135}$.
$\frac{105}{135} = \frac{21}{27} = \frac{7}{9}$. *Ans.*

3. Reduce to lowest terms $\frac{928}{1320}$.
$\frac{928}{1320} = \frac{116}{165}$. *Ans.*

4. Reduce to lowest terms $\frac{1728}{2448}$.
$\frac{1728}{2448} = \frac{216}{306} = \frac{108}{153} = \frac{12}{17}$. *Ans.*

5. Reduce to lowest terms $\frac{1296}{6561}$.
$\frac{1296}{6561} = \frac{144}{729} = \frac{16}{81}$. *Ans.*

6. Reduce to lowest terms $\frac{2310}{3080}$.
$\frac{2310}{3080} = \frac{231}{308} = \frac{21}{28} = \frac{3}{4}$. *Ans.*

7. Reduce to lowest terms $\frac{1848}{2352}$.
$\frac{1848}{2352} = \frac{231}{294} = \frac{77}{88} = \frac{7}{8}$. *Ans.*

8. Reduce to lowest terms $\frac{3960}{12672}$.
$\frac{3960}{12672} = \frac{495}{1584} = \frac{55}{176} = \frac{5}{16}$. *Ans.*

9. Reduce to lowest terms $\frac{1848}{3003}$.
$\frac{1848}{3003} = \frac{616}{1001} = \frac{88}{143} = \frac{8}{13}$. *Ans.*

10. Reduce to lowest terms $\frac{924}{1092}$.
$\frac{924}{1092} = \frac{231}{273} = \frac{77}{91} = \frac{11}{13}$. *Ans.*

11. Reduce to lowest terms $\frac{2640}{2970}$.
$\frac{2640}{2970} = \frac{88}{99} = \frac{8}{9}$. *Ans.*

12. Reduce to lowest terms $\frac{324}{1092}$.
$\frac{324}{1092} = \frac{81}{273} = \frac{27}{91}$. *Ans.*

13 Reduce to lowest terms $\frac{6732}{9108}$.
$\frac{6732}{9108} = \frac{1683}{2277} = \frac{153}{207} = \frac{17}{23}$. *Ans.*

14. Reduce to lowest terms $\frac{6840}{27360}$.
$\frac{6840}{27360} = \frac{171}{684} = \frac{19}{76} = \frac{1}{4}$. *Ans.*

15. Reduce to lowest terms $\frac{5760}{7000}$.
$\frac{5760}{7000} = \frac{576}{700} = \frac{144}{175}$. *Ans.*

16. Reduce to lowest terms $\frac{875}{10000}$.
$\frac{875}{10000} = \frac{7}{80}$. *Ans.*

17. Reduce to lowest terms $\frac{2208}{4140}$.
$\frac{2208}{4140} = \frac{552}{1035} = \frac{184}{345} = \frac{8}{15}$. *Ans.*

18. Reduce to lowest terms $\frac{1015}{1566}$.
$\frac{1015}{1566} = \frac{35}{54}$. *Ans.*

19. Reduce to lowest terms $\frac{516}{2107}$.
$\frac{516}{2107} = \frac{12}{49}$. *Ans.*

20. Reduce to lowest terms $\frac{3872}{92807}$.
$\frac{3872}{92807} = \frac{352}{8437} = \frac{32}{767}$.

21. Reduce to lowest terms $\frac{78473}{94653}$.

```
78473)94653(1
      78473
   10|16180
    2| 1618
      809)78473(97
          7281
          ----
          5663
          5663
```

G. C. M. = 809.

$\frac{78473}{94653} = \frac{97}{117}$. *Ans.*

22. Reduce to lowest terms $\frac{17596}{26145}$.

```
    4|17596
      4399)26145(5
           21995
        10|4150
         5| 415
           83)17596(212
              166
               99
               83
               166
               166
```

G. C. M. = 83.

$\frac{17596}{26145} = \frac{212}{315}$. *Ans.*

23. Reduce to lowest terms $\frac{44323}{61087}$.

```
 44323)61087(1
       44323
     4|16764
     3| 4191
    11| 1397
      127)44323(349
          381
           622
           508
           1143
           1143
```

G. C. M. = 127.

$\frac{44323}{61087} = \frac{349}{481}$. *Ans.*

24. Reduce to lowest terms $\frac{339}{1243}$.

```
   3|339
     113)1243(11
         1243
```

G. C. M. = 113.

$\frac{339}{1243} = \frac{3}{11}$. *Ans.*

25. Reduce to lowest terms $\frac{1177}{2675}$.

```
   11|1177
      107)2675(25
          214
           535
           535
```

G. C. M. = 107.

$\frac{1177}{2675} = \frac{11}{25}$. *Ans.*

26. Reduce to lowest terms $\frac{11445}{15369}$.

$\frac{11445}{15369} = \frac{3815}{5123} = \frac{35}{47}$. *Ans.*

```
     5|3815
     7| 763
G. C. M. = 109)5123(47
               436
                763
                763
```

27. Reduce to lowest terms $\frac{14141}{16289}$.

```
 14141)16289(1
       14141
    12|2148
       179)14141(79
           1253
            1611
            1611
```

G. C. M. = 179.

$\frac{14141}{16289} = \frac{79}{91}$. *Ans.*

28. Reduce to lowest terms $\frac{881496}{1047663}$.

$\frac{881496}{1047663} = \frac{293832}{349221}$. *Ans.*

29. Reduce to lowest terms $\frac{65065}{999999}$.

$\frac{65065}{999999} = \frac{65}{999}$. *Ans.*

Divide both terms by 1001.

30. Reduce to lowest terms $\frac{428571}{999999}$.

$\frac{428571}{999999} = \frac{3}{7}$. *Ans.*

Divide both terms by 142857.

EXERCISE XXI.

1. Find the product of $\frac{3}{4} \times 2$.

$$\frac{3}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{1} = \frac{3}{2} = 1\tfrac{1}{2}. \text{ *Ans.*}$$

2. Find the product of $\frac{3}{4} \times 9$.

$$\frac{3}{4} \times \frac{9}{1} = \frac{27}{4} = 6\tfrac{3}{4}. \text{ *Ans.*}$$

3. Find the product of $10 \times \frac{2}{5}$.

$$\frac{\overset{2}{\cancel{10}}}{1} \times \frac{2}{\cancel{5}} = 4. \text{ *Ans.*}$$

4. Find the product of $15 \times \frac{3}{3}$.

$$\frac{15}{1} \times \frac{\cancel{3}}{\cancel{3}} = 15. \text{ *Ans.*}$$

5. Find the product of $\frac{9}{21} \times 7$.

$$\frac{\overset{3}{\cancel{9}}}{\underset{\cancel{3}}{\cancel{21}}} \times \frac{\cancel{7}}{1} = 3. \text{ *Ans.*}$$

6. Find the product of $16 \times \frac{5}{8}$.

$$\frac{\overset{2}{\cancel{16}}}{1} \times \frac{5}{\cancel{8}} = 10. \text{ *Ans.*}$$

7. Find the product of $\frac{5}{8} \times 2$.

$$\frac{5}{\underset{4}{\cancel{8}}} \times \frac{\cancel{2}}{1} = \frac{5}{4} = 1\tfrac{1}{4}. \text{ *Ans.*}$$

8. Find the product of $\frac{2}{15} \times 5$.

$$\frac{2}{\underset{3}{\cancel{15}}} \times \frac{\cancel{5}}{1} = \tfrac{2}{3}. \text{ *Ans.*}$$

9. Find the product of $27 \times \frac{5}{9}$.

$$\frac{\overset{3}{\cancel{27}}}{1} \times \frac{5}{\cancel{9}} = 15. \text{ *Ans.*}$$

10. Find the product of $\frac{13}{20} \times 2$.

$$\frac{13}{\underset{10}{\cancel{20}}} \times \frac{\cancel{2}}{1} = \frac{13}{10} = 1\tfrac{3}{10}. \text{ *Ans.*}$$

11. Find the product of $\frac{13}{20} \times 3$.

$$\frac{13}{20} \times \frac{3}{1} = \frac{39}{20} = 1\tfrac{19}{20}. \text{ *Ans.*}$$

12. Find the product of $\frac{13}{20} \times 4$.

$$\frac{13}{\underset{5}{\cancel{20}}} \times \frac{\cancel{4}}{1} = \frac{13}{5} = 2\tfrac{3}{5}. \text{ *Ans.*}$$

13. Find the product of $5 \times \frac{13}{20}$.

$$\frac{\cancel{5}}{1} \times \frac{13}{\underset{4}{\cancel{20}}} = \frac{13}{4} = 3\tfrac{1}{4}. \text{ *Ans.*}$$

14. Find the product of $6 \times \frac{13}{20}$.

$$\frac{\overset{3}{\cancel{6}}}{1} \times \frac{13}{\underset{10}{\cancel{20}}} = \frac{39}{10} = 3\tfrac{9}{10}. \text{ *Ans.*}$$

15. Find the product of $7 \times \frac{13}{20}$.

$$\frac{7}{1} \times \frac{13}{20} = \frac{91}{20} = 4\tfrac{11}{20}. \text{ Ans.}$$

16. Find the product of $8 \times \frac{13}{20}$.

$$\frac{\overset{2}{\cancel{8}}}{1} \times \frac{13}{\underset{5}{\cancel{20}}} = \frac{26}{5} = 5\tfrac{1}{5}. \text{ Ans.}$$

17. Find the product of $\frac{13}{20} \times 10$.

$$\frac{13}{\underset{2}{\cancel{20}}} \times \frac{\cancel{10}}{1} = \frac{13}{2} = 6\tfrac{1}{2}. \text{ Ans.}$$

18. Find the product of $\frac{13}{20} \times 12$.

$$\frac{13}{\underset{5}{\cancel{20}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{39}{5} = 7\tfrac{4}{5}. \text{ Ans.}$$

19. Find the product of $\frac{13}{20} \times 15$.

$$\frac{13}{\underset{4}{\cancel{20}}} \times \frac{\overset{3}{\cancel{15}}}{1} = \frac{39}{4} = 9\tfrac{3}{4}. \text{ Ans.}$$

20. Find the product of $\frac{13}{20} \times 20$.

$$\frac{13}{\cancel{20}} \times \frac{\cancel{20}}{1} = 13. \text{ Ans.}$$

Exercise XXII.

1. Simplify $\frac{2}{3}$ of $\frac{7}{11}$.

$$\frac{2}{3} \times \frac{7}{11} = \frac{14}{33}. \text{ Ans.}$$

2. Simplify $\frac{3}{7}$ of $2\frac{1}{10}$.

$$\frac{3}{7} \times 2\tfrac{1}{10} = \frac{3}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{10} = \frac{9}{10}. \text{ Ans.}$$

3. Simplify $\frac{3}{7}$ of $\frac{5}{9}$.

$$\frac{\cancel{3}}{7} \times \frac{5}{\underset{3}{\cancel{9}}} = \frac{5}{21}. \text{ Ans.}$$

4. Simplify $2\frac{2}{5} \times 2\frac{1}{2}$.

$$2\tfrac{2}{5} \times 2\tfrac{1}{2} = \frac{\overset{6}{\cancel{12}}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}} = 6. \text{ Ans.}$$

5. Simplify $4\frac{4}{5} \times 2\frac{1}{7}$.

$$4\tfrac{4}{5} \times 2\tfrac{1}{7} = \frac{24}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{7} = \frac{72}{7} = 10\tfrac{2}{7}. \text{ Ans.}$$

6. Simplify $4\frac{5}{6} \times 9\frac{1}{3}$.

$$4\tfrac{5}{6} \times 9\tfrac{1}{3} = \frac{29}{\underset{3}{\cancel{6}}} \times \frac{\overset{14}{\cancel{28}}}{3} = \frac{406}{9} = 45\tfrac{1}{9}. \text{ Ans.}$$

7. Simplify $\frac{1}{3}$ of $\frac{3}{5}$ of 10.

$$\frac{1}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{1} = 2. \text{ Ans.}$$

8. Simplify $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{3}$.

$$\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\underset{2}{\cancel{4}}}\times\frac{\cancel{2}}{3}=\frac{1}{3}.\ \textit{Ans.}$$

9. Simplify $\frac{4}{5}\times\frac{5}{6}\times\frac{3}{7}$ of $4\frac{1}{5}$.

$$\frac{4}{5}\times\frac{5}{6}\times\frac{3}{7}\times 4\tfrac{1}{5}=\frac{\overset{2}{\cancel{4}}}{\cancel{5}}\times\frac{\cancel{5}}{\underset{2}{\cancel{6}}}\times\frac{\cancel{3}}{\cancel{7}}\times\frac{\overset{3}{\cancel{21}}}{5}=\frac{6}{5}=1\tfrac{1}{5}.\ \textit{Ans.}$$

10. Simplify $\frac{5}{6}$ of $4\frac{1}{2}$.

$$\frac{5}{6}\times 4\tfrac{1}{2}=\frac{5}{\underset{2}{\cancel{6}}}\times\frac{\overset{3}{\cancel{9}}}{2}=\frac{15}{4}=3\tfrac{3}{4}.\ \textit{Ans.}$$

11. Simplify $\frac{8}{9}$ of $\frac{9}{10}$ of $\frac{5}{7}$ of $\frac{3}{4}$ of $\frac{1}{5}$ of $15\frac{3}{4}$.

$$\frac{8}{9}\times\frac{9}{10}\times\frac{5}{7}\times\frac{3}{4}\times\frac{1}{5}\times 15\tfrac{3}{4}=\frac{\overset{2}{\cancel{8}}}{\cancel{9}}\times\frac{\cancel{9}}{\underset{5}{\cancel{10}}}\times\frac{\cancel{5}}{\cancel{7}}\times\frac{3}{\cancel{4}}\times\frac{1}{\cancel{5}}\times\frac{\overset{9}{\cancel{63}}}{4}=\frac{27}{20}=1\tfrac{7}{20}.\ \textit{Ans.}$$

12. Simplify $5\frac{3}{4}\times 8\frac{2}{5}$.

$$5\tfrac{3}{4}\times 8\tfrac{2}{5}=\frac{23}{\underset{2}{\cancel{4}}}\times\frac{\overset{21}{\cancel{42}}}{5}=\frac{483}{10}=48\tfrac{3}{10}.\ \textit{Ans.}$$

13. Simplify $\frac{2}{3}\times\frac{4}{7}\times\frac{7}{15}\times 7\frac{1}{2}$.

$$\frac{2}{3}\times\frac{4}{7}\times\frac{7}{15}\times 7\tfrac{1}{2}=\frac{\cancel{2}}{3}\times\frac{4}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{15}}\times\frac{\cancel{15}}{\cancel{2}}=\frac{4}{3}=1\tfrac{1}{3}.\ \textit{Ans.}$$

14. Simplify $\frac{3}{5}$ of $\frac{10}{27}$ of $\frac{9}{20}$ of $8\frac{1}{3}$.

$$\frac{3}{5}\times\frac{10}{27}\times\frac{9}{20}\times 8\tfrac{1}{3}=\frac{\cancel{3}}{\cancel{5}}\times\frac{\cancel{10}}{\underset{3}{\cancel{27}}}\times\frac{\cancel{9}}{\underset{2}{\cancel{20}}}\times\frac{\overset{5}{\cancel{25}}}{\cancel{3}}=\frac{5}{6}.\ \textit{Ans.}$$

15. Simplify $\frac{8}{11}\times\frac{20}{21}\times\frac{35}{48}\times 2\frac{10}{19}$.

$$\frac{8}{11}\times\frac{20}{21}\times\frac{35}{48}\times 2\tfrac{10}{19}=\frac{8}{11}\times\frac{20}{\underset{3}{\cancel{21}}}\times\frac{\overset{5}{\cancel{35}}}{\cancel{48}}\times\frac{\cancel{48}}{19}=\frac{800}{627}=1\tfrac{173}{627}.\ \textit{Ans.}$$

16. Simplify $\frac{42}{43}\times\frac{13}{105}\times 1\frac{7}{308}$.

$$\frac{42}{43}\times\frac{13}{105}\times 1\tfrac{7}{308}=\frac{\overset{21}{\cancel{42}}}{43}\times\frac{13}{\underset{\cancel{21}}{\cancel{105}}}\times\frac{\overset{9}{\overset{\cancel{63}}{\cancel{315}}}}{\underset{22}{\underset{\cancel{154}}{\cancel{308}}}}=\frac{117}{946}.\ \textit{Ans.}$$

17. Simplify $\frac{5}{8}\times\frac{120}{121}\times\frac{66}{85}\times 17$.

$$\frac{\cancel{5}}{\cancel{8}}\times\frac{\overset{15}{\cancel{120}}}{\underset{11}{\cancel{121}}}\times\frac{\overset{6}{\cancel{66}}}{\underset{\cancel{17}}{\cancel{85}}}\times\frac{\cancel{17}}{\cancel{1}}=\frac{90}{11}=8\tfrac{2}{11}.\ \textit{Ans.}$$

18. Simplify $\frac{38}{39}\times\frac{52}{57}\times\frac{69}{86}\times 1\frac{20}{23}$.

$$\frac{38}{39}\times\frac{52}{57}\times\frac{69}{86}\times 1\tfrac{20}{23}=\frac{\overset{\cancel{19}}{\cancel{38}}}{\underset{\cancel{13}}{\cancel{39}}}\times\frac{\overset{4}{\cancel{52}}}{\underset{3}{\cancel{57}}}\times\frac{\overset{\cancel{3}}{\cancel{69}}}{\underset{\cancel{2}}{\cancel{86}}}\times\frac{\cancel{43}}{\cancel{23}}=\frac{4}{3}=1\tfrac{1}{3}.\ \textit{Ans.}$$

19. Simplify $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 10.

$$\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{\cancel{5}}{\cancel{6}}\times\frac{\cancel{6}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{8}}\times\frac{\cancel{8}}{\cancel{9}}\times\frac{\cancel{9}}{\cancel{10}}\times\frac{\cancel{10}}{1}=1.\ \textit{Ans.}$$

20. Simplify $\frac{7}{25}$ of $\frac{8}{11}$ of 30.

$$\frac{7}{\underset{5}{\cancel{25}}}\times\frac{8}{11}\times\frac{\overset{6}{\cancel{30}}}{1}=\frac{336}{55}=6\tfrac{6}{55}.\ \textit{Ans.}$$

21. Simplify $\frac{113}{355}\times\frac{85}{226}\times\frac{12}{35}\times 1\frac{3}{4}$.

$$\frac{113}{355}\times\frac{85}{226}\times\frac{12}{35}\times 1\tfrac{3}{4}=\frac{\cancel{113}}{\underset{71}{\cancel{355}}}\times\frac{\overset{17}{\cancel{85}}}{\underset{\cancel{2}}{\cancel{226}}}\times\frac{\overset{3}{\overset{\cancel{6}}{\cancel{12}}}}{\underset{5}{\cancel{35}}}\times\frac{\cancel{7}}{\underset{2}{\cancel{4}}}=\frac{51}{710}.\ \textit{Ans.}$$

22. Simplify $\frac{7}{8}\times\frac{3}{4}\times\frac{8}{21}\times\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{3}{4}$ of 8.

$$\frac{\cancel{7}}{\cancel{8}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{8}}{\underset{\cancel{3}}{\cancel{21}}}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{\cancel{5}}{\cancel{8}}\times\frac{3}{4}\times\frac{\cancel{8}}{1}=\frac{3}{4}.\ \textit{Ans.}$$

23. Simplify $\frac{2}{13}$ of $\frac{39}{40}$ of $\frac{52}{117}$.

$$\frac{\cancel{2}}{\cancel{13}}\times\frac{\overset{\cancel{3}}{\cancel{39}}}{\underset{5}{\underset{\cancel{20}}{\cancel{40}}}}\times\frac{\overset{\cancel{13}}{\cancel{52}}}{\underset{3}{\underset{\cancel{9}}{\cancel{117}}}}=\frac{1}{15}.\ \textit{Ans.}$$

24. Simplify $\frac{9}{11} \times \frac{7}{12} \times \frac{22}{63} \times 48$.

$$\frac{\cancel{9}}{\cancel{11}} \times \frac{\cancel{7}}{\underset{6}{\cancel{12}}} \times \frac{\overset{2}{\cancel{22}}}{\underset{7}{\cancel{63}}} \times \frac{\overset{8}{\cancel{48}}}{\cancel{1}} = 8. \textit{ Ans.}$$

25. Simplify $\frac{39}{40}$ of $\frac{7}{65}$ of $\frac{25}{42}$ of 12.

$$\frac{\overset{3}{\cancel{39}}}{\underset{\underset{4}{\cancel{8}}}{\cancel{40}}} \times \frac{\cancel{7}}{\underset{\cancel{5}}{\cancel{65}}} \times \frac{\overset{\cancel{5}}{\cancel{25}}}{\underset{\underset{\cancel{2}}{\cancel{14}}}{\cancel{42}}} \times \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{12}}}{1} = \frac{3}{4}. \textit{ Ans.}$$

26. Simplify $1\frac{2}{3}$ of $4\frac{1}{2}$ of $\frac{3}{8}$.

$$1\tfrac{2}{3} \times 4\tfrac{1}{2} \times \frac{3}{8} = \frac{5}{\cancel{3}} \times \frac{9}{2} \times \frac{\cancel{3}}{8} = \frac{45}{16} = 2\tfrac{13}{16}. \textit{ Ans.}$$

27. Simplify $2\frac{4}{9} \times 1\frac{3}{7} \times 1\frac{13}{15} \times 8$.

$$2\tfrac{4}{9} \times 1\tfrac{3}{7} \times 1\tfrac{13}{15} \times 8 = \frac{22}{9} \times \frac{\overset{2}{\cancel{10}}}{\cancel{7}} \times \frac{\overset{4}{\cancel{28}}}{\underset{3}{\cancel{15}}} \times \frac{8}{1} = \frac{1408}{27} = 52\tfrac{4}{27}. \textit{ Ans.}$$

28. Simplify $3\frac{5}{7}$ of $2\frac{1}{2}$ of $1\frac{8}{13}$ of $1\frac{4}{11}$.

$$3\tfrac{5}{7} \times 2\tfrac{1}{2} \times 1\tfrac{8}{13} \times 1\tfrac{4}{11} = \frac{\overset{2}{\cancel{26}}}{\cancel{7}} \times \frac{5}{\cancel{2}} \times \frac{\overset{3}{\cancel{21}}}{\cancel{13}} \times \frac{15}{11} = \frac{225}{11} = 20\tfrac{5}{11}. \textit{ Ans}$$

29. Simplify $\frac{11}{12} \times 5\frac{1}{3} \times 4\frac{1}{8} \times \frac{7}{22} \times 5$.

$$\frac{11}{12} \times 5\tfrac{1}{3} \times 4\tfrac{1}{8} \times \frac{7}{22} \times 5 = \frac{\cancel{11}}{12} \times \frac{\overset{2}{\cancel{16}}}{\cancel{3}} \times \frac{\overset{11}{\cancel{33}}}{\cancel{8}} \times \frac{7}{\underset{\cancel{2}}{\cancel{22}}} \times \frac{5}{1} = \frac{385}{12} = 32\tfrac{1}{12}. \textit{ Ans.}$$

30. Simplify $\frac{2}{5}$ of $\frac{7}{15} \times 8\frac{2}{7} \times \frac{6}{29}$ of $1\frac{17}{18}$.

$$\frac{2}{5} \times \frac{7}{15} \times 8\tfrac{2}{7} \times \frac{6}{29} \times 1\tfrac{17}{18} = \frac{\cancel{2}}{\cancel{5}} \times \frac{\cancel{7}}{\underset{5}{\cancel{15}}} \times \frac{\overset{2}{\cancel{58}}}{\cancel{7}} \times \frac{\overset{2}{\cancel{6}}}{\cancel{29}} \times \frac{\overset{7}{\cancel{35}}}{\underset{9}{\cancel{18}}} = \frac{28}{45}. \textit{ Ans.}$$

31. Simplify $\frac{17}{82} \times \frac{27}{38} \times \frac{123}{153}$.

$$\frac{\cancel{17}}{\underset{2}{\cancel{82}}} \times \frac{\overset{9}{\cancel{27}}}{38} \times \frac{\overset{\cancel{41}}{\cancel{123}}}{\underset{\underset{\cancel{17}}{\cancel{51}}}{\cancel{153}}} = \frac{9}{76}. \quad \textit{Ans.}$$

32. Simplify $\frac{292}{465} \times \frac{573}{657} \times \frac{248}{1719}$.

$$\frac{\overset{4}{\cancel{292}}}{\underset{\underset{5}{\cancel{155}}}{\cancel{465}}} \times \frac{\overset{\cancel{191}}{\cancel{573}}}{\underset{9}{\cancel{657}}} \times \frac{\overset{8}{\cancel{248}}}{\underset{9}{\cancel{1719}}} = \frac{32}{405}. \quad \textit{Ans.}$$

33. Simplify $\frac{1205}{2763}$ of $\frac{712}{2169}$ of $\frac{1535}{1068}$.

$$\frac{\overset{5}{\cancel{1205}}}{\underset{9}{\cancel{2763}}} \times \frac{\overset{\overset{2}{\cancel{8}}}{\cancel{712}}}{\underset{9}{\cancel{2169}}} \times \frac{\overset{5}{\cancel{1535}}}{\underset{\underset{3}{\cancel{12}}}{\cancel{1068}}} = \frac{50}{243}. \quad \textit{Ans.}$$

Exercise XXIII.

1. Divide $\frac{24}{35}$ by 6.

$$\frac{24}{35} \div 6 = \frac{1}{\cancel{6}} \times \frac{\overset{4}{\cancel{24}}}{35} = \frac{4}{35}. \quad \textit{Ans.}$$

2. Divide $\frac{10}{11}$ by 5.

$$\frac{10}{11} \div 5 = \frac{\overset{2}{\cancel{10}}}{11} \times \frac{1}{\cancel{5}} = \frac{2}{11}. \quad \textit{Ans.}$$

3. Divide $\frac{3}{7}$ by 8.

$$\frac{3}{7} \div 8 = \frac{1}{8} \times \frac{3}{7} = \frac{3}{56}. \quad \textit{Ans.}$$

4. Divide $18\frac{2}{3}$ by 7.

$$18\tfrac{2}{3} \div 7 = \frac{1}{\cancel{7}} \times \frac{\overset{8}{\cancel{56}}}{3} = 2\tfrac{2}{3}. \quad \textit{Ans.}$$

5. Divide $\frac{5}{8}$ by $\frac{3}{4}$.

$$\frac{5}{8} \div \frac{3}{4} = \frac{\cancel{4}}{3} \times \frac{5}{\underset{2}{\cancel{8}}} = \frac{5}{6}. \quad \textit{Ans.}$$

6. Divide $\frac{12}{16}$ by $\frac{3}{8}$.

$$\frac{12}{16} \div \frac{3}{8} = \frac{\overset{2}{\cancel{8}}}{\cancel{3}} \times \frac{\overset{\cancel{3}}{\cancel{12}}}{\underset{\cancel{4}}{\cancel{16}}} = 2. \quad \textit{Ans.}$$

7. Divide $1\frac{3}{4}$ by $3\frac{1}{3}$.

$$1\tfrac{3}{4} \div 3\tfrac{1}{3} = \frac{7}{4} \div \frac{10}{3} = \frac{3}{10} \times \frac{7}{4} = \frac{21}{40}. \quad \textit{Ans.}$$

8. Divide $5\frac{1}{5}$ by $4\frac{2}{3}$.

$$5\tfrac{1}{5} \div 4\tfrac{2}{3} = \frac{26}{5} \div \frac{14}{3} = \frac{3}{\underset{7}{\cancel{14}}} \times \frac{\overset{13}{\cancel{26}}}{5} = \frac{39}{35} = 1\tfrac{4}{35}. \textit{ Ans.}$$

9. Divide $8\frac{2}{9}$ by $4\frac{1}{9}$.

$$8\tfrac{2}{9} \div 4\tfrac{1}{9} = \frac{74}{9} \div \frac{37}{9} = \frac{\cancel{9}}{\cancel{37}} \times \frac{\overset{2}{\cancel{74}}}{\cancel{9}} = 2. \textit{ Ans.}$$

10. Divide $7\frac{1}{5}$ by $4\frac{2}{7}$.

$$7\tfrac{1}{5} \div 4\tfrac{2}{7} = \frac{36}{5} \div \frac{30}{7} = \frac{7}{\underset{5}{\cancel{30}}} \times \frac{\overset{6}{\cancel{36}}}{5} = \frac{42}{25} = 1\tfrac{17}{25}. \textit{ Ans.}$$

11. Divide $6\frac{3}{4}$ by $9\frac{1}{2}$.

$$6\tfrac{3}{4} \div 9\tfrac{1}{2} = \frac{27}{4} \div \frac{19}{2} = \frac{\cancel{2}}{19} \times \frac{27}{\underset{2}{\cancel{4}}} = \frac{27}{38}. \textit{ Ans.}$$

12. Divide $8\frac{2}{3}$ by $4\frac{2}{3}$.

$$8\tfrac{2}{3} \div 4\tfrac{2}{3} = \frac{26}{3} \div \frac{14}{3} = \frac{\cancel{3}}{\underset{7}{\cancel{14}}} \times \frac{\overset{13}{\cancel{26}}}{\cancel{3}} = \frac{13}{7} = 1\tfrac{6}{7}. \textit{ Ans.}$$

13. Divide $3\frac{8}{9}$ by $\frac{14}{27}$.

$$3\tfrac{8}{9} \div \tfrac{14}{27} = \frac{35}{9} \div \frac{14}{27} = \frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{14}}} \times \frac{\overset{5}{\cancel{35}}}{\cancel{9}} = \frac{15}{2} = 7\tfrac{1}{2}. \textit{ Ans.}$$

14. Divide $4\frac{3}{7}$ by $6\frac{8}{9}$.

$$4\tfrac{3}{7} \div 6\tfrac{8}{9} = \frac{31}{7} \div \frac{62}{9} = \frac{9}{\underset{2}{\cancel{62}}} \times \frac{\cancel{31}}{7} = \frac{9}{14}. \textit{ Ans.}$$

15. Divide 5 by $4\frac{2}{7}$.

$$5 \div 4\tfrac{2}{7} = 5 \div \frac{30}{7} = \frac{7}{\underset{6}{\cancel{30}}} \times \frac{\cancel{5}}{1} = \frac{7}{6} = 1\tfrac{1}{6}. \textit{ Ans.}$$

16. Divide $3\frac{4}{5}$ of $2\frac{1}{2}$ by $1\frac{1}{2}$ of $2\frac{1}{9}$.

$$3\tfrac{4}{5} \text{ of } 2\tfrac{1}{2} \div 1\tfrac{1}{2} \text{ of } 2\tfrac{1}{9} = \frac{19}{5} \text{ of } \frac{5}{2} \div \frac{3}{2} \text{ of } \frac{19}{9} = \frac{\cancel{19}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{19}} = 3. \textit{ Ans.}$$

17. Divide $2\frac{2}{7}$ by $3\frac{1}{3}$ of $1\frac{1}{15}$.

$$2\tfrac{2}{7} \div 3\tfrac{1}{3} \text{ of } 1\tfrac{1}{15} = \frac{16}{7} \div \frac{10}{3} \text{ of } \frac{16}{15} = \frac{\cancel{16}}{7} \times \frac{3}{\underset{2}{\cancel{10}}} \times \frac{\overset{3}{\cancel{15}}}{\cancel{16}} = \frac{9}{14}. \textit{ Ans.}$$

18. Divide $2\frac{9}{11}$ of $5\frac{1}{2}$ by $7\frac{3}{4}$.

$$2\tfrac{9}{11} \text{ of } 5\tfrac{1}{2} \div 7\tfrac{3}{4} = \frac{31}{11} \text{ of } \frac{11}{2} \div \frac{31}{4} = \frac{\cancel{31}}{\cancel{11}} \times \frac{\cancel{11}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{31}} = 2. \textit{ Ans.}$$

19. Divide $5\frac{3}{5}$ of $8\frac{1}{3}$ of $1\frac{4}{7}$ by $2\frac{1}{10}$ of $5\frac{5}{9}$.

$$5\tfrac{3}{5} \text{ of } 8\tfrac{1}{3} \text{ of } 1\tfrac{4}{7} \div 2\tfrac{1}{10} \text{ of } 5\tfrac{5}{9} = \frac{28}{5} \text{ of } \frac{25}{3} \text{ of } \frac{11}{7} \div \frac{21}{10} \text{ of } \frac{50}{9}$$

$$= \frac{\overset{4}{\cancel{28}}}{\cancel{5}} \times \frac{\cancel{25}}{\cancel{3}} \times \frac{11}{\cancel{7}} \times \frac{\overset{\cancel{2}}{\cancel{10}}}{\underset{7}{\cancel{21}}} \times \frac{\overset{\cancel{3}}{\cancel{9}}}{\underset{\cancel{2}}{\cancel{50}}} = \frac{44}{7} = 6\tfrac{2}{7}. \textit{ Ans.}$$

Exercise XXIV.

1. Express with least common denominator $\frac{1}{2}$, $\frac{2}{5}$, $\frac{5}{6}$.

$$\text{L. C. D.} = 2 \times 3 \times 5 = 30.$$

$$\frac{1}{2}, \frac{2}{5}, \frac{5}{6} = \frac{15 \quad 12 \quad 25}{30}. \textit{ Ans.}$$

2. Express with least common denominator $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{8}$, $\frac{9}{10}$.

$$\text{L. C. D.} = 2^3 \times 3^2 \times 5 = 360.$$

$$\frac{2}{3}, \frac{5}{9}, \frac{7}{8}, \frac{9}{10} = \frac{240 \quad 200 \quad 315 \quad 324}{360}. \textit{ Ans.}$$

3. Express with least common denominator $\frac{5}{6}$, $\frac{1}{8}$, $\frac{5}{21}$, $\frac{19}{35}$.

$$\text{L. C. D.} = 2^3 \times 3 \times 5 \times 7 = 840.$$

$$\frac{5}{6}, \frac{1}{8}, \frac{5}{21}, \frac{19}{35} = \frac{700 \quad 105 \quad 200 \quad 456}{840}. \textit{ Ans.}$$

4. Express with least common denominator $\frac{2}{15}$, $\frac{7}{20}$, $\frac{3}{25}$, $\frac{8}{45}$.

$$\text{L. C. D.} = 2^2 \times 3^2 \times 5^2 = 800.$$

$$\frac{2}{15}, \frac{7}{20}, \frac{3}{25}, \frac{8}{45} = \frac{120 \quad 315 \quad 108 \quad 160}{800}. \textit{ Ans.}$$

5. Express with least common denominator $\frac{12}{25}$, $\frac{17}{40}$, $\frac{13}{60}$, $\frac{19}{75}$.

L. C. D. $= 2^3 \times 3 \times 5^2 = 600$.

$$\frac{12}{25}, \frac{17}{40}, \frac{13}{60}, \frac{19}{75} = \frac{288 \quad 255 \quad 130 \quad 152}{600}. \textit{ Ans.}$$

6. Express with least common denominator $\frac{3}{8}$, $\frac{7}{30}$, $\frac{4}{35}$, $\frac{3}{28}$, $\frac{19}{24}$.

L. C. D. $= 2^3 \times 3 \times 5 \times 7 = 840$.

$$\frac{3}{8}, \frac{7}{30}, \frac{4}{35}, \frac{3}{28}, \frac{19}{24} = \frac{315 \quad 196 \quad 96 \quad 90 \quad 665}{840}. \textit{ Ans.}$$

7. Express with least common denominator $\frac{11}{16}$, $\frac{7}{18}$, $\frac{13}{20}$, $\frac{23}{30}$, $\frac{17}{54}$.

L. C. D. $= 2^4 \times 3^3 \times 5 = 2160$.

$$\frac{11}{16}, \frac{7}{18}, \frac{13}{20}, \frac{23}{30}, \frac{17}{54} = \frac{1485 \quad 840 \quad 1404 \quad 1656 \quad 680}{2160}. \textit{ Ans.}$$

8. Which is the greater, $\frac{13}{20}$ or $\frac{17}{28}$? $\frac{5}{6}$ or $\frac{7}{9}$? $\frac{3}{5}$ or $\frac{7}{12}$?

L. C. D. $= 2^2 \times 5^2 = 100$. L. C. D. $= 2 \times 3^2 = 18$. L. C. D. $= 5 \times 12 = 60$.

$$\frac{13}{20} = \frac{65}{100}, \qquad \frac{5}{6} = \frac{15}{18}, \qquad \frac{3}{5} = \frac{36}{60},$$

$$\frac{17}{28} = \frac{68}{100}. \qquad \frac{7}{9} = \frac{14}{18}. \qquad \frac{7}{12} = \frac{35}{60}.$$

$\therefore$ $\frac{17}{28}$ is the greater. $\therefore$ $\frac{5}{6}$ is the greater. $\therefore$ $\frac{3}{5}$ is the greater.

9. Arrange the fractions $\frac{7}{12}$, $\frac{11}{18}$, $\frac{13}{24}$ in order of magnitude.

L. C. D. $= 2^3 \times 3^2 = 72$.

$$\frac{7}{12} = \frac{42}{72}, \quad \frac{11}{18} = \frac{44}{72}, \quad \frac{13}{24} = \frac{39}{72}. \qquad \frac{13}{24}, \frac{7}{12}, \frac{11}{18}. \textit{ Ans.}$$

10. Arrange the fractions $\frac{5}{12}$, $\frac{8}{15}$, $\frac{4}{11}$, $\frac{7}{18}$ in order of magnitude.

L. C. D. $= 2^2 \times 3^2 \times 5 \times 11 = 1980$.

$$\frac{5}{12}, \frac{8}{15}, \frac{4}{11}, \frac{7}{18} = \frac{825 \quad 1056 \quad 720 \quad 770}{1980}.$$

$$\frac{4}{11}, \frac{7}{18}, \frac{5}{12}, \frac{8}{15}. \textit{ Ans.}$$

Exercise XXV.

1. Find the sum of $\frac{1}{2} + \frac{3}{2}$.

$$\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2. \textit{ Ans.}$$

2. Find the sum of $\frac{1}{3} + \frac{2}{3} + \frac{1}{3}$.

$$\frac{1}{3} + \frac{2}{3} + \frac{1}{3} = \frac{4}{3} = 1\tfrac{1}{3}. \textit{ Ans.}$$

3. Find the sum of $\frac{1}{4} + \frac{1}{4} + \frac{3}{4}$.

$$\frac{1}{4} + \frac{1}{4} + \frac{3}{4} = \frac{5}{4} = 1\tfrac{1}{4}. \textit{ Ans.}$$

4. Find the sum of $1\frac{1}{2} + 2\frac{1}{2}$.

$$1\tfrac{1}{2} + 2\tfrac{1}{2} = 3\tfrac{1+1}{2} = 4. \textit{ Ans.}$$

5. Find the sum of $1\frac{1}{3} + 2\frac{2}{3}$.

$$1\tfrac{1}{3} + 2\tfrac{2}{3} = 3\tfrac{1+2}{3} = 4. \textit{ Ans.}$$

6. Find the sum of $3\frac{1}{4} + \frac{3}{4}$.

$$3\tfrac{1}{4} + \tfrac{3}{4} = 3\tfrac{1+3}{4} = 4. \textit{ Ans.}$$

7. Find the sum of $2\frac{3}{5} + 3\frac{4}{5}$.

$$2\tfrac{3}{5} + 3\tfrac{4}{5} = 5\tfrac{3+4}{5} = 6\tfrac{2}{5}. \textit{ Ans.}$$

8. Find the sum of $1\frac{7}{8} + \frac{3}{8}$.

$$1\tfrac{7}{8} + \tfrac{3}{8} = 1\tfrac{7+3}{8} = 2\tfrac{1}{4}. \textit{ Ans.}$$

9. Find the sum of $\frac{9}{17} + \frac{3}{17} + \frac{14}{17} + \frac{11}{17}$.

$$\frac{9}{17} + \frac{3}{17} + \frac{14}{17} + \frac{11}{17} = \frac{37}{17} = 2\tfrac{3}{17}. \textit{ Ans.}$$

10. Find the sum of $8\frac{9}{17} + 6\frac{3}{17} + 5\frac{14}{17} + \frac{11}{17}$.

$$8\tfrac{9}{17} + 6\tfrac{3}{17} + 5\tfrac{14}{17} + \tfrac{11}{17} = 19\tfrac{37}{17} = 21\tfrac{3}{17}. \textit{ Ans.}$$

11. Find the sum of $\frac{4}{5} + \frac{5}{6}$.

$$\frac{4}{5} + \frac{5}{6} = \frac{24 + 25}{30} = 1\tfrac{19}{30}. \textit{ Ans.}$$

12. Find the sum of $\frac{3}{4} + \frac{7}{8}$.

$$\frac{3}{4} + \frac{7}{8} = \frac{6 + 7}{8} = 1\tfrac{5}{8}. \textit{ Ans.}$$

13. Find the sum of $\frac{1}{2} + \frac{1}{6}$.

$$\frac{1}{2} + \frac{1}{6} = \frac{3 + 1}{6} = \frac{2}{3}. \textit{ Ans.}$$

14. Find the sum of $\frac{4}{15} + \frac{11}{20}$.

L. C. D. $= 2^2 \times 3 \times 5 = 60$.

$$\frac{16 + 33}{60} = \frac{49}{60}. \textit{ Ans.}$$

15. Find the sum of $\frac{5}{16} + \frac{11}{24}$.

L. C. D. $= 2^4 \times 3 = 48$.

$$\frac{15 + 22}{48} = \frac{37}{48}. \textit{ Ans.}$$

16. Find the sum of $12\frac{5}{8} + 7\frac{3}{16}$.

$$12\tfrac{5}{8} + 7\tfrac{3}{16} = 19\tfrac{10+3}{16} = 19\tfrac{13}{16}. \textit{ Ans}$$

17. Find the sum of $85\frac{7}{12} \times 27\frac{11}{18}$.

$$85\tfrac{7}{12} \times 27\tfrac{11}{18} = 112\tfrac{21+22}{36} = 113\tfrac{7}{36}. \textit{ Ans.}$$

18. Find the sum of $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$.

$$\text{L. C. D.} = 2^2 \times 3 \times 5 = 60.$$

$$\frac{30+20+15+12}{60} = \frac{77}{60} = 1\tfrac{17}{60}. \ \textit{Ans.}$$

19. Find the sum of $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$.

$$\text{L. C. D.} = 2^2 \times 3 \times 5 = 60.$$

$$\frac{30+40+45+48}{60} = \frac{163}{60} = 2\tfrac{43}{60}. \ \textit{Ans.}$$

20. Find the sum of $\frac{5}{6}+\frac{11}{12}+\frac{8}{15}+\frac{7}{20}+\frac{13}{30}$.

$$\text{L. C. D.} = 2^2 \times 3 \times 5 = 60.$$

$$\frac{50+55+32+21+26}{60} = \frac{184}{60} = 3\tfrac{1}{15}. \ \textit{Ans.}$$

21. Find the sum of $5\frac{17}{20}+11\frac{19}{30}+24\frac{21}{40}+\frac{9}{50}+17\frac{8}{15}+14+11\frac{5}{12}$.

$$\text{L. C. D.} = 2^3 \times 3 \times 5^2 = 600.$$

$$82\tfrac{1883}{600} = 85\tfrac{83}{600}. \ \textit{Ans.}$$

22. Find the sum of

$$9\tfrac{4}{7}+15\tfrac{11}{28}+163\tfrac{17}{63}+1\tfrac{11}{42}+10\tfrac{1}{4}.$$

$$\text{L. C. D.} = 2^2 \times 3^2 \times 7 = 252.$$

$$198\tfrac{440}{252} = 199\tfrac{47}{63}. \ \textit{Ans.}$$

23. Find the sum of

$$3\tfrac{3}{5}+4\tfrac{2}{3}+1\tfrac{5}{6}+2.$$

$$\text{L. C. D.} = 2+3+5 = 30.$$

$$12\tfrac{3}{30} = 12\tfrac{1}{10}.$$

24. Find the sum of

$$1\tfrac{3}{20}+2\tfrac{2}{25}+5\tfrac{7}{30}+\tfrac{4}{15}.$$

$$\text{L. C. D.} = 2^2 \times 3 \times 5^2 = 300.$$

$$8\tfrac{219}{300} = 8\tfrac{73}{100}.$$

25. Find the sum of

$$\tfrac{2}{7}+1\tfrac{4}{9}+2+3\tfrac{3}{8}+4\tfrac{5}{12}.$$

$$\text{L. C. D.} = 2^3 \times 3^3 \times 7 = 504.$$

$$10\tfrac{767}{504} = 11\tfrac{263}{504}. \ \textit{Ans.}$$

26. Find the sum of

$$4\tfrac{4}{9}+3\tfrac{3}{8}+2\tfrac{2}{7}+1\tfrac{1}{6}+\tfrac{9}{14}.$$

$$\text{L. C. D.} = 2^3 \times 3^2 \times 7 = 504.$$

$$10\tfrac{965}{504} = 11\tfrac{461}{504}. \ \textit{Ans.}$$

27. Find the sum of

$$\tfrac{11}{35}+\tfrac{7}{40}+10+\tfrac{23}{60}.$$

$$\text{L. C. D.} = 2^3 \times 3 \times 5 \times 7 = 840.$$

$$10\tfrac{733}{840}. \ \textit{Ans.}$$

28. Find the sum of

$$\tfrac{27}{50}+\tfrac{29}{60}+\tfrac{31}{80}+\tfrac{33}{100}+\tfrac{37}{240}.$$

$$\text{L. C. D.} = 2^4 \times 3^2 \times 5^2 = 3600.$$

$$\tfrac{6822}{3600} = 1\tfrac{179}{200}. \ \textit{Ans.}$$

29. Find the sum of

$$2+\tfrac{2}{3}+1\tfrac{3}{4}+4\tfrac{8}{9}+5\tfrac{13}{24}.$$

$$\text{L. C. D.} = 2^3 \times 3^2 = 72.$$

$$12\tfrac{205}{72} = 14\tfrac{61}{72}. \ \textit{Ans.}$$

30. Find the sum of

$3\frac{5}{8} + 6 + \frac{4}{11} + 2\frac{3}{10} + 5\frac{5}{16} + \frac{9}{20}$.

L. C. D. $= 2^4 \times 5 \times 11 = 880$.

$16\frac{1805}{880} = 18\frac{9}{176}$. *Ans.*

31. Find the sum of

$\frac{8}{15} + \frac{7}{18} + 3\frac{17}{20} + 1\frac{19}{24} + 2\frac{19}{120}$.

L. C. D. $= 2^3 \times 3^2 \times 5 = 360$.

$6\frac{980}{360} = 8\frac{13}{18}$. *Ans.*

32. Find the sum of

$\frac{5}{14} + \frac{6}{11} + 9\frac{1}{2}$.

L. C. D. $= 2 \times 7 \times 11 = 154$.

$9\frac{216}{154} = 10\frac{31}{77}$. *Ans.*

33. Find the sum of

$20\frac{5}{12} + 11\frac{7}{20} + 5\frac{1}{8} + 305$.

L. C. D. $= 2^3 \times 3 \times 5 = 120$.

$341\frac{107}{120}$. *Ans.*

34. Find the sum of

$\frac{11}{38} + \frac{14}{57} + \frac{17}{76}$.

L. C. D. $= 2^2 \times 3 \times 19 = 228$.

$\frac{173}{228}$. *Ans.*

35. Find the sum of

$\frac{5}{17} + \frac{11}{34} + \frac{14}{51} + \frac{19}{68}$.

L. C. D. $= 2^2 \times 3 \times 17 = 204$.

$\frac{239}{204} = 1\frac{35}{204}$.

36. Find the sum of

$317\frac{2}{5} + 17\frac{3}{51} + 4\frac{9}{10} + \frac{7}{15} + 6\frac{2}{3} + \frac{5}{17}$.

L. C. D. $= 2 \times 3 \times 5 \times 17 = 510$.

$344\frac{1421}{510} = 346\frac{401}{510}$. *Ans.*

37. Find the sum of

$4\frac{7}{15} + 8\frac{5}{21} + 4\frac{7}{11} + 5\frac{2}{7} + 5\frac{4}{5} + \frac{2}{3}$.

L. C. D. $= 3 \times 5 \times 7 \times 11 = 1155$.

$26\frac{3573}{1155} = 29\frac{36}{385}$. *Ans.*

38. Find the sum of

$3\frac{2}{3} + 5\frac{3}{40} + 8\frac{7}{240} + \frac{39}{60} + 1\frac{29}{2880}$.

L. C. D. $= 2880$.

$17\frac{4121}{2880} = 18\frac{1241}{2880}$. *Ans.*

39. Find the sum of

$4\frac{5}{13} + 7\frac{5}{39} + 5\frac{47}{78} + 275\frac{37}{156} + 2\frac{57}{91}$.

L. C. D. $= 2^2 \times 3 \times 7 \times 13 = 1092$.

$293\frac{2161}{1092} = 294\frac{1069}{1092}$. *Ans.*

40. Find the sum of

$\frac{17}{33} + 7\frac{5}{12} + 6\frac{8}{44} + 400\frac{3}{28} + 51\frac{25}{56}$.

L. C. D. $= 2^3 \times 7 \times 3 \times 11 = 1848$.

$464\frac{3081}{1848} = 465\frac{411}{616}$. *Ans.*

Exercise XXVI.

1. Find the value of $52\frac{1}{8} - 46$.

$52\frac{1}{8} - 46 = 6\frac{1}{8}$. *Ans.*

2. Find the value of $\frac{6}{9} - \frac{3}{9}$.

$$\frac{6}{9} - \frac{3}{9} = \frac{6-3}{9} = \frac{3}{9} = \frac{1}{3}. \textit{ Ans.}$$

3. Find the value of $\frac{3}{4} - \frac{2}{3}$.

$$\frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}. \textit{ Ans.}$$

4. Find the value of $\frac{8}{15} - \frac{5}{12}$.

$$\frac{8}{15} - \frac{5}{12} = \frac{32-25}{60} = \frac{7}{60}. \textit{ Ans.}$$

5. Find the value of $\frac{11}{18} - \frac{3}{14}$.

$$\frac{11}{18} - \frac{3}{14} = \frac{77 - 27}{126} = \frac{50}{126} = \frac{25}{63}$$ *Ans.*

6. Find the value of $4 - \frac{1}{2}$.

$4 - \frac{1}{2} = 3\frac{1}{2}$. *Ans.*

7. Find the value of $7 - \frac{2}{3}$.

$7 - \frac{2}{3} = 6\frac{1}{3}$. *Ans.*

8. Find the value of $3 - \frac{5}{6}$.

$3 - \frac{5}{6} = 2\frac{1}{6}$. *Ans.*

9. Find the product of $8 - \frac{3}{7}$.

$8 - \frac{3}{7} = 7\frac{4}{7}$. *Ans.*

10. Find the product of $5 - \frac{4}{5}$.

$5 - \frac{4}{5} = 4\frac{1}{5}$. *Ans.*

11. Find the value of $5 - \frac{7}{9}$.

$5 - \frac{7}{9} = 4\frac{2}{9}$. *Ans.*

12. Find the value of $6\frac{1}{3} - 5\frac{1}{6}$.

$6\frac{1}{3} - 5\frac{1}{6} = 1\frac{2-1}{6} = 1\frac{1}{6}$. *Ans.*

13. Find the value of $4\frac{2}{5} - 3\frac{3}{7}$.

$4\frac{2}{5} - 3\frac{3}{7} = 1\frac{14-15}{35} = \frac{34}{35}$. *Ans.*

14. Find the value of $7\frac{1}{3} - 2\frac{3}{10}$.

$7\frac{1}{3} - 2\frac{3}{10} = 5\frac{10-9}{30} = 5\frac{1}{30}$. *Ans.*

15. Find the value of $7\frac{2}{5} - 4\frac{8}{9}$.

$7\frac{2}{5} - 4\frac{8}{9} = 3\frac{18-40}{45} = 2\frac{23}{45}$. *Ans.*

16. Find the value of $6\frac{2}{3} - 2\frac{3}{4}$.

$6\frac{2}{3} - 2\frac{3}{4} = 4\frac{8-9}{12} = 3\frac{11}{12}$. *Ans.*

17. Find the value of $9\frac{4}{5} - 4\frac{5}{6}$.

$9\frac{4}{5} - 4\frac{5}{6} = 5\frac{24-25}{30} = 4\frac{29}{30}$. *Ans.*

18. Find the value of $4\frac{2}{3} - \frac{1}{2}$.

$4\frac{2}{3} - \frac{1}{2} = 4\frac{4-3}{6} = 4\frac{1}{6}$. *Ans.*

19. Find the value of $6\frac{3}{4} - 4\frac{2}{3}$.

$6\frac{3}{4} - 4\frac{2}{3} = 2\frac{9-8}{12} = 2\frac{1}{12}$.

20. Find the value of $7\frac{1}{2} - 2\frac{3}{4}$.

$7\frac{1}{2} - 2\frac{3}{4} = 5\frac{2-3}{4} = 4\frac{3}{4}$. *Ans.*

21. Find the value of $8\frac{1}{5} - 4\frac{4}{7}$.

$8\frac{1}{5} - 4\frac{4}{7} = 4\frac{7-20}{35} = 3\frac{22}{35}$.

22. Find the value of $85\frac{7}{22} - 27\frac{11}{18}$.

$85\frac{7}{22} - 27\frac{11}{18} = 58\frac{63-121}{198} = 57\frac{140}{198} = 57\frac{70}{99}$. *Ans.*

23. Find the value of $8\frac{7}{10} - 2\frac{11}{16}$.

$8\frac{7}{10} - 2\frac{11}{16} = 6\frac{56-55}{80} = 6\frac{1}{80}$. *Ans.*

24. Find the value of $10 - 3\frac{5}{8}$.

$10 - 3\frac{5}{8} = 6\frac{3}{8}$. *Ans.*

25. Find the value of $120\frac{21}{32} - 110\frac{13}{24}$.

$120\frac{21}{32} - 110\frac{13}{24} = 10\frac{63-52}{96} = 10\frac{11}{96}$. *Ans.*

26. Find the value of $5\frac{17}{28} - \frac{27}{35}$.

$5\frac{17}{28} - \frac{27}{35} = 5\frac{85-108}{140} = 4\frac{117}{140}$. *Ans.*

27. Find the value of $13\frac{3}{40} - 2\frac{15}{44}$.

$$13\tfrac{3}{40} - 2\tfrac{15}{44} = 11\tfrac{33-150}{440} = 10\tfrac{323}{440}.\ \textit{Ans.}$$

28. Find the value of $2\frac{151}{240} - 1\frac{163}{192}$.

$$2\tfrac{151}{240} - 1\tfrac{163}{192} = 1\tfrac{604-815}{960} = \tfrac{749}{960}.\ \textit{Ans.}$$

29. Find the value of $4 - 1\frac{2317}{4000}$.

$$4 - 1\tfrac{2317}{4000} = 2\tfrac{4000-2317}{4000} = 2\tfrac{1683}{4000}.\ \textit{Ans.}$$

30. Find the value of $1473 - 279\frac{11}{12}$.

$$1473 - 279\tfrac{11}{12} = 1193\tfrac{1}{12}.\ \textit{Ans.}$$

31. Find the value of $1473\frac{5}{13} - 279\frac{11}{12}$.

$$1473\tfrac{5}{13} - 279\tfrac{11}{12} = 1194\tfrac{60-143}{156} = 1193\tfrac{73}{156}.\ \textit{Ans.}$$

32. Find the value of $1473\frac{7}{18} - 279\frac{11}{12}$.

$$1473\tfrac{7}{18} - 279\tfrac{11}{12} = 1194\tfrac{14-33}{36} = 1193\tfrac{17}{36}.\ \textit{Ans.}$$

33. Find the value of $278\frac{15}{16} - 30\frac{5}{12}$.

$$278\tfrac{15}{16} - 30\tfrac{5}{12} = 248\tfrac{45-20}{48} = 248\tfrac{25}{48}.\ \textit{Ans.}$$

34. Find the value of $125\frac{5}{22} - 10\frac{17}{33}$.

$$125\tfrac{5}{22} = 10\tfrac{17}{33} = 115\tfrac{15-34}{66} = 114\tfrac{47}{66}.\ \textit{Ans.}$$

35. Find the value of $118\frac{5}{11} - 17\frac{3}{14}$.

$$118\tfrac{5}{11} - 17\tfrac{3}{14} = 101\tfrac{70-33}{154} = 101\tfrac{37}{154}.\ \textit{Ans.}$$

36. Find the value of $94\frac{5}{11} - 91\frac{13}{14}$.

$$94\tfrac{5}{11} - 91\tfrac{13}{14} = 3\tfrac{70-143}{154} = 2\tfrac{81}{154}.\ \textit{Ans.}$$

37. Find the value of $7\frac{5}{21} - 2\frac{11}{14}$.

$$7\tfrac{5}{21} - 2\tfrac{11}{14} = 5\tfrac{10-33}{42} = 4\tfrac{19}{42}.\ \textit{Ans.}$$

38. Find the value of $\frac{235}{357} - \frac{13}{51}$.

$$\frac{235}{357} - \frac{13}{51} = \frac{235-91}{357} = \frac{48}{119}.\ \textit{Ans.}$$

39. Find the value of $\frac{17}{63} - \frac{29}{108}$.

$$\frac{17}{63} - \frac{29}{108} = \frac{204 - 203}{756} = \frac{1}{756}. \textit{ Ans.}$$

40. Find the value of $\frac{9}{38} - \frac{43}{209}$.

$$\frac{9}{38} - \frac{43}{209} = \frac{99 - 86}{418} = \frac{13}{418}. \textit{ Ans.}$$

41. Find the value of $\frac{146}{273} - \frac{268}{637}$.

$$\frac{146}{273} - \frac{268}{637} = \frac{1032 - 804}{1911} = \frac{218}{1911}. \textit{ Ans.}$$

42. Find the value of $\frac{359}{360} - \frac{199}{200}$.

$$\frac{359}{360} - \frac{199}{200} = \frac{1795 - 1791}{1800} = \frac{1}{450}. \textit{ Ans.}$$

EXERCISE XXVII.

1. Simplify $3\frac{2}{5} - 2\frac{5}{8} + 4\frac{3}{10} + 1\frac{7}{9} - 5\frac{8}{15}$.

Sum of plus terms $= 9\frac{43}{90}$

Sum of minus terms $= 8\frac{19}{120}$

Difference $= 1\frac{23}{72}$. *Ans.*

2. Simplify $1\frac{5}{11} - \frac{11}{12} + 7\frac{3}{8} - 2\frac{1}{3} - 1\frac{1}{16}$.

Sum of plus terms $= 8\frac{73}{88}$

Sum of minus terms $= 4\frac{15}{16}$

Difference $= 3\frac{157}{176}$. *Ans.*

3. Simplify $12 - 3\frac{2}{7} - 1\frac{3}{10} - 4\frac{5}{28} + 2\frac{13}{20} - 4\frac{3}{5}$.

Sum of plus terms $= 14\frac{13}{20}$

Sum of minus terms $= 13\frac{51}{140}$

Difference $= 1\frac{2}{7}$. *Ans.*

4. Simplify $43\frac{7}{15} - 1\frac{1}{3} - 1\frac{21}{48} - 1\frac{23}{24} - 2\frac{13}{48} - 2\frac{7}{12} - 2\frac{43}{48} - 3\frac{}{12}$.

Sum of plus terms $= 43\frac{7}{15}$

Sum of minus terms $= 16\frac{5}{48}$

Difference $= 27\frac{29}{80}$. *Ans.*

5. Simplify $\frac{1}{2}+\frac{4}{13}+7\frac{9}{40}+8\frac{14}{39}+7\frac{1}{4}+8\frac{3}{10}+4\frac{1}{12}-36\frac{1}{40}$.

Sum of plus terms $=36\frac{1}{40}$
Sum of minus terms $=36\frac{1}{40}$

Difference $=0$. *Ans.*

6. Simplify $(8\frac{5}{18}+1\frac{10}{27}+17\frac{11}{36}+40)-(30\frac{13}{40}+11\frac{11}{20})$.

Sum of plus terms $=66\frac{103}{108}$
Sum of minus terms $=41\frac{7}{8}$

Difference $=25\frac{17}{216}$. *Ans.*

7. Simplify $(172\frac{19}{78}+93\frac{14}{117})+172\frac{19}{78}-93\frac{14}{117}$.

$$(172\tfrac{19}{78}+93\tfrac{14}{117})+(172\tfrac{19}{78}-93\tfrac{14}{117})$$
$$=172\tfrac{19}{78}+93\tfrac{14}{117}+172\tfrac{19}{78}-93\tfrac{14}{117}$$
$$=172\tfrac{19}{78}+172\tfrac{19}{78}=344\tfrac{19+19}{78}=344\tfrac{19}{39}.\ \textit{Ans.}$$

8. Simplify $(172\frac{19}{78}+93\frac{14}{117})-(172\frac{19}{78}-93\frac{14}{117})$.

$$(172\tfrac{19}{78}+93\tfrac{14}{117})-(172\tfrac{19}{78}-93\tfrac{14}{117})$$
$$=172\tfrac{19}{78}+93\tfrac{14}{117}-172\tfrac{19}{78}+93\tfrac{14}{117}$$
$$=93\tfrac{14}{117}+93\tfrac{14}{117}=186\tfrac{28}{117}.\ \textit{Ans.}$$

9. Simplify $(\frac{3}{13}-\frac{2}{39})+(\frac{5}{78}+\frac{7}{156})$.

$$\left(\frac{3}{13}-\frac{2}{39}\right)+\left(\frac{5}{78}+\frac{7}{156}\right)=\frac{3}{13}-\frac{2}{39}+\frac{5}{78}+\frac{7}{156}$$
$$=\frac{36-8+10+7}{156}=\frac{15}{52}.\ \textit{Ans.}$$

10. Simplify $\frac{4}{9}-\frac{3}{11}-2\frac{3}{4}+3\frac{2}{3}+7\frac{7}{12}-1\frac{3}{5}-\frac{3}{22}$.

Sum of plus terms $=11\frac{25}{36}$
Sum of minus terms $=4\frac{167}{220}$

Difference $=6\frac{463}{495}$. *Ans.*

11. Simplify $\frac{3}{10}-\frac{7}{100}-\frac{9}{1000}-\frac{5}{10000}$.

$$\frac{3}{10}-\frac{7}{100}-\frac{9}{1000}-\frac{5}{10000}=\frac{3000-700-90-5}{10000}=\frac{441}{2000}.\ \textit{Ans.}$$

12. Simplify $9\frac{5}{8}-7-\frac{3}{4}-\frac{5}{6}$.

$$9\tfrac{5}{8}-7-\tfrac{3}{4}-\tfrac{5}{6}=2\tfrac{15-18-20}{24}=1\tfrac{1}{24}.\ \textit{Ans.}$$

13. Simplify $5\frac{2}{3} + 8\frac{3}{4} - 1\frac{3}{5} - 4\frac{7}{9}$.

Sum of plus terms $= 14\frac{5}{12}$
Sum of minus terms $= 6\frac{17}{45}$

Difference $= 8\frac{7}{180}$. *Ans.*

14. Simplify $6\frac{3}{4} - 5\frac{2}{3} + 4\frac{2}{5} - 4\frac{5}{12}$.

Sum of plus terms $= 11\frac{3}{20}$
Sum of minus terms $= 10\frac{1}{12}$

Difference $= 1\frac{1}{15}$. *Ans.*

15. Simplify $14\frac{7}{18} + 9\frac{3}{5} - 6\frac{3}{4} - 12\frac{4}{5} - 3\frac{3}{5}$.

Sum of plus terms $= 23\frac{89}{90}$
Sum of minus terms $= 23\frac{3}{20}$

Difference $= \frac{151}{180}$. *Ans.*

16. Simplify $20\frac{2}{3} - 2\frac{5}{8} - 9\frac{5}{9} + 10\frac{3}{10} - 14\frac{7}{12}$.

Sum of plus terms $= 30\frac{29}{30}$
Sum of minus terms $= 26\frac{55}{72}$

Difference $= 4\frac{73}{360}$. *Ans.*

17. Simplify $95\frac{2}{3} - 9\frac{7}{20} - 8\frac{3}{7} - 14\frac{6}{15} + 74\frac{2}{9}$.

Sum of plus terms $= 169\frac{8}{9}$
Sum of minus terms $= 32\frac{5}{28}$

Difference $= 137\frac{179}{252}$. *Ans.*

18. Simplify $12\frac{3}{4} + 23\frac{3}{8} - (4\frac{3}{10} + 12\frac{2}{5} + 7\frac{14}{15})$.

Sum of plus terms $= 36\frac{1}{8}$
Sum of minus terms $= 24\frac{19}{30}$

Difference $= 11\frac{59}{120}$. *Ans.*

19. Simplify $16\frac{2}{15} + 18\frac{5}{24} - (5\frac{2}{5} + 9\frac{9}{40} + 14\frac{5}{24})$.

Terms outside parenthesis $= 34\frac{41}{120}$
Terms inside parenthesis $= 28\frac{100}{120}$

Difference $= 5\frac{61}{120}$. *Ans.*

20. Simplify $97\frac{8}{9} - (20 + 9\frac{3}{4} + 18\frac{3}{25} + 24\frac{47}{50})$.

Terms outside parenthesis $= 97\frac{8}{9}$

Terms inside parenthesis $= 72\frac{81}{100}$

Difference $= 25\frac{71}{900}$. *Ans.*

21. Simplify $2\frac{12}{16} + 3\frac{15}{35} - (1\frac{53}{70} + 1\frac{31}{32} + \frac{49}{56})$.

Terms outside parenthesis $= 6\frac{5}{28}$

Terms inside parenthesis $= 4\frac{673}{1120}$

Difference $= 1\frac{647}{1120}$. *Ans.*

22. Simplify $\frac{143}{100} + \frac{2471}{1000} - \frac{82643}{100000}$.

$$\frac{143}{100} + \frac{2471}{1000} - \frac{82643}{100000} = \frac{143000 + 247100 - 82643}{100000}$$

$$= \frac{307457}{100000} = 3\frac{7457}{100000}. \textit{ Ans.}$$

Exercise XXVIII.

1. Simplify $\frac{2\frac{3}{11}}{3\frac{3}{4}}$.

Multiply by 44.

$\frac{100}{165} = \frac{20}{33}$. *Ans.*

2. Simplify $\frac{3}{7\frac{1}{8}}$.

Multiply by 8.

$\frac{24}{57} = \frac{8}{19}$. *Ans.*

3. Simplify $\frac{17\frac{1}{7}}{13\frac{1}{3}}$.

Multiply by 21.

$\frac{360}{280} = 1\frac{2}{7}$. *Ans.*

4. Simplify $\frac{\frac{5}{9}}{8\frac{1}{3}}$.

Multiply by 27.

$\frac{15}{225} = \frac{1}{15}$. *Ans.*

5. Simplify $\frac{5\frac{1}{9}}{8\frac{4}{11}}$.

Multiply by 99.

$\frac{253}{414} = \frac{11}{18}$. *Ans.*

6. Simplify $\frac{1\frac{4}{5} \text{ of } 3\frac{1}{7}}{4\frac{1}{8} \text{ of } \frac{9}{10}}$.

$$\frac{\cancel{9}}{\cancel{5}} \times \frac{\overset{2}{\cancel{22}}}{7} \times \frac{8}{\underset{3}{\cancel{33}}} \times \frac{\overset{2}{\cancel{10}}}{\cancel{9}} = \frac{32}{21} = 1\frac{11}{21}.$$

Ans.

7. Simplify $\dfrac{2\frac{1}{2} - 1\frac{5}{9}}{1\frac{5}{6} - 1\frac{5}{8}}$.

Multiply by 72.

$$\frac{180 - 112}{132 - 117} = \frac{68}{15} = 4\tfrac{8}{15}. \textit{ Ans.}$$

8. Simplify $\dfrac{10\frac{2}{5} - 1\frac{5}{7}}{7\frac{1}{8} - 3\frac{3}{40}}$.

Multiply by 280.

$$\frac{2912 - 480}{1995 - 861} = \frac{2432}{1134} = 2\tfrac{82}{567}. \textit{ Ans.}$$

9. Simplify $\dfrac{\frac{3}{7} \text{ of } 2\frac{1}{17}}{1\frac{2}{3} \div 2\frac{3}{7}}$.

$$\frac{3}{\cancel{7}} \times \frac{\overset{5}{\cancel{35}}}{\cancel{17}} \times \frac{\cancel{17}}{7} \times \frac{3}{\cancel{5}} = 1\tfrac{2}{7}. \textit{ Ans.}$$

10. Simplify $\dfrac{6\frac{3}{4} - 1\frac{5}{14}}{2\frac{1}{6} + 1\frac{3}{7}}$.

Multiply by 84.

$$\frac{567 - 114}{182 + 120} = \frac{453}{302} = 1\tfrac{1}{2}. \textit{ Ans.}$$

11. Simplify $\dfrac{5\frac{4}{5} + 2\frac{3}{7}}{4\frac{2}{3} - 3\frac{11}{13}}$.

$$\frac{8\frac{8}{35}}{\frac{32}{39}} = \frac{\overset{9}{\cancel{288}}}{35} \times \frac{39}{\cancel{32}} = \frac{351}{35} = 10\tfrac{1}{35}. \textit{ Ans.}$$

12. Simplify $\dfrac{8\frac{3}{4}}{14} - \dfrac{\frac{2}{3}}{1\frac{1}{7}}$.

$$\frac{\overset{5}{\cancel{35}}}{4} \times \frac{1}{\underset{2}{\cancel{14}}} - \frac{\cancel{2}}{3} \times \frac{7}{\underset{4}{\cancel{8}}} = \frac{5}{8} - \frac{7}{12} = \frac{1}{24}. \textit{ Ans.}$$

13. Simplify $\dfrac{3\frac{3}{7}}{11\frac{1}{4}}$ of $\dfrac{3\frac{3}{8}}{2\frac{2}{5}}$.

$$\frac{\overset{2}{\cancel{24}}}{7} \times \frac{\cancel{4}}{\underset{9}{\cancel{45}}} \times \frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{5}}{\cancel{12}} = \frac{3}{7}. \textit{ Ans.}$$

14. Simplify $\dfrac{5\frac{8}{9} - 4\frac{11}{12}}{5\frac{3}{8} - 2\frac{17}{18}}$.

$$\frac{\frac{35}{36}}{2\frac{31}{72}} = \frac{70}{175} = \frac{2}{5}. \textit{ Ans.}$$

15. Simplify $\dfrac{2\frac{3}{4} + 2\frac{7}{8}}{4\frac{3}{4} - 3\frac{1}{7}}$.

$$\frac{5\frac{5}{8}}{1\frac{17}{28}} = \frac{\cancel{45}}{\underset{2}{\cancel{8}}} \times \frac{\overset{7}{\cancel{28}}}{\cancel{45}} = 3\tfrac{1}{2}. \textit{ Ans.}$$

16. Simplify $\dfrac{2\frac{3}{5} \times \frac{9}{11}}{3\frac{5}{7} \div 4\frac{1}{8}}$.

$$\frac{\cancel{13}}{5} \times \frac{9}{\cancel{11}} \times \frac{\overset{3}{\cancel{33}}}{8} \times \frac{7}{\underset{2}{\cancel{26}}} = \frac{189}{80} = 2\tfrac{29}{80}. \textit{ Ans.}$$

17. Simplify $\dfrac{\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}}{\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}}$.

Multiply by 60.

$$\frac{51 + 44 + 42 + 48}{51 - 44 + 42 - 48} = 185. \textit{ Ans.}$$

18. Simplify $\dfrac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}}$.

Multiply by 28.

$$\frac{116 - 63}{182 - 60} = \frac{53}{122}. \textit{ Ans.}$$

19. Simplify $\dfrac{2\frac{27}{40} - 4\frac{4}{7} + 3\frac{1}{8}}{5\frac{6}{7} - 4\frac{7}{8} + \frac{2}{5}}$.

Multiply by 280.

$$\frac{749 - 1280 + 875}{1640 - 1365 + 112} = \frac{344}{387} = \frac{8}{9}. \textit{ Ans.}$$

20. Simplify $\dfrac{1\frac{1}{4} \times 1\frac{2}{7} + \frac{1}{3} \text{ of } 2\frac{1}{4} - \frac{13}{28} \times 2}{\frac{13}{28} \text{ of } 2 + \frac{1}{3} \text{ of } 2\frac{1}{4} - 1\frac{1}{4} \text{ of } 1\frac{2}{7}}$.

$$\frac{\frac{45}{28} + \frac{3}{4} - \frac{13}{14}}{\frac{13}{14} + \frac{3}{4} - \frac{45}{28}} = \frac{45 + 21 - 26}{26 + 21 - 45} = \frac{40}{2} = 20. \textit{ Ans.}$$

21. Simplify $2\frac{1}{4} \times \dfrac{10\frac{1}{3} - 4\frac{11}{12}}{6\frac{3}{16} + 7\frac{2}{3}} \times \dfrac{3\frac{5}{11}}{1\frac{2}{5} \times 9\frac{1}{11}}$.

$$\frac{9}{4} \times \frac{516 - 236}{297 + 368} \times \frac{38}{140} = \frac{9}{\cancel{4}} \times \frac{\overset{14}{\cancel{56}}}{\underset{7}{\cancel{133}}} \times \frac{\cancel{19}}{\underset{5}{\cancel{70}}} = \frac{9}{35}. \textit{ Ans.}$$

22. Simplify $\dfrac{8\frac{7}{8} - 7\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5}}{9\frac{9}{10} - 8\frac{13}{15} + 7\frac{7}{8} - 6\frac{6}{7}}$.

$$\frac{7455 - 6600 + 4900 - 4032}{8316 - 7448 + 6615 - 5760} = \frac{1723}{1723} = 1. \textit{ Ans.}$$

Exercise XXIX.

1. What fraction of 8 is 3?

$\frac{3}{8}$. *Ans.*

2. What fraction of 3 is 8?

$\frac{8}{3} = 2\frac{2}{3}$. *Ans.*

3. What fraction of 9 is 7?

$\frac{7}{9}$. *Ans.*

4. What fraction of 7 is 9?

$\frac{9}{7} = 1\frac{2}{7}$. *Ans.*

5. What fraction of 8 is 12?

$\frac{12}{8} = 1\frac{1}{2}$. *Ans.*

6. What fraction of 12 is 8?

$\frac{8}{12} = \frac{2}{3}$. *Ans.*

7. What fraction of $2\frac{1}{5}$ is $\frac{3}{5}$.

$\dfrac{\frac{3}{5}}{2\frac{1}{5}} = \frac{3}{11}$. *Ans.*

8. What fraction of $\frac{3}{5}$ is $2\frac{1}{5}$?

$\dfrac{2\frac{1}{5}}{\frac{3}{5}} = 3\frac{2}{3}$. *Ans.*

9. What fraction of $2\frac{3}{4}$ is $1\frac{1}{4}$?

$$\frac{1\frac{1}{4}}{2\frac{3}{4}} = \frac{5}{11}. \text{ } \textit{Ans.}$$

10. What fraction of $1\frac{1}{4}$ is $2\frac{3}{4}$?

$$\frac{2\frac{3}{4}}{1\frac{1}{4}} = 2\frac{1}{5}. \text{ } \textit{Ans.}$$

11. What fraction of $2\frac{1}{3}$ is $7\frac{1}{8}$?

$$\frac{7\frac{1}{8}}{2\frac{1}{3}} = 3\frac{3}{56}. \text{ } \textit{Ans.}$$

12. What fraction of $7\frac{1}{3}$ is $2\frac{1}{8}$?

$$\frac{2\frac{1}{8}}{7\frac{1}{3}} = \frac{51}{176}. \text{ } \textit{Ans.}$$

13. What fraction of $3\frac{1}{3}$ is $8\frac{1}{7}$?

$$\frac{8\frac{1}{7}}{3\frac{1}{3}} = 2\frac{31}{70}. \text{ } \textit{Ans.}$$

14. What fraction of \$2 is \$$1\frac{1}{2}$?

$$\frac{\$1\frac{1}{2}}{\$2} = \frac{3}{4}. \text{ } \textit{Ans.}$$

15. What fraction of \$$2\frac{1}{2}$ is \$5?

$$\frac{\$5}{\$2\frac{1}{2}} = 2. \text{ } \textit{Ans.}$$

16. What fraction of \$$\frac{3}{4}$ is \$$\frac{1}{4}$?

$$\frac{\$\frac{1}{4}}{\$\frac{3}{4}} = \frac{1}{3}. \text{ } \textit{Ans.}$$

17. What fraction of \$$\frac{5}{4}$ is \$$\frac{3}{8}$?

$$\frac{\$\frac{3}{8}}{\$\frac{5}{4}} = \frac{3}{10}. \text{ } \textit{Ans.}$$

18. What fraction of \$$2\frac{3}{4}$ is \$$\frac{1}{3}$?

$$\frac{\$\frac{1}{3}}{\$2\frac{3}{4}} = \frac{4}{33}. \text{ } \textit{Ans.}$$

19. What fraction of \$$\frac{1}{2}$ is \$$\frac{1}{10}$?

$$\frac{\$\frac{1}{10}}{\$\frac{1}{2}} = \frac{1}{5}. \text{ } \textit{Ans.}$$

20. What fraction of \$1 is \$$\frac{7}{8}$?

$$\frac{\$\frac{7}{8}}{\$1} = \frac{7}{8}. \text{ } \textit{Ans.}$$

21. What fraction of \$10 is \$$\frac{2}{3}$?

$$\frac{\$\frac{2}{3}}{\$10} = \frac{1}{15}. \text{ } \textit{Ans.}$$

22. What fraction of \$100 is \$6?

$$\frac{\$6}{\$100} = \frac{3}{50}. \text{ } \textit{Ans.}$$

23. What fraction of \$100 is \$$4\frac{1}{2}$?

$$\frac{\$4\frac{1}{2}}{\$100} = \frac{9}{200}. \text{ } \textit{Ans.}$$

24. What fraction of \$4 is \$25?

$$\frac{\$25}{\$4} = 6\frac{1}{4}. \text{ } \textit{Ans.}$$

25. What fraction of $100\frac{5}{9}$ is $8\frac{4}{9}$?

$$\frac{8\frac{4}{9}}{100\frac{5}{9}} = \frac{76}{905}. \text{ } \textit{Ans.}$$

26. What fraction of 21 is $\frac{15}{19}$ of $3\frac{4}{5}$?

$$\frac{\frac{15}{19} \times 3\frac{4}{5}}{21} = \frac{1}{7}. \text{ } \textit{Ans.}$$

27. What fraction of

$18\frac{117}{256}$ is $\frac{5}{8}$ of $33\frac{3}{4}$?

$$\frac{\frac{5}{8} \times 33\frac{3}{4}}{18\frac{117}{256}} = 1\frac{1}{7}. \ \textit{Ans.}$$

28. What fraction of

$3\frac{1}{3}$ is $\frac{2}{3} \times 1\frac{1}{3}$?

$$\frac{\frac{2}{3} \times 1\frac{1}{3}}{3\frac{1}{3}} = \frac{4}{15}. \ \textit{Ans.}$$

29. What fraction of

$3\frac{1}{11} \times 5\frac{1}{27}$ is 1720?

$$\frac{1720}{3\frac{1}{11} \times 5\frac{1}{27}} = 110\frac{275}{578}. \ \textit{Ans.}$$

30. What fraction of

$3\frac{1}{2} \times \frac{8}{9}$ of $\frac{4}{7}$ is $1\frac{3}{5}$?

$$\frac{1\frac{3}{5}}{3\frac{1}{2} \times \frac{8}{9} \times \frac{4}{7}} = \frac{9}{10}. \ \textit{Ans.}$$

31. What part of

$\frac{28}{59} \times \frac{59}{63}$ is $\frac{1}{6} \times 4 \times \frac{2}{3}$?

$$\frac{\frac{1}{6} \times 4 \times \frac{2}{3}}{\frac{28}{59} \times \frac{59}{63}} = 1. \ \textit{Ans.}$$

32. What part of

$13\frac{5}{8} \times \frac{2}{3} \times \frac{9}{65}$ is $\frac{2}{3}$ of $1\frac{44}{65}$ of $1\frac{1}{8}$?

$$\frac{\frac{2}{3} \times 1\frac{44}{65} \times 1\frac{1}{8}}{13\frac{5}{8} \times \frac{2}{3} \times \frac{9}{65}} = 1. \ \textit{Ans.}$$

33. What part of $\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}$ is $\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}$?

$$\frac{\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}}{\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}}.$$

Multiply by 60.

$$\frac{51 - 44 + 42 - 48}{51 + 44 + 42 + 48} = \frac{1}{185}. \ \textit{Ans.}$$

34. What part of $4\frac{1}{7} - 2\frac{1}{4}$ is $6\frac{1}{2} - 2\frac{1}{7}$?

$$\frac{6\frac{1}{2} - 2\frac{1}{7}}{4\frac{1}{7} - 2\frac{1}{4}} = \frac{182 - 60}{116 - 63} = \frac{122}{53} = 2\frac{16}{53}. \ \textit{Ans.}$$

35. What part of $17\frac{2}{3} - 12\frac{6}{7}$ is $5 - \frac{1}{13} - \frac{4}{39} - \frac{1}{25}$?

$$\frac{5 - \frac{1}{13} - \frac{4}{39} - \frac{1}{25}}{17\frac{2}{3} - 12\frac{6}{7}}.$$

Multiply by 6825.

$$\frac{34125 - 525 - 700 - 273}{120575 - 87750} = \frac{32627}{32825}. \ \textit{Ans.}$$

36. What part of $24 - 17\frac{4}{13}$ is $7 + \frac{2}{15} - \frac{5}{81} - \frac{11}{45}$?

$$\frac{7 + \frac{2}{15} - \frac{5}{81} - \frac{11}{45}}{24 - 17\frac{4}{13}}.$$

Multiply by 5265.

$$\frac{36855 + 702 - 325 - 1287}{126360 - 91125} = \frac{35945}{35235} = 1\frac{142}{7047}. \ \textit{Ans.}$$

37. What part of $\frac{3}{7}$ of $2\frac{1}{17}$ is $1\frac{2}{3} \div 2\frac{3}{7}$?

$$\frac{1\frac{2}{3} \div 2\frac{3}{7}}{\frac{3}{7} \times 2\frac{1}{17}} = \frac{\cancel{7}}{\cancel{17}} \times \frac{\cancel{5}}{3} \times \frac{7}{3} \times \frac{\cancel{17}}{\underset{\cancel{7}}{\cancel{35}}} = \frac{7}{9}. \textit{ Ans.}$$

38. What part of

$$\left(\frac{7}{4-\frac{5}{6}} - \frac{5}{6-\frac{3}{8}}\right) \div \left(\frac{4}{7-\frac{4}{7}} + \frac{2}{4-\frac{2}{5}}\right) \text{ is}$$

$$\left(14 - \frac{1}{\frac{1}{2}-\frac{6}{31}}\right) \div \left(\frac{1}{\frac{1}{2}-\frac{27}{59}} - 13\right)?$$

$$\frac{\left(14 - \frac{1}{\frac{1}{2}-\frac{6}{31}}\right) \div \left(\frac{1}{\frac{1}{2}-\frac{27}{59}} - 13\right)}{\left(\frac{7}{4-\frac{5}{6}} - \frac{5}{6-\frac{3}{8}}\right) \div \left(\frac{4}{7-\frac{4}{7}} + \frac{2}{4-\frac{2}{5}}\right)}$$

$$= \frac{(14 - \frac{62}{19}) \div (\frac{118}{5} - 13)}{(\frac{42}{19} - \frac{8}{9}) \div (\frac{28}{45} + \frac{5}{9})} = \frac{\frac{204}{19} \times \frac{5}{53}}{\frac{226}{171} \times \frac{45}{53}} = \frac{102}{113}. \textit{ Ans.}$$

Exercise XXX.

1. Reduce to common fractions in their lowest terms 0.125.

$0.125 = \frac{125}{1000} = \frac{1}{8}$. *Ans.*

2. Reduce to common fractions in their lowest terms 0.625.

$0.625 = \frac{625}{1000} = \frac{5}{8}$. *Ans.*

3. Reduce to common fractions in their lowest terms 0.675.

$0.675 = \frac{675}{1000} = \frac{27}{40}$. *Ans.*

4. Reduce to common fractions in their lowest terms 10.864.

$10.864 = 10\frac{864}{1000} = 10\frac{108}{125}$. *Ans.*

5. Reduce to common fractions in their lowest terms 50.84.

$50.84 = 50\frac{84}{100} = 50\frac{21}{25}$. *Ans.*

6. Reduce to common fractions in their lowest terms 3.00025.

$3.00025 = 3\frac{25}{100000} = 3\frac{1}{4000}$. *Ans.*

7. Reduce to common fractions in their lowest terms 8.1075.

$8.1075 = 8\frac{1075}{10000} = 8\frac{43}{4000}$. *Ans.*

8. Reduce to common fractions in their lowest terms 35.01024.

$35.01024 = 35\frac{1024}{100000} = 35\frac{32}{3125}$. *Ans.*

9. Reduce to common fractions in their lowest terms 7.015625.

$7.015625 = 7\frac{15625}{1000000} = 7\frac{1}{64}$. *Ans.*

10. Reduce to common fractions in their lowest terms 20.100256.

$20.100256 = 20\frac{100256}{1000000} = 20\frac{3133}{31250}$. *Ans.*

11. Reduce to common fractions in their lowest terms 10.012575.

$$10.012575 = 10\tfrac{12575}{1000000} = 10\tfrac{503}{40000}. \textit{ Ans.}$$

12. Reduce to common fractions in their lowest terms 104.235.

$$104.235 = 104\tfrac{235}{1000} = 104\tfrac{47}{200}. \textit{ Ans.}$$

13. Reduce to common fractions in their lowest terms 50.0004.

$$50.0004 = 50\tfrac{4}{10000} = 50\tfrac{1}{2500}. \textit{ Ans.}$$

14. Reduce to common fractions in their lowest terms 100.001.

$$100.001 = 100\tfrac{1}{1000}. \textit{ Ans.}$$

15. Reduce to common fractions in their lowest terms 8.00725.

$$8.00725 = 8\tfrac{725}{100000} = 8\tfrac{29}{4000}. \textit{ Ans.}$$

16. Reduce to common fractions in their lowest terms 20.018375.

$$20.018375 = 20\tfrac{18375}{1000000} = 20\tfrac{147}{8000}. \textit{ Ans.}$$

17. Reduce to common fractions in their lowest terms 125.6048.

$$125.6048 = 125\tfrac{6048}{10000} = 125\tfrac{378}{625}. \textit{ Ans.}$$

18. Reduce to common fractions in their lowest terms 0.128.

$$0.128 = \tfrac{128}{1000} = \tfrac{16}{125}. \textit{ Ans.}$$

19. Reduce to common fractions in their lowest terms 0.73125.

$$0.73125 = \tfrac{73125}{100000} = \tfrac{117}{160}. \textit{ Ans.}$$

20. Reduce to common fractions in their lowest terms 1.1875.

$$1.1875 = 1\tfrac{1875}{10000} = 1\tfrac{3}{16}. \textit{ Ans.}$$

21. Reduce to common fractions in their lowest terms 0.603125.

$$0.603125 = \tfrac{603125}{1000000} = \tfrac{193}{320}. \textit{ Ans.}$$

22. Reduce to common fractions in their lowest terms 6.03125.

$$6.03125 = 6\tfrac{3125}{100000} = 6\tfrac{1}{32}. \textit{ Ans.}$$

23. Reduce to common fractions in their lowest terms 60.3125.

$$60.3125 = 60\tfrac{3125}{10000} = 60\tfrac{5}{16}. \textit{ Ans.}$$

24. Reduce to common fractions in their lowest terms 7.0315.

$$7.0315 = 7\tfrac{315}{10000} = 7\tfrac{63}{2000}. \textit{ Ans.}$$

EXERCISE XXXI.

1. Reduce to decimals $\frac{7}{8}$.

$$8\overline{)7.000}\quad 0.875$$

2. Reduce to decimals $\frac{15}{16}$.

$$16\overline{)15.0000}\quad 0.9375$$

3. Reduce to decimals $\frac{9}{32}$.

$$32\overline{)9.00000}\quad 0.28125$$

4. Reduce to decimals $\frac{9}{25}$.

$$25\overline{)9.00}\quad 0.36$$

5. Reduce to decimals $\frac{5}{64}$.

$$64\overline{)5.000000}\quad 0.078125$$

6. Reduce to decimals $4\frac{11}{800}$.

$$8\overline{)0.11000}\quad 0.01375$$

4.01375. *Ans.*

7. Reduce to decimals $5\frac{5}{32000}$.

$$32\overline{)0.00500000}\quad 0.00015625$$

5.00015625. *Ans.*

8. Reduce to decimals $9\frac{123}{25600}$.

$$256\overline{)1.2300000000}\quad 0.0048046875$$

9.0048046875. *Ans.*

9. Reduce to decimals $11\frac{19}{4000}$.

$$4\overline{)0.01900}\quad 0.00475$$

11.00475. *Ans.*

10. Reduce to decimals $\frac{9}{125}$.

$$125\overline{)9.000}\quad 0.072$$

11. Reduce to decimals $\frac{17}{4000}$.

$$4\overline{)0.01700}\quad 0.00425$$

12. Reduce to decimals $\frac{119}{128}$.

$$128\overline{)119.0000000}\quad 0.9296875$$

13. Reduce to decimals $\frac{13}{625}$.

$$625\overline{)13.0000}\quad 0.0208$$

14. Reduce to decimals $\frac{11}{256}$.

$$256\overline{)11.00000000}\quad 0.04296875$$

15. Reduce to decimals $\frac{3}{160}$.

$$16\overline{)0.30000}\quad 0.01875$$

16. Reduce to decimals $\frac{124}{16}$.

$$16\overline{)124.00}\quad 7.75$$

17. Reduce to decimals $\frac{2}{3}$ of $1\frac{4}{5}$.

$$\frac{2}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{5} = \frac{6}{5}.$$

$$5\overline{)6.0}\quad 1.2$$

18. Reduce to decimals $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{7}{10}$.

$$\frac{3}{4} \times \frac{\cancel{5}}{8} \times \frac{7}{\underset{2}{\cancel{10}}} = \frac{21}{64}.$$

$$64\overline{)21.000000}\quad 0.328125$$

19. Reduce to decimals $3\frac{3}{5}$ of $4\frac{1}{9}$.

$$\frac{\overset{2}{\cancel{18}}}{5} \times \frac{37}{\cancel{9}} = \frac{74}{5}.$$

$$5\overline{)74.0}\quad 14.8$$

20. Reduce to decimals $\frac{29}{32}$ of $\frac{49}{64}$.

$$\frac{29}{32} \times \frac{49}{64} = \frac{1421}{2048}.$$

$$2048\overline{)1421.00000000000}\quad 0.69384765625$$

Exercise XXXII.

1. In like manner simplify $7\frac{2}{5} + 4\frac{5}{8} + 9\frac{13}{20} + 11\frac{29}{32}$.

$7\frac{2}{5} + 4\frac{5}{8} + 9\frac{13}{20} + 11\frac{29}{32} = 7.4 + 4.625 + 9.65 + 11.90625 = 33.58125.$ (1)

$7\frac{2}{5} + 4\frac{5}{8} + 9\frac{13}{20} + 11\frac{29}{32} = 31\frac{413}{160} = 33\frac{93}{160} = 33.58125.$ (2)

2. In like manner simplify $84\frac{13}{20} + 19\frac{11}{21} - \frac{41}{50}$.

$84\frac{13}{20} + 19\frac{11}{21} + \frac{41}{50} = 84.65 + 19.523809\frac{11}{21} + 0.82 = 104.993809\frac{11}{21}.$ (1)

$84\frac{13}{20} + 19\frac{11}{21} + \frac{41}{50} = 103\frac{1365+1100+1722}{2100} = 103\frac{4187}{2100} = 104\frac{2087}{2100}$
$= 104.993809\frac{11}{21}.$ (2)

3. In like manner simplify $4\frac{27}{64} + 13\frac{17}{20} + 42\frac{37}{50} + 2\frac{13}{16} + 1\frac{1}{2}$.

$4\frac{27}{64} + 13\frac{17}{20} + 42\frac{37}{50} + 2\frac{13}{16} + 1\frac{1}{2} = 4.421875 + 13.85 + 42.74 + 2.8125 + 1.5$
$= 65.324375.$ (1)

$4\frac{27}{64} + 13\frac{17}{20} + 42\frac{37}{50} + 2\frac{13}{16} + 1\frac{1}{2} = 62\frac{675+1360+1184+1300+800}{1600}$
$= 62\frac{5319}{1600} = 65\frac{519}{1600} = 65.324375.$ (2)

4. In like manner simplify $5\frac{7}{8} + 13\frac{4}{5} + 19\frac{7}{16} + 7\frac{3}{20}$.

$5\frac{7}{8} + 13\frac{4}{5} + 19\frac{7}{16} + 7\frac{3}{20} = 5.875 + 13.8 + 19.4375 + 7.15 = 46.2625.$ (1)

$5\frac{7}{8} + 13\frac{4}{5} + 19\frac{7}{16} + 7\frac{3}{20} = 44\frac{70+64+35+12}{80} = 44\frac{181}{80}$
$= 46\frac{21}{80} = 46.2625.$ (2)

5. In like manner simplify $5\frac{5}{10} + \frac{2}{3}$ of $1\frac{4}{5} + \frac{7}{8}$ of $2\frac{2}{7} + \frac{3}{4}$ of $\frac{5}{8}$.

$$5\tfrac{5}{10} + \tfrac{2}{3} \times 1\tfrac{4}{5} + \tfrac{7}{8} \times 2\tfrac{2}{7} + \tfrac{3}{4} \times \tfrac{5}{8}$$
$$= 5.5 + 0.666\tfrac{2}{3} \times 1.8 + 0.875 \times 2.285714\tfrac{2}{7} + 0.75 \times 0.625$$
$$= 5.5 + 1.2 + 2 + 0.46875 = 9.16875. \qquad (1)$$

$$5\tfrac{5}{10} + \tfrac{2}{3} \times 1\tfrac{4}{5} + \tfrac{7}{8} \times 2\tfrac{2}{7} + \tfrac{3}{4} \times \tfrac{5}{8} = 5\tfrac{1}{2} + 1\tfrac{1}{5} + 2 + \tfrac{15}{32} = 8\tfrac{80+32+75}{160}$$
$$= 8\tfrac{187}{160} = 9\tfrac{27}{160} = 9.16875. \qquad (2)$$

6. In like manner simplify $1\frac{5}{12}$ of $2\frac{5}{8}$.

$$1\tfrac{5}{12} \times 2\tfrac{5}{8} = 1.4166\tfrac{2}{3} \times 2.625 = 3.71875. \qquad (1)$$

$$1\tfrac{5}{12} \times 2\tfrac{5}{8} = \frac{17}{\underset{4}{\cancel{12}}} \times \frac{\overset{7}{\cancel{21}}}{8} = \frac{119}{32} = 3\tfrac{23}{32} = 3.71875. \qquad (2)$$

7. In like manner simplify $3\frac{5}{16} + 2\frac{19}{20}$.

$$3\tfrac{5}{16} + 2\tfrac{19}{20} = 3.3125 + 2.95 = 6.2625. \qquad (1)$$

$$3\tfrac{5}{16} + 2\tfrac{19}{20} = 5\tfrac{25+76}{80} = 5\tfrac{101}{80} = 6\tfrac{21}{80} = 6.2625. \qquad (2)$$

8. In like manner simplify $7\frac{2}{5} - 4\frac{5}{8}$.

$$7\tfrac{2}{5} - 4\tfrac{5}{8} = 7.4 - 4.625 = 2.775. \qquad (1)$$

$$7\tfrac{2}{5} - 4\tfrac{5}{8} = 3\tfrac{16-25}{40} = 2\tfrac{31}{40} = 2.775. \qquad (2)$$

9. In like manner simplify $82\frac{1}{5} - 37\frac{11}{16}$.

$$82\tfrac{1}{5} - 37\tfrac{11}{16} = 82.2 - 37.6875 = 44.5125. \qquad (1)$$

$$82\tfrac{1}{5} - 37\tfrac{11}{16} = 45\tfrac{16-55}{80} = 44\tfrac{41}{80} = 44.5125. \qquad (2)$$

10. In like manner simplify $100 - 17\frac{113}{625}$.

$$100 - 17\tfrac{113}{625} = 100 - 17.1808 = 82.8192. \qquad (1)$$

$$100 - 17\tfrac{113}{625} = 82\tfrac{512}{625} = 82.8192. \qquad (2)$$

11. In like manner simplify $5\frac{1}{2} - 1\frac{1}{2}$ of $1\frac{13}{24}$.

$$5\tfrac{1}{2} - 1\tfrac{1}{2} \times 1\tfrac{13}{24} = 5.5 - 1.5 \times 1.5416\tfrac{2}{3} = 5.5 - 2.3125 = 3.1875. \qquad (1)$$

$$5\tfrac{1}{2} - 1\tfrac{1}{2} \times 1\tfrac{13}{24} = 5\tfrac{1}{2} - 2\tfrac{5}{16} = 3\tfrac{3}{16} = 3.1875. \qquad (2)$$

12. In like manner simplify $\frac{14}{25} - \frac{11}{64}$.

$\frac{14}{25} - \frac{11}{64} = 0.56 - 0.171875 = 0.388125.$ (1)

$\frac{14}{25} - \frac{11}{64} = \frac{896-275}{1600} = \frac{621}{1600} = 0.388125.$ (2)

13. In like manner simplify $8\frac{1}{5} - 1\frac{1}{2} \times \frac{3}{16}$.

$8\frac{1}{5} - 1\frac{1}{2} \times \frac{3}{16} = 8.2 - 1.5 \times 0.1875 = 8.2 - 0.28125 = 7.91875.$ (1)

$8\frac{1}{5} - 1\frac{1}{2} \times \frac{3}{16} = 8\frac{1}{5} - \frac{9}{32} = 8\frac{32-45}{160} = 7\frac{147}{160} = 7.91875.$ (2)

14. In like manner simplify $\frac{19}{64} \times 1000$.

$\frac{19}{64} \times 1000 = 0.296875 \times 1000 = 296.875.$ (1)

$\frac{19}{64} \times \frac{1000}{1} = \frac{2375}{8} = 296\frac{7}{8} = 296.875.$ (2)

Exercise XXXIII.

1. Reduce to decimals $\frac{5}{9}$.

$$\begin{array}{r} 0.\dot{5} \\ 9\overline{)5.0} \end{array}$$

2. Reduce to decimals $\frac{5}{11}$.

$$\begin{array}{r} 0.\dot{4}\dot{5} \\ 11\overline{)5.00} \end{array}$$

3. Reduce to decimals $3\frac{5}{12}$.

$$\begin{array}{r} 0.41\dot{6} \\ 12\overline{)5.000} \end{array}$$

$3\frac{5}{12} = 3.41\dot{6}$. *Ans.*

4. Reduce to decimals $\frac{11}{60}$.

$$\begin{array}{r} 0.18\dot{3} \\ 6\overline{)1.10\dot{0}} \end{array}$$

5. Reduce to decimals $3\frac{17}{48}$.

$$\begin{array}{r} 0.3541\dot{6} \\ 48\overline{)17.00000} \end{array}$$

$3\frac{17}{48} = 3.3541\dot{6}$. *Ans.*

6. Reduce to decimals $2\frac{5}{37}$.

$$\begin{array}{r} 0.\dot{1}3\dot{5} \\ 37\overline{)5.000} \end{array}$$

$2\frac{5}{37} = 2.\dot{1}3\dot{5}$. *Ans.*

7. Reduce to decimals $\frac{3}{3700}$.

$$\begin{array}{r} 0.00\dot{0}8\dot{1} \\ 37\overline{)0.03000} \end{array}$$

8. Reduce to decimals $11\frac{11}{84}$.

$$\begin{array}{r} 0.13\dot{0}9523\dot{8} \\ 84\overline{)11.00000000} \end{array}$$

$11\frac{11}{84} = 11.13\dot{0}9523\dot{8}$. *Ans.*

9. Reduce to decimals $9\frac{11}{108}$.

$$\begin{array}{r} 0.10\dot{1}8\dot{5} \\ 108\overline{)11.00000} \end{array}$$

$9\frac{11}{108} = 9.10\dot{1}8\dot{5}$. *Ans.*

10. Reduce to decimals $11\frac{4}{35}$.

$$\begin{array}{r} 0.1\dot{1}42857\dot{1} \\ 35\overline{)4.0000000} \end{array}$$

$11\frac{4}{35} = 11.1\dot{1}42857\dot{1}$. *Ans.*

11. Reduce to decimals $\frac{15}{56}$.

$$\begin{array}{r} 0.267\dot{8}5714\dot{2} \\ 56\overline{)15.000000000} \end{array}$$

12. Reduce to decimals $\frac{8}{21}$.

$$\begin{array}{r} 0.\dot{3}8095\dot{2} \\ 21\overline{)8.000000} \end{array}$$

13. Reduce to decimals $\frac{13}{33}$.

$$\begin{array}{r} 0.\dot{3}\dot{9} \\ 33\overline{)13.00} \end{array}$$

14. Reduce to decimals $\frac{37}{70}$.

$$\begin{array}{r} 0.5\dot{2}8571\dot{4} \\ 7\overline{)3.7000000} \end{array}$$

15. Reduce to decimals $2\frac{58}{255}$.

$$\begin{array}{r} 0.2\dot{2}74509803921568\dot{6} \\ 255\overline{)58.0000000000000000} \end{array}$$

$2\frac{58}{255} = 2.2\dot{2}74509803921568\dot{6}$.

16. Reduce to decimals $5\frac{6}{26}$.

$$\begin{array}{r} 0.\dot{2}3076\dot{9} \\ 13\overline{)3.000000} \end{array}$$

$5\frac{6}{26} = 5\frac{3}{13} = 5.\dot{2}3076\dot{9}$. *Ans.*

17. If $\dfrac{117}{5^7 \times 2^3}$ be expressed as a decimal, the quotient will contain how many decimal places?

As 7 is the highest power of 2 or 5 in the denominator, and as there are no other factors than 2 or 5, there will be seven decimal places in the quotient.

18. If $\dfrac{119}{2^5 \times 13}$ be expressed as a decimal, how many decimal places will *precede* the recurring period?

As 5 is the highest power of 2 or 5 in the denominator, and as there is another factor than 2 or 5, five decimal places will precede the repetend.

19. If $\dfrac{57}{5^2 \times 7}$ be reduced to a decimal, how many decimal places will precede the recurring period?

As 2 is the highest power of 2 or 5 in the denominator, and as there is another factor than 2 or 5, two decimal places will precede the repetend.

Exercise XXXIV.

1. Reduce to common fractions in their lowest terms $0.2\dot{4}\dot{5}$.

$0.2\dot{4}\dot{5} = \frac{243}{990} = \frac{27}{110}$. *Ans.*

2. Reduce to common fractions in their lowest terms $0.4\dot{2}\dot{5}$.

$0.4\dot{2}\dot{5} = \frac{421}{990}$. *Ans.*

3. Reduce to common fractions in their lowest terms $53.00\dot{2}4\dot{3}$.

$53.00\dot{2}4\dot{3} = 53\frac{243}{99900} = 53\frac{9}{3700}$. *Ans.*

4. Reduce to common fractions in their lowest terms $7.\dot{2}01\dot{1}$.

$7.\dot{2}01\dot{1} = 7\frac{2011}{9999}$. *Ans.*

5. Reduce to common fractions in their lowest terms $2.5\dot{3}0\dot{6}$.

$2.5\dot{3}0\dot{6} = 2\frac{5301}{9990} = 2\frac{1767}{3330}$. *Ans.*

6. Reduce to common fractions in their lowest terms $0.004\dot{2}\dot{6}$.

$0.004\dot{2}\dot{6} = \frac{422}{99000} = \frac{211}{49500}$. *Ans.*

7. Reduce to common fractions in their lowest terms $31.2\dot{0}\dot{3}$.

$31.2\dot{0}\dot{3} = 31\frac{201}{990} = 31\frac{67}{330}$. *Ans.*

8. Reduce to common fractions in their lowest terms $0.\dot{3}5\dot{1}$.

$0.\dot{3}5\dot{1} = \frac{351}{999} = \frac{13}{37}$. *Ans.*

9. Reduce to common fractions in their lowest terms $1.41\dot{6}$.

$1.41\dot{6} = 1\frac{375}{900} = 1\frac{5}{12}$. *Ans.*

10. Reduce to common fractions in their lowest terms $0.5\dot{5}7\dot{5}$.

$0.5\dot{5}7\dot{5} = \frac{5570}{9990} = \frac{557}{999}$. *Ans.*

11. Reduce to common fractions in their lowest terms $2.0\dot{8}\dot{1}$.

$2.0\dot{8}\dot{1} = 2\frac{81}{990} = 2\frac{9}{110}$. *Ans.*

12. Reduce to common fractions in their lowest terms $5.12\dot{2}9\dot{7}$.

$5.12\dot{2}9\dot{7} = 5\frac{12285}{99900} = 5\frac{91}{740}$. *Ans.*

13. Reduce to common fractions in their lowest terms $0.35\dot{9}\dot{0}$

$0.35\dot{9}\dot{0} = \frac{3555}{9900} = \frac{79}{220}$. *Ans.*

14. Reduce to common fractions in their lowest terms $4.3\dot{1}6\dot{2}$.

$4.3\dot{1}6\dot{2} = 4\frac{3159}{9990} = 4\frac{117}{370}$. *Ans.*

15. Reduce to common fractions in their lowest terms $0.7\dot{2}8\dot{3}$.

$$0.7\dot{2}8\dot{3} = \tfrac{7276}{9990} = \tfrac{3638}{4995}. \textit{ Ans.}$$

16. Reduce to common fractions in their lowest terms $5.1\dot{4}2857\dot{7}$.

$$5.1\dot{4}285\dot{7} = 5\tfrac{142856}{999990} = 5\tfrac{71428}{499995}. \textit{ Ans.}$$

17. Reduce to common fractions in their lowest terms $0.2\dot{3}6\dot{8}$.

$$0.2\dot{3}6\dot{8} = \tfrac{2366}{9990} = \tfrac{1183}{4995}. \textit{ Ans.}$$

18. Reduce to common fractions in their lowest terms $1.1\dot{3}\dot{6}$.

$$1.1\dot{3}\dot{6} = 1\tfrac{135}{990} = 1\tfrac{27}{198} = 1\tfrac{3}{22}. \textit{ Ans.}$$

19. Reduce to common fractions in their lowest terms $1.\dot{5}3\dot{1}$.

$$1.\dot{5}3\dot{1} = 1\tfrac{531}{999} = 1\tfrac{59}{111}. \textit{ Ans.}$$

20. Reduce to common fractions in their lowest terms $3.28\dot{9}6\dot{3}$.

$$3.28\dot{9}6\dot{3} = 3\tfrac{28935}{99900} = 3\tfrac{643}{2220}. \textit{ Ans.}$$

21. Reduce to common fractions in their lowest terms $5.878\dot{3}$.

$$5.878\dot{3} = 5\tfrac{7905}{9000} = 5\tfrac{527}{600}. \textit{ Ans.}$$

22. Reduce to common fractions in their lowest terms $1.\dot{6}940\dot{8}$.

$$1.\dot{6}940\dot{8} = 1\tfrac{69402}{99990} = 1\tfrac{11567}{16665}. \textit{ Ans.}$$

23. Reduce to common fractions in their lowest terms $0.48\dot{3}2\dot{4}$.

$$0.48\dot{3}2\dot{4} = \tfrac{48276}{99900} = \tfrac{447}{925}. \textit{ Ans.}$$

24. Reduce to common fractions in their lowest terms $0.00\dot{1}221\dot{3}$.

$$0.00\dot{1}221\dot{3} = \tfrac{12213}{9999900} = \tfrac{1357}{1111100}. \textit{ Ans.}$$

EXERCISE XXXV.

1. Find the G.C.M. and L.C.M. of $\frac{7}{9}$, $\frac{14}{27}$, $\frac{18}{45}$.

$\frac{18}{45} = \frac{2}{5}$.

G.C.M. of 7, 14, 2 = 1.
L.C.M. of 9, 27, 5 = 105.
∴ G.C.M. of fractions = $\frac{1}{105}$.
L.C.M. of 7, 14, 2 = 14.
G.C.M. of 9, 27, 5 = 1.
∴ L.C.M. of fractions = 14.

2. Find the G.C.M. and L.C.M. of $2\frac{2}{9}$, $2\frac{2}{5}$, $\frac{4}{40}$.

$2\frac{2}{9} = \frac{20}{9}$, $2\frac{2}{5} = \frac{12}{5}$, $\frac{4}{40} = \frac{1}{10}$.

G.C.M. of 20, 12, 1 = 1.
L.C.M. of 9, 5, 10 = 90.
∴ G.C.M. of fractions = $\frac{1}{90}$.
L.C.M. of 20, 12, 1 = 60.
G.C.M. of 9, 5, 10 = 1.
∴ L.C.M. of fractions = 60.

3. Find the G.C.M. and L.C.M of $33\frac{3}{7}$, $50\frac{5}{8}$.

$33\frac{3}{7} = \frac{234}{7}$, $50\frac{5}{8} = \frac{405}{8}$.

G.C.M. of 234, 405 = 9.
L.C.M. of 7, 8 = 56.
∴ G.C.M. of fractions = $\frac{9}{56}$.
L.C.M. of 234, 405 = 10,530.
G.C.M. of 7, 8 = 1.
∴ L.C.M. of fractions = 10,530.

4. Find the G.C.M. and L.C.M. of $\frac{7}{24}$, $\frac{35}{36}$, $\frac{49}{60}$.

G.C.M. of 7, 35, 49 = 7.
L.C.M. of 24, 36, 60 = 360.
∴ G.C.M. of fractions = $\frac{7}{360}$.
L.C.M. of 7, 35, 49 = 245.
G.C.M. of 24, 36, 60 = 12.
∴ L.C.M. of fractions = $\frac{245}{12}$
= $20\frac{5}{12}$.

5. Find the G.C.M. and L.C.M. of $5\frac{1}{2}$, $7\frac{1}{3}$, $8\frac{1}{4}$, $4\frac{8}{9}$, $9\frac{1}{6}$, $6\frac{5}{12}$.

$5\frac{1}{2}$, $7\frac{1}{3}$, $8\frac{1}{4}$, $4\frac{8}{9}$, $9\frac{1}{6}$, $6\frac{5}{12} = \frac{11}{2}$, $\frac{22}{3}$, $\frac{33}{4}$, $\frac{44}{9}$, $\frac{55}{6}$, $\frac{77}{12}$.

G.C.M. of 11, 22, 33, 44, 55, 77 = 11.
L.C.M. of 2, 3, 4, 9, 6, 12 = 36.
∴ G.C.M. of fractions = $\frac{11}{36}$.
L.C.M. of 11, 22, 33, 44, 55, 77 = 4620.
G.C.M. of 2, 3, 4, 9, 6, 12 = 1.
∴ L.C.M. of fractions = 4620.

6. Find the G.C.M. and L.C.M. of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{12}$.

G.C.M. of 1, 1, 1, 1, 1, 1, 1 = 1.
L.C.M. of 2, 3, 4, 5, 6, 10, 12 = 60.
∴ G.C.M. of fractions = $\frac{1}{60}$.
L.C.M. of 1, 1, 1, 1, 1, 1, 1 = 1.
G.C.M. of 2, 3, 4, 5, 6, 10, 12 = 1.
∴ L.C.M. of fractions = 1.

7. Find the G. C. M. and L. C. M. of $50\frac{1}{2}$, $67\frac{1}{3}$, $44\frac{8}{9}$, $84\frac{1}{6}$, 707.

$$50\tfrac{1}{2},\ 67\tfrac{1}{3},\ 44\tfrac{8}{9},\ 84\tfrac{1}{6},\ 707 = \tfrac{101}{2},\ \tfrac{202}{3},\ \tfrac{404}{9},\ \tfrac{505}{6},\ 707.$$

G. C. M. of 101, 202, 404, 505, 707 = 101.
L. C. M. of 2, 3, 9, 6, 1 = 18.
$\therefore$ G. C. M. of fractions $= \frac{101}{18} = 5\frac{11}{18}$.
L. C. M. of 101, 202, 404, 505, 707 = 14, 140.
G. C. M. of 2, 3, 9, 6, 1 = 1.
$\therefore$ L. C. M. of fractions = 14, 140.

8. Find the G. C. M. and L. C. M. of $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$.

G. C. M. of 4, 5, 6, 7, 8, 9 = 1.
L. C. M. of 5, 6, 7, 8, 9, 10 = 2520.
$\therefore$ G. C. M. of fractions $= \frac{1}{2520}$.
L. C. M. of 4, 5, 6, 7, 8, 9 = 2520.
G. C. M. of 5, 6, 7, 8, 9, 10 = 1.
$\therefore$ L. C. M. of fractions = 2520.

9. Find the G. C. M. of $1\frac{1}{14}$, $1\frac{19}{21}$, $4\frac{2}{7}$, $\frac{25}{42}$

$$1\tfrac{1}{14},\ 1\tfrac{19}{21},\ 4\tfrac{2}{7},\ \tfrac{25}{42} = \tfrac{15}{14},\ \tfrac{40}{21},\ \tfrac{30}{7},\ \tfrac{25}{42}.$$

G. C. M. of 15, 40, 30, 25 = 5.
L. C. M. of 14, 21, 7, 42 = 42.
$\therefore$ G. C. M. of fractions $= \frac{5}{42}$.
L. C. M. of 15, 40, 30, 25 = 600.
G. C. M. of 14, 21, 7, 42 = 7.
$\therefore$ L. C. M. of fractions $= \frac{600}{7} = 85\frac{5}{7}$.

10. Find the G. C. M. and L. C. M. of $18\frac{2}{5}$, $57\frac{1}{2}$.

$$18\tfrac{2}{5} = \tfrac{92}{5},\ 57\tfrac{1}{2} = \tfrac{115}{2}.$$

G. C. M. of 92, 115 = 23.
L. C. M. of 5, 2 = 10.
$\therefore$ G. C. M. of fractions $= \frac{23}{10} = 2\frac{3}{10}$.
L. C. M. of 92, 115 = 460.
G. C. M. of 5, 2 = 1.
$\therefore$ L. C. M. of fractions = 460.

11. Find the G. C. M. and L. C. M. of $134\frac{3}{4}$, $128\frac{1}{3}$, $115\frac{1}{2}$.

$$134\tfrac{3}{4},\ 128\tfrac{1}{3},\ 115\tfrac{1}{2} = \tfrac{539}{4},\ \tfrac{385}{3},\ \tfrac{231}{2}.$$

G. C. M. of 539, 385, 231 = 77.
L. C. M. of 4, 3, 2 = 12.
$\therefore$ G. C. M. of fractions $= \frac{77}{12} = 6\frac{5}{12}$.
L. C. M. of 539, 385, 231 = 8085.
G. C. M. of 4, 3, 2, = 1.
$\therefore$ L. C. M. of fractions = 8085.

12. Find the G. C. M. and L. C. M. of $2\frac{22}{25}$, $1\frac{37}{75}$, $\frac{63}{100}$.

$$2\tfrac{22}{25},\ 1\tfrac{37}{75},\ \tfrac{63}{100} = \tfrac{72}{25},\ \tfrac{112}{75},\ \tfrac{63}{100}.$$

G. C. M. of 72, 112, 63 = 1.
L. C. M. of 25, 75, 100 = 300.
$\therefore$ G. C. M. of fractions $= \frac{1}{300}$.
L. C. M. of 72, 112, 63 = 1008.
G. C. M. of 25, 75, 100 = 25.
$\therefore$ L. C. M. of fractions $= \frac{1008}{25} = 40\frac{8}{25}$.

13. A, B, and C start together and travel round a circular island, in the same direction. It takes A $2\frac{1}{3}$ days, B $2\frac{5}{6}$, C $2\frac{7}{8}$ days to walk round the island. They travel until they all meet at the point of starting. In how many days will they be together at the point of starting?

$$2\tfrac{1}{3},\ 2\tfrac{5}{6},\ 2\tfrac{7}{8} = \tfrac{7}{3},\ \tfrac{17}{6},\ \tfrac{23}{8}.$$

L. C. M. of 7, 17, 23 = 2737.
G. C. M. of 3, 6, 8 = 1.
$\therefore$ L. C. M. = 2737.
2737 days. *Ans.*

14. If the step of a man be $2\frac{1}{3}$ft., and that of a horse be $2\frac{3}{4}$ft., find the smallest number of feet which is an exact number of man-paces and of horse-paces.

$$2\tfrac{1}{3},\ 2\tfrac{3}{4} = \tfrac{7}{3},\ \tfrac{11}{4}.$$

L. C. M. of 7, 11 = 77.
G. C. M. of 3, 4 = 1.
$\therefore$ L. C. M. = 77.
77 ft. *Ans.*

15. Find the largest number that is contained without remainder in $2\frac{5}{9}$, $6\frac{7}{18}$, $11\frac{1}{2}$, and $19\frac{1}{6}$.

$$2\tfrac{5}{9},\ 6\tfrac{7}{18},\ 11\tfrac{1}{2},\ 19\tfrac{1}{6} = \tfrac{23}{9},\ \tfrac{115}{18},\ \tfrac{23}{2},\ \tfrac{115}{6}.$$

G. C. M. of 23, 115, 23, 115 = 23.

L. C. M. of 9, 18, 2, 6 = 18.

$\therefore$ G. C. M. $= \frac{23}{18} = 1\frac{5}{18}$.

Exercise XXXVI.

1. Simplify $\frac{2709}{6966}$, $\frac{43785}{56835}$, $\frac{2436}{567216}$, $\frac{4087}{5063}$.

$$\frac{2709}{6966} = \frac{7}{18}. \qquad \frac{43785}{56835} = \frac{973}{1263}. \qquad \frac{2436}{567216} = \frac{203}{47268}. \qquad \frac{4087}{5063} = \frac{67}{83}.$$

2. Which is greater, and how much, $\frac{7}{9}$ or $\frac{19}{24}$?

$$\frac{7}{9},\ \frac{19}{24} = \frac{56,\quad 57}{72}. \qquad \therefore\ \frac{19}{24} \text{ is greater by } \frac{1}{72}.$$

3. Find the sum of $3\frac{2}{5}$, $2\frac{4}{11}$, $5\frac{1}{2}$, $7\frac{7}{10}$, $1\frac{3}{22}$.

$$3\tfrac{2}{5} + 2\tfrac{4}{11} + 5\tfrac{1}{2} + 7\tfrac{7}{10} + 1\tfrac{3}{22} = 18\tfrac{44+40+55+77+15}{110} = 18\tfrac{231}{110} = 20\tfrac{1}{10}. \textit{ Ans.}$$

4. Simplify $5\frac{1}{2} - 3\frac{3}{7} + 2\frac{9}{10} - 1\frac{3}{5}$.

Sum of plus terms $= 8\frac{2}{5}$.

Sum of minus terms $= 5\frac{1}{35}$.

Difference $= 3\frac{13}{35}$.

5. Simplify $1\frac{4}{5} + 3\frac{5}{6} - 2\frac{7}{12} + 4\frac{3}{20} - 3\frac{7}{15}$.

Sum of plus terms $= 9\frac{47}{60}$.

Sum of minus terms $= 6\frac{3}{60}$.

Difference $= 3\frac{11}{15}$.

6. Simplify $\dfrac{3\frac{1}{2} + 3\frac{5}{6}}{4\frac{1}{3} - 2\frac{7}{12}}$.

$$\frac{3\frac{1}{2} + 3\frac{5}{6}}{4\frac{1}{3} - 2\frac{7}{12}} = \frac{42 + 46}{52 - 31} = \frac{88}{21} = 4\tfrac{4}{21}. \textit{ Ans.}$$

7. Simplify the expressions: $7 \div 2\frac{3}{4}$; $\dfrac{7}{1\frac{3}{8}}$; $\dfrac{95\frac{1}{2}}{8\frac{7}{11}}$; $15 \div \frac{2}{3}$; $\dfrac{16}{5\frac{1}{3}}$; $7\frac{4}{11} \div 9$; $43\frac{1}{4} \div 37\frac{1}{3}$; $\dfrac{6\frac{7}{11}}{18\frac{1}{4}}$; $5\frac{4}{5} \div 4\frac{5}{6}$; $\dfrac{\frac{3}{8} \text{ of } 4\frac{1}{2}}{\frac{7}{4} \times \frac{3}{2}}$; $106 \div 8\frac{5}{6}$; $\dfrac{17}{4\frac{7}{17}}$

$7 \div 2\frac{3}{4} = \frac{4}{11} \times \frac{7}{1} = \frac{28}{11} = 2\frac{6}{11}$;

$\dfrac{7}{1\frac{3}{8}} = \frac{8}{11} \times \frac{7}{1} = \frac{56}{11} = 5\frac{1}{11}$;

$\dfrac{95\frac{1}{2}}{8\frac{7}{11}} = \frac{11}{95} \times \frac{191}{2} = \frac{2101}{190} = 11\frac{11}{190}$;

$15 \div \frac{2}{3} = \frac{3}{2} \times \frac{15}{1} = \frac{45}{2} = 22\frac{1}{2}$;

$\dfrac{16}{5\frac{1}{3}} = \frac{3}{\cancel{16}} \times \frac{\cancel{16}}{1} = 3$;

$7\frac{4}{11} \div 9 = \frac{1}{\cancel{9}} \times \frac{\overset{9}{\cancel{81}}}{11} = \frac{9}{11}$;

$43\frac{1}{4} \div 37\frac{1}{3} = \frac{3}{112} \times \frac{173}{4} = \frac{519}{448} = 1\frac{71}{448}$;

$\dfrac{6\frac{7}{11}}{18\frac{1}{4}} = \frac{4}{\cancel{73}} \times \frac{\cancel{73}}{11} = \frac{4}{11}$;

$5\frac{4}{5} \div 4\frac{5}{6} = \frac{6}{\cancel{29}} \times \frac{\cancel{29}}{5} = \frac{6}{5} = 1\frac{1}{5}$;

$\dfrac{\frac{3}{8} \text{ of } 4\frac{1}{2}}{\frac{7}{4} \times \frac{3}{2}} = \frac{\cancel{3}}{\underset{2}{\cancel{8}}} \times \frac{9}{\cancel{2}} \times \frac{\cancel{4}}{7} \times \frac{\cancel{2}}{\cancel{3}} = \frac{9}{14}$;

$106 \div 8\frac{5}{6} = \frac{6}{\cancel{53}} \times \frac{\overset{2}{\cancel{106}}}{1} = 12$;

$\dfrac{17}{4\frac{7}{17}} = \frac{17}{75} \times \frac{17}{1} = \frac{289}{75} = 3\frac{64}{75}$.

8. Simplify the expressions: $7\frac{19}{32} \times 8$; $43\frac{13}{53} \times 6\frac{5}{8}$; $6\frac{5}{6} \div 8\frac{1}{5}$; $5\frac{1}{17} \times 51$; $\frac{11}{12}$ of $\frac{11}{13}$; $\frac{30}{49}$ of $\frac{7}{15}$ of $\frac{7}{8}$ of $\frac{2}{3}$ of $\frac{2}{5}$; $\frac{17}{19}$ of $\frac{228}{361}$; $\frac{1}{2} \times \frac{3}{4} \times \frac{7}{11} \times \frac{8}{9} \times \frac{3}{7}$.

$$7\tfrac{19}{32} \times 8 = \frac{243}{\underset{4}{\cancel{32}}} \times \frac{\cancel{8}}{1} = \frac{243}{4} = 60\tfrac{3}{4};$$

$$43\tfrac{13}{53} \times 6\tfrac{5}{8} = \frac{\overset{573}{\cancel{2292}}}{\cancel{53}} \times \frac{\cancel{53}}{\underset{2}{\cancel{8}}} = \frac{573}{2} = 286\tfrac{1}{2};$$

$6\frac{5}{6} \div 8\frac{1}{5} = \frac{5}{\cancel{41}} \times \frac{\cancel{41}}{6} = \frac{5}{6}$;

$5\frac{1}{17} \times 51 = \frac{86}{\cancel{17}} \times \frac{\overset{3}{\cancel{51}}}{1} = 258$;

$\frac{11}{12} \times \frac{11}{13} = \frac{121}{156}$;

$\frac{\cancel{17}}{\cancel{19}} \times \frac{\overset{\overset{4}{\cancel{12}}}{\cancel{228}}}{\underset{\underset{11}{\cancel{33}}}{\cancel{361}}} = \frac{4}{11}$;

$\frac{\overset{2}{\cancel{30}}}{\underset{7}{\cancel{49}}} \times \frac{\cancel{7}}{\cancel{15}} \times \frac{\cancel{7}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{8}}} \times \frac{\cancel{2}}{3} \times \frac{\cancel{2}}{5} = \frac{1}{15}$;

$\frac{1}{\cancel{2}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{7}}{11} \times \frac{\overset{\cancel{2}}{\cancel{8}}}{\underset{\cancel{3}}{\cancel{9}}} \times \frac{\cancel{3}}{\cancel{7}} = \frac{1}{11}$.

9. By what must $\frac{1}{6}$ be multiplied to obtain $\frac{1}{2}$? $\frac{1}{6}$ to obtain $\frac{2}{3}$? $\frac{1}{2}$ to obtain $\frac{5}{6}$? $\frac{2}{3}$ to obtain $\frac{5}{6}$? $\frac{3}{5}$ to obtain $\frac{7}{8}$?

$$\frac{1}{2} \div \frac{1}{6} = \frac{\overset{3}{\cancel{6}}}{1} \times \frac{1}{\cancel{2}} = 3;$$

$$\frac{5}{6} \div \frac{1}{2} = \frac{\cancel{2}}{1} \times \frac{5}{\underset{3}{\cancel{6}}} = \frac{5}{3} = 1\tfrac{2}{3};$$

$$\frac{2}{3} \div \frac{1}{6} = \frac{\overset{2}{\cancel{6}}}{1} \times \frac{2}{\cancel{3}} = 4;$$

$$\frac{5}{6} \div \frac{2}{3} = \frac{\cancel{3}}{2} \times \frac{5}{\underset{2}{\cancel{6}}} = \frac{5}{4} = 1\tfrac{1}{4};$$

$$\frac{7}{8} \div \frac{3}{5} = \frac{5}{3} \times \frac{7}{8} = \frac{35}{24} = 1\tfrac{11}{24}.$$

10. By what must $\frac{1}{6}$ be divided to obtain $\frac{1}{2}$? $\frac{2}{3}$ to obtain $\frac{1}{6}$? $\frac{5}{6}$ to obtain $\frac{5}{6}$? $\frac{7}{8}$ to obtain $\frac{4}{5}$? $\frac{3}{5}$ to obtain $\frac{7}{8}$? 8 to obtain $7\frac{19}{32}$?

$$\frac{1}{6} \div \frac{1}{2} = \frac{\cancel{2}}{1} \times \frac{1}{\underset{3}{\cancel{6}}} = \frac{1}{3};$$

$$\frac{7}{8} \div \frac{4}{5} = \frac{5}{4} \times \frac{7}{8} = \frac{35}{32} = 1\tfrac{3}{32};$$

$$\frac{2}{3} \div \frac{1}{6} = \frac{\overset{2}{\cancel{6}}}{1} \times \frac{2}{\cancel{3}} = 4;$$

$$\frac{3}{5} \div \frac{7}{8} = \frac{8}{7} \times \frac{3}{5} = \frac{24}{35};$$

$$\frac{5}{6} \div \frac{5}{6} = \frac{\cancel{6}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} = 1;$$

$$8 \div 7\tfrac{19}{32} = \frac{32}{243} \times \frac{8}{1} = \frac{256}{243} = 1\tfrac{13}{243}.$$

11. What number exceeds $5\frac{2}{9}$ by $4\frac{7}{8}$?

$$5\tfrac{2}{9} + 4\tfrac{7}{8} = 9\tfrac{16+63}{72} = 9\tfrac{79}{72} = 10\tfrac{7}{72}.$$

12. From what must $6\frac{3}{5}$ be subtracted to leave $\frac{1}{2}$ of $3\frac{1}{9}$?

$$\frac{1}{2} \times 3\tfrac{1}{9} = \frac{1}{\cancel{2}} \times \frac{\overset{14}{\cancel{28}}}{9} = \frac{14}{9} = 1\tfrac{5}{9};$$

$$6\tfrac{3}{5} + 1\tfrac{5}{9} = 7\tfrac{27+25}{45} = 7\tfrac{52}{45} = 8\tfrac{7}{45}.$$

13. What fraction falls short of $\frac{7}{12}$ by $\frac{3}{20}$?

$$\frac{7}{12} - \frac{3}{20} = \frac{35-9}{60} = \frac{26}{60} = \frac{13}{30}.$$

14. What fraction is that to which $\frac{5}{76}$ must be added to give $\frac{11}{57}$?

$$\frac{11}{57} - \frac{5}{76} = \frac{44-15}{228} = \frac{29}{228}.$$

15. Convert into decimals: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{3}{4}$; $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$; $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$; $\frac{1}{6}$, $\frac{5}{6}$; $\frac{3}{7}$; $\frac{5}{9}$; $\frac{3}{11}$; $\frac{7}{40}$.

$$2)\underline{1.0} \quad 0.5$$
$$4)\underline{1.00} \quad 0.25$$
$$3)\underline{1.0} \quad 0.\dot{3}$$
$$4)\underline{3.00} \quad 0.75$$
$$8)\underline{1.000} \quad 0.125$$
$$8)\underline{3.000} \quad 0.375$$
$$8)\underline{5.000} \quad 0.625$$

$$8)\underline{7.000} \quad 0.875$$
$$16)\underline{1.0000} \quad 0.0625$$
$$\begin{array}{r} 0.0625 \\ \underline{3} \\ 0.1875 \end{array} \qquad \begin{array}{r} 0.0625 \\ \underline{5} \\ 0.3125 \end{array} \qquad \begin{array}{r} 0.0625 \\ \underline{7} \\ 0.4375 \end{array}$$

$$\begin{array}{r} 0.0625 \\ \underline{9} \\ 0.5625 \end{array} \qquad \begin{array}{r} 0.0625 \\ \underline{11} \\ 0.6875 \end{array} \qquad \begin{array}{r} 0.0625 \\ \underline{13} \\ 0.8125 \end{array} \qquad \begin{array}{r} 0.0625 \\ \underline{15} \\ 0.9375 \end{array}$$

$$6)\underline{1.00} \quad 0.1\dot{6}$$
$$6)\underline{5.00} \quad 0.8\dot{3}$$
$$7)\underline{3.000000} \quad 0.\dot{4}2857\dot{1}$$
$$9)\underline{5.0} \quad 0.\dot{5}$$
$$11)\underline{3.00} \quad 0.\dot{2}\dot{7}$$
$$4)\underline{0.700} \quad 0.175$$

16. Convert into common fractions: 0.16; 0.016; 0.125; 0.13; 0.725; 0.625; 0.00625; 0.8125; 0.03125; 0.08; $0.\dot{5}\dot{4}$; $0.01\dot{6}$; $0.5\dot{4}3\dot{7}$; $0.0\dot{2}\dot{7}$; $0.27\dot{7}$; $0.68\dot{4}9\dot{4}$; $1.3\dot{4}\dot{5}$.

$0.16 = \frac{16}{100} = \frac{4}{25}$;

$0.016 = \frac{16}{1000} = \frac{2}{125}$;

$0.125 = \frac{125}{1000} = \frac{1}{8}$;

$0.13 = \frac{16}{100}$;

$0.725 = \frac{725}{1000} = \frac{29}{40}$;

$0.625 = \frac{625}{1000} = \frac{5}{8}$;

$0.00625 = \frac{625}{100000} = \frac{1}{160}$;

$0.8125 = \frac{8125}{10000} = \frac{13}{16}$;

$0.03125 = \frac{3125}{10000} = \frac{1}{32}$;

$0.08 = \frac{8}{100} = \frac{2}{25}$;

$0.\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$;

$0.01\dot{6} = \frac{15}{900} = \frac{1}{60}$;

$0.5\dot{4}3\dot{7} = \frac{5432}{9990} = \frac{2716}{4995}$;

$0.0\dot{2}\dot{7} = \frac{27}{990} = \frac{3}{110}$;

$0.27\dot{7} = \frac{250}{900} = \frac{5}{18}$;

$0.68\dot{4}9\dot{4} = \frac{68426}{99900} = \frac{34213}{49950}$;

$1.3\dot{4}\dot{5} = 1\frac{342}{990} = 1\frac{19}{55}$.

17. Simplify $\dfrac{2.8 \times 2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}}$.

$$\frac{2.8 \times 2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}} = \frac{2\frac{4}{5} \times 2\frac{3}{11}}{1\frac{3}{22}} = \frac{14}{5} \times \frac{\cancel{25}}{\cancel{11}} \times \frac{\overset{2}{\cancel{22}}}{\cancel{25}} = \frac{28}{5} = 5\tfrac{3}{5} = 5.6. \ \textit{Ans.}$$

18. Multiply $6.9\dot{5}\dot{4}$ by $5.3\dot{0}\dot{3}$, and express the result as a whole number and common fraction.

$$6.9\dot{5}\dot{4} = 6\tfrac{21}{22};\quad 5.3\dot{0}\dot{3} = 5\tfrac{10}{33};\quad \frac{\overset{51}{\cancel{153}}}{22} \times \frac{175}{\underset{11}{\cancel{33}}} = \frac{8925}{242} = 36\tfrac{213}{242}.$$

19. Simplify $1\frac{1}{2}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4}$ and reduce the result to a decimal.

$$1\tfrac{1}{2} \times 2\tfrac{4}{5} + 6\tfrac{7}{8} \div 2\tfrac{3}{4} = \tfrac{3}{2} \times \tfrac{14}{5} + \tfrac{4}{11} \times \tfrac{55}{8} = 4\tfrac{1}{5} + 2\tfrac{1}{2} = 6\tfrac{2+5}{10} = 6\tfrac{7}{10} = 6.7.$$

20. From what number can $4\frac{17}{36}$ be taken 9 times and leave no remainder?

$$4\tfrac{17}{36} \times 9 = \frac{161}{\underset{4}{\cancel{36}}} \times \frac{\cancel{9}}{1} = \frac{161}{4} = 40\tfrac{1}{4}.$$

21. Of what fraction is $17\frac{1}{3}$ the 7th part?

$$17\tfrac{1}{3} \times 7 = \frac{52}{3} \times \frac{7}{1} = \frac{364}{3} = 121\tfrac{1}{3}.$$

22. Add $\frac{4}{5}$, 0.35, $\frac{5}{8}$, $\frac{3}{4}$, 0.112, 45.28.

$$\tfrac{4}{5} + 0.35 + \tfrac{5}{8} + \tfrac{3}{4} + 0.112 + 45.28$$
$$= 0.8 + 0.35 + 0.625 + 0.75 + 0.112 + 45.28 = 47.917.$$

23. Convert into decimals $\frac{13}{15}$; $\frac{3}{11}$; $\frac{4}{35}$; $\frac{17}{60}$; $\frac{14}{19}$; $\frac{5}{13}$; $\frac{7}{53}$.

$$15\overline{)13.}\;=0.8\dot{6} \qquad 11\overline{)3.00}\;=0.\dot{2}\dot{7} \qquad 35\overline{)4.0000000}\;=0.1\dot{1}4285\dot{7} \qquad 60\overline{)17.000}\;=0.28\dot{3}$$

$$19\overline{)14.000000000000000000}\;=0.\dot{7}3684210526315789\dot{4} \qquad 13\overline{)5.000000}\;=0.\dot{3}8461\dot{5} \qquad 53\overline{)7.0000000000000}\;=0.\dot{1}32075471698\dot{1}$$

24. What part of $\frac{15}{73}$ is $\frac{3}{1241}$?

$$\frac{3}{1241} \div \frac{15}{73} = \frac{\cancel{73}}{\underset{5}{\cancel{15}}} \times \frac{\cancel{3}}{\underset{17}{\cancel{1241}}} = \frac{1}{85}.$$

25. Divide 0.0015 by 0.012, and express the result as a common fraction in lowest terms.

$$12\overline{)1.500}\;=0.125 \qquad 0.125 = \tfrac{1}{8}.$$

26. Convert into decimals: $\frac{3}{32}$; $\frac{3}{32000}$; $\frac{17}{74}$; $\frac{1}{7}$.

$$32\overline{)3.00000}\ \ 0.09375 \qquad 32\overline{)0.00300000}\ \ 0.00009375 \qquad 74\overline{)17.0000}\ \ 0.2\dot{2}9\dot{7} \qquad 7\overline{)1.000000}\ \ 0.\dot{1}4185\dot{7}$$

27. If the product of two factors is $\frac{5}{8}$, and one factor is $1\frac{1}{4}$, find the other factor.

$$\frac{5}{8} \div 1\frac{1}{4} = \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\underset{2}{\cancel{8}}} = \frac{1}{2}.$$

28. If the dividend is $\frac{11}{12}$ and the quotient $6\frac{1}{2}$, find the divisor.

$$\frac{11}{12} \div 6\frac{1}{2} = \frac{\cancel{2}}{13} \times \frac{11}{\underset{6}{\cancel{12}}} = \frac{11}{78}.$$

29. The dividend is $12\frac{17}{72}$, quotient 3, remainder $1\frac{5}{12}$; find the divisor.

$$(12\tfrac{17}{72} - 1\tfrac{5}{12}) \div 3 = 10\tfrac{59}{72} \div 3 = \tfrac{1}{3} \times \tfrac{779}{72} = \tfrac{779}{216} = 3\tfrac{131}{216}.$$

30. Find the G. C. M. and the L. C. M of 833, 1127, 1421, 343.

$$\begin{array}{r|rrrr} 7 & 833 & 1127 & 1421 & 343 \\ \hline 7 & 119 & 161 & 203 & 49 \\ \hline & 17 & 23 & 29 & 7 \end{array}$$

G. C. M. $= 7 \times 7 = 49$.

L. C. M. $= 7^3 \times 17 \times 23 \times 29 = 3,889,277$.

31. Arrange in order of magnitude $\frac{9}{11}$, $\frac{32}{45}$, $\frac{19}{27}$, $\frac{17}{20}$, $\frac{20}{33}$.

$\frac{9}{11} = \frac{4860}{5940}$. $\frac{32}{45} = \frac{4224}{5940}$. $\frac{19}{27} = \frac{4180}{5940}$. $\frac{17}{20} = \frac{5049}{5940}$. $\frac{20}{33} = \frac{3600}{5940}$.

$\therefore$ the order of magnitude is $\frac{17}{20}$, $\frac{9}{11}$, $\frac{32}{45}$, $\frac{19}{27}$, $\frac{20}{33}$.

32. Find the L. C. M. of $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$.

L. C. M. of 15, 26, 65 $= 390$.

G. C. M. of 17, 51, 102 $= 17$.

$\therefore$ L.C.M. of fractions $= \frac{390}{17} = 22\frac{16}{17}$.

33. Find the G. C. M. of $\frac{65}{68}$, $\frac{39}{2}$, $\frac{91}{64}$, and $6\frac{1}{2}$.

G. C. M. of 65, 39, 91, 13 $= 13$.

L. C. M. of 68, 2, 64, 2 $= 1088$.

$\therefore$ G. C. M. of fractions $= \frac{13}{1088}$.

34. Convert into common fractions in lowest terms: $7.\dot{2}01\dot{1}$; $6.9\dot{5}\dot{4}$; $5.\dot{3}0\dot{3}$; $21.\dot{3}9\dot{6}$.

$$7.\dot{2}01\dot{1} = 7\tfrac{2011}{9999}. \qquad 6.9\dot{5}\dot{4} = 6\tfrac{945}{990} = 6\tfrac{21}{22}.$$

$$5.\dot{3}0\dot{3} = 5\tfrac{300}{990} = 5\tfrac{10}{33}. \qquad 21.\dot{3}9\dot{6} = 21\tfrac{396}{999} = 21\tfrac{44}{111}.$$

35. Simplify $\dfrac{3\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}$.

$$\frac{3\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}} = \frac{4 + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - \frac{5}{18} + \frac{1}{3}} = \frac{4\frac{25}{48}}{5\frac{1}{6}} = \frac{7}{8}.$$

36. Simplify $\dfrac{6\frac{3}{4} + 5\frac{1}{2} \times 3\frac{1}{7} - 7\frac{1}{4}}{3\frac{1}{5} + 2\frac{1}{2} - 4\frac{1}{10}}$.

$$\frac{6\frac{3}{4} + 5\frac{1}{2} \times 3\frac{1}{7} - 7\frac{1}{4}}{3\frac{1}{5} + 2\frac{1}{2} - 4\frac{1}{10}} = \frac{6\frac{3}{4} + 17\frac{2}{7} - 7\frac{1}{4}}{3\frac{1}{5} + 2\frac{1}{2} - 4\frac{1}{10}}$$

$$= \frac{945 + 2420 - 1015}{448 + 350 - 574} = \frac{2350}{224} = 10\tfrac{55}{112}.$$

37. Simplify $\dfrac{2\frac{4}{5} - 1\frac{1}{2} + 9\frac{1}{11}}{4\frac{1}{5} - 2\frac{1}{4} + 13\frac{7}{11}}$.

$$\frac{2\frac{4}{5} - 1\frac{1}{2} + 9\frac{1}{11}}{4\frac{1}{5} - 2\frac{1}{4} + 13\frac{7}{11}} = \frac{616 - 330 + 2000}{924 - 495 + 3000} = \frac{2286}{3429} = \frac{2}{3}.$$

38. Simplify $\dfrac{(3.71 - 1.908) \times 7.03}{2.\dot{2} - \frac{74}{333}}$.

$$\frac{(3.71 - 1.908) \times 7.03}{2.\dot{2} - \frac{74}{333}} = \frac{1.802 \times 7.03}{2\frac{2}{9} - \frac{74}{333}} = \frac{12.66806}{2} = 6.33403.$$

39. Simplify $\dfrac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{2}{5} \text{ of } \dfrac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}$.

$$\frac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{2}{5} \text{ of } \frac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}$$

$$= \frac{45}{8} \times \frac{3}{2} \times \frac{5}{6} \times \frac{9}{5} \times \frac{31}{3} \times \frac{2}{5} \times \frac{3}{2} \times \frac{37}{9} \times \frac{8}{111} \times \frac{3}{16} = \frac{279}{64} = 4\tfrac{23}{64}.$$

40. Simplify $1\frac{1}{2}$ of $2\frac{4}{5}+6\frac{7}{8}\div 2\frac{3}{4}+\left(5\frac{1}{2}+\frac{0.24+0.5\dot{3}}{2.2-0.6\dot{4}}\right)$.

$$1\tfrac{1}{2} \text{ of } 2\tfrac{4}{5}+6\tfrac{7}{8}\div 2\tfrac{3}{4}+\left(5\tfrac{1}{2}+\frac{0.24+0.5\dot{3}}{2.2-0.6\dot{4}}\right)=4\tfrac{1}{5}+2\tfrac{1}{2}+5\tfrac{1}{2}+\frac{\frac{58}{75}}{1\frac{5}{9}}$$

$$=4\tfrac{1}{5}+2\tfrac{1}{2}+5\tfrac{1}{2}+\tfrac{261}{525}=11\tfrac{210+525+525+522}{1050}$$

$$=11\tfrac{1782}{1050}=12\tfrac{122}{175}.$$

41. Simplify 0.9 of $\frac{5}{8}$ of $\frac{4}{7}$ of $15\frac{3}{4}$.

$$0.9 \text{ of } \tfrac{5}{8} \text{ of } \tfrac{4}{7} \text{ of } 15\tfrac{3}{4}=\frac{9}{\underset{2}{\cancel{10}}}\times\frac{\cancel{5}}{\underset{2}{\cancel{8}}}\times\frac{\cancel{4}}{\cancel{7}}\times\frac{\overset{9}{\cancel{63}}}{4}=\frac{81}{16}=5\tfrac{1}{16}.$$

42. What part of $\frac{2}{3}$ is $\frac{1}{2}$.

$$\frac{\frac{1}{2}}{\frac{2}{3}}=\frac{1}{2}\times\frac{3}{2}=\frac{3}{4}.$$

43. What part of 0.390625 is $0.0\dot{5}$?

$$\frac{0.05}{0.390625}=\frac{\frac{1}{20}}{\frac{25}{64}}=\frac{\overset{16}{\cancel{64}}}{25}\times\frac{1}{\underset{5}{\cancel{20}}}=\frac{16}{125}$$

44. $0.\dot{0}\dot{9}$ is what fraction of $0.20\dot{4}\dot{5}$?

$$\frac{0.\dot{0}\dot{9}}{0.20\dot{4}\dot{5}}=\frac{\frac{9}{99}}{\frac{2025}{9900}}=\frac{\frac{1}{11}}{\frac{9}{44}}=\frac{4}{9}.$$

45. Convert into decimals $\frac{49}{67}$; $\frac{14}{37}$; $\frac{62}{73}$.

$$67\overline{)49.000000000000000000000000000000000}\quad 0.\dot{7}3134328358208955223880597014925\dot{3}$$

$$37\overline{)14.000}\quad 0.\dot{3}7\dot{8} \qquad 73\overline{)62.00000000}\quad 0.\dot{8}493150\dot{6}$$

46. The G. C. M. of three numbers is 15, and their L. C. M. is 450. What are the numbers?

$$450=(5\times 3)\times 2\times 3\times 5$$
$$5\times 3=15=\text{G. C. M.}$$
$$15\times 2=30.$$
$$15\times 3=45.$$
$$15\times 5=75.$$

EXERCISE XXXVII.

1. Reduce 3 yds. 2 ft. to inches.

3 yds. 2 ft.
$\times 3$
9
2
11 ft.
$\times 12$
132 in.

2. Reduce 4 mi. 124 rds. to feet.

4 mi. 124 rds.
$\times 320$
1280
124
1404 rds.
$\times 16\frac{1}{2}$
23166 ft.

3. Reduce 27 rds. $4\frac{1}{4}$ yds. to inches.

27 rds. $4\frac{1}{4}$ yds.
$\times 5\frac{1}{2}$
$148\frac{1}{2}$
$\times 4\frac{1}{4}$
$152\frac{3}{4}$ yds.
$\times 36$
5499 in.

4. Reduce 290 leagues to feet.

290 leagues.
$\times 3$
870 knots.
$\times 6086$
5294820 ft.

5. Reduce 82,976,432 in. to miles.

12 | 82976432 in.
3 | 6914702 ft. 8 in.
$5\frac{1}{2}$ | 2304900 yds. 2 ft.
2
11 | 4609800
320 | 419072 rds. 8 half-yds. = 4 [yds.
1309 mi. 192 rds.

1309 mi. 192 rds. 4 yds. 2 ft. 8 in.

6. Reduce 7 mi. $3\frac{1}{2}$ yds. to inches.

7 mi. $3\frac{1}{2}$ yds.
$\times 1760$
12320
$3\frac{1}{2}$
$12323\frac{1}{2}$ yds.
$\times 36$
443646 in.

7. Reduce 27 mi. 222 rds. to inches.

27 mi. 222 rds.
$\times 320$
8640
222
8862 rds.
$\times 5\frac{1}{2}$
48741 yds.
$\times 36$
1754676 in.

8. Reduce 712 mi. to inches.

$$\begin{array}{r} 712 \text{ mi.} \\ \times\, 5280 \\ \hline 3759360 \text{ ft.} \\ \times\, 12 \\ \hline 45112320 \text{ in.} \end{array}$$

9. Reduce 540,451 ft. to miles.

$$\begin{array}{r|l} 3 & 540451 \text{ ft.} \\ 5\frac{1}{2} & 180150 \text{ yds. 1 ft.} \\ & \quad\;\; 2 \\ \hline 11 & 360300 \\ 320 & \;\; 32754 \text{ rds. 6 half-yards} = 3 \text{ yds.} \\ & \quad 102 \text{ mi. } 114 \text{ rds.} \end{array}$$

102 mi. 114 rds. 3 yds. 1 ft.

10. Reduce 271,256 in. to miles.

$$\begin{array}{r|l} 12 & 271256 \text{ in.} \\ 3 & \;\; 22604 \text{ ft. } \ldots\ldots \; 8 \text{ in.} \\ 5\frac{1}{2} & \quad 7534 \text{ yds. } \ldots \; 2 \text{ ft.} \\ & \qquad\; 2 \\ \hline 11 & 15068 \\ 320 & \;\; 1369 \text{ rds. } \ldots\ldots \; 9 \text{ half-yards} = 4\frac{1}{2} \text{ yds.} \\ & \quad\; 4 \text{ mi. } \ldots\ldots \; 89 \text{ rds.} \end{array}$$

$$\begin{array}{r} 4 \text{ mi. } 89 \text{ rds. } 4\frac{1}{2} \text{ yds.} \quad 2 \text{ ft. } 8 \text{ in.} \\ \frac{1}{2} \text{ yd.} = 1 \text{ ft. } 6 \text{ in.} \\ \hline 4 \text{ mi. } 89 \text{ rds. } 5 \text{ yds.} \quad 1 \text{ ft. } 2 \text{ in.} \end{array}$$

11. Reduce 723,964 ft. to miles.

$$\begin{array}{r|l} 3 & 723964 \text{ ft.} \\ 5\frac{1}{2} & 241321 \text{ yds. 1 ft.} \\ & \quad\;\; 2 \\ \hline 11 & 482642 \\ 320 & \;\; 43876 \text{ rds. 6 half-yards} = 3 \text{ yds.} \\ & \quad 137 \text{ mi. } 36 \text{ rds.} \end{array}$$

137 mi. 36 rds. 3 yds. 1 ft.

12. Reduce 233,205 in. to miles.

$$\begin{array}{r|l}
12 & 233205 \text{ in.} \\
3 & 19433 \text{ ft. . . . . 9 in.} \\
5\frac{1}{2} & 6477 \text{ yds. . . . 2 ft.} \\
 & 2 \\
11 & 12954 \\
320 & 1177 \text{ rds. . . . 7 half-yards} = 3\frac{1}{2} \text{ yds.} \\
 & 3 \text{ mi. . . . 217 rds.}
\end{array}$$

3 mi. 217 rds. $3\frac{1}{2}$ yds. 2 ft. 9 in.
$\frac{1}{2}$ yd. = 1 ft. 6 in.
3 mi. 217 rds. 4 yds. 1 ft. 3 in.

13. Reduce 10 chains to inches.

$$\begin{array}{r}
10 \text{ ch.} \\
\times 4 \\
\hline
40 \text{ rds.} \\
\times 16\frac{1}{2} \\
\hline
660 \text{ ft.} \\
\times 12 \\
\hline
7920 \text{ in.}
\end{array}$$

14. Reduce 233,185 in. to fathoms.

$12\underline{)233185}$ in.
$6\underline{)19432}$ ft. 1 in.
3238 fath. 4 ft.

3238 fath. 4 ft. 1 in.

15. If the height of a horse be 16 hands, how many feet is his height?

16 hands
$\times 4$
$12\underline{)64}$ in.
5 ft. 4 in.

16. If a train move 40 ft. in a second, what is its rate in miles per hour? (One hour = 3600 seconds.)

$$\begin{array}{r}
3600 \\
40 \text{ ft.} \\
\hline
5280\underline{)144000} \text{ ft.} \\
27\frac{1440}{5280} \text{ mi.} \\
= 27\frac{3}{11} \text{ mi.}
\end{array}$$

Exercise XXXVIII.

1. Reduce 92,638 sq. yds. to inches.

$$\begin{array}{r}
92638 \text{ sq. yds.} \\
\times 9 \\
\hline
833742 \text{ sq. ft.} \\
\times 144 \\
\hline
120058848 \text{ sq. in.}
\end{array}$$

2. Reduce 1,223,527 sq. in. to yards.

$144\underline{)1223527}$ sq. in.
$9\underline{)8496}$ sq. ft. 103 sq. in.
944 sq. yds.

944 sq. yds. 103 sq. in.

3. Reduce 721 sq. mi. to rods.

721 sq. mi.
× 640
461440 A.
× 160
73830400 sq. rds.

4. Reduce 34,729 sq. yds. to rods.

$30\frac{1}{4}$)34729 sq. yds.
4
121)138916
1148 sq. rds. 8 quarter-sq. yds. = 2 sq. yds.

1148 sq. rds. 2 sq. yds.

6. Reduce 1 A. to feet.

1 A.
160
160 sq. rds.
× $30\frac{1}{4}$
4840 sq. yds.
× 9
43560 sq. ft.

5. Reduce 3 A. 107 sq. rds. 27 sq. yds. 7 sq. ft. 23 sq. in. to inches.

3 A.
× 160
480
107
587 sq. rds.
× $30\frac{1}{4}$
$17756\frac{3}{4}$
27
$17783\frac{3}{4}$ sq. yds.
× 9
$160053\frac{3}{4}$
7
$160060\frac{3}{4}$ sq. ft.
× 144
23048748
23
23048771 sq. in.

7. Reduce 99,894,712 sq. in. to acres.

144)99894712 sq. in.
9)693713 sq. ft. 40 sq. in.
$30\frac{1}{4}$)77079 sq. yds. 2 sq. ft.
4
121)308316
160)2548 sq. rds. 8 quarter-yds. = 2 sq. yds.
15 A. 148 sq. rds.

15 A. 148 sq. rds. 2 sq. yds. 2 sq. ft. 40 sq. in.

8. Reduce 15,376 sq. yds. to acres.

$30\frac{1}{4})\underline{15376}$ sq. yds.
$\underline{\quad 4}$
$121)\underline{61504}$
$160)\underline{508}$ sq. rds. 36 quarter-sq. yds = 9 sq. yds.
3 A. 28 sq. rds.

3 A. 28 sq. rds. 9 sq. yds.

9. Reduce 562,934 sq. in. to rods.

$144)\underline{562934}$ sq. in.
$9)\underline{3909}$ sq. ft. 38 sq. in.
$30\frac{1}{4})434$ sq. yds. 3 sq. ft.
$\underline{\quad 4}$
$121)\underline{1736}$
14 sq. rds. 42 quarter-sq. yds. = $10\frac{1}{2}$ sq. yds.

14 sq. rds. $10\frac{1}{2}$ sq. yds. 3 sq. ft. 38 sq. in.
= 14 sq. rds. 10 sq. yds. 7 sq. ft. 110 sq. in.

EXERCISE XXXIX.

1. Reduce 7 cu. yds. 13 cu. ft. to cubic feet.

7 cu. yds. 13 cu. ft.
$\times \underline{27}$
189
$\underline{13}$
202 cu. ft.

2. Reduce 25 cu. yds. 5 cu. ft. 143 cu. in. to cubic inches.

25 cu. yds. 5 cu. ft. 143 cu. in.
$\times \underline{27}$
675
$\underline{5}$
680 cu. ft.
$\times \underline{1728}$
1175040
$\underline{143}$
1175183 cu. in.

3. Reduce 74,325 cu. in. to cubic feet.

$1728)\underline{74325}$ cu. in.
43 cu. ft. 21 cu. in.
43 cu. ft. 21 cu. in.

4. Reduce 439,000 cu. in. to cubic yards.

$1728)\underline{439000}$ cu. in.
$27)\underline{254}$ cu. ft. 88 cu. in.
9 cu. yds. 11 cu. ft.
9 cu. yds. 11 cu. ft. 88 cu. in.

5. Reduce 921,730 cu. in. to cubic yards.

$1728)\underline{921730}$ cu. in.
$27)\underline{533}$ cu. ft. 706 cu. in.
19 cu. yds. 20 cu. ft.
19 cu. yds. 20 cu. ft. 706 cu. in.

6. Wood cut in lengths of 4 ft. is piled to a height of $3\frac{1}{2}$ ft. How long must the pile be to contain a cord?

$$\begin{array}{r} 3\frac{1}{2} \text{ ft.} \\ \times 4 \text{ ft.} \\ \hline 14 \text{ sq. ft.} \end{array}$$

$$\begin{array}{r} 9\frac{1}{7} \text{ ft.} \\ 14\overline{)128} \\ 126 \\ \hline 2 \end{array}$$

7. A pile of wood 127 ft. long, 4 ft. wide, and 3 ft. 8 in. high is sold for \$7 a cord. How much money is received for it?

$$\begin{array}{r} 127 \text{ ft.} \\ \times 4 \text{ ft.} \\ \hline 508 \text{ sq. ft.} \\ \times 3\frac{2}{3} \\ \hline 128\overline{)1862\frac{2}{3}} \\ 14\frac{35}{64} \text{ cd.} \\ \times \$7 \\ \hline \$101\frac{53}{64} = \$101.86. \end{array}$$

Exercise XL.

1. Reduce 3 pks. 5 qts. 1 pt. to pints.

$$\begin{array}{l} 3 \text{ pks. } 5 \text{ qts. } 1 \text{ pt.} \\ \times 8 \\ \hline 24 \\ 5 \\ \hline 29 \text{ qts.} \\ \times 2 \\ \hline 58 \\ 1 \\ \hline 59 \text{ pts.} \end{array}$$

2. Reduce 4234 pts. to bushels.

$$\begin{array}{r|l} 2 & 4234 \text{ pts.} \\ 8 & 2117 \text{ qts.} \\ 4 & 264 \text{ pks. } 5 \text{ qts.} \\ & 66 \text{ bu.} \end{array}$$

66 bu. 5 qts.

3. Reduce 24 gals. 2 qts. 1 pt. 2 gi. to gills.

$$\begin{array}{l} 24 \text{ gal. } 2 \text{ qts. } 1 \text{ pt. } 2 \text{ gi.} \\ \times 4 \\ \hline 96 \\ 2 \\ \hline 98 \text{ qts.} \\ \times 2 \\ \hline 196 \\ 1 \\ \hline 197 \text{ pts.} \\ \times 4 \\ \hline 788 \\ 2 \\ \hline 790 \text{ gi.} \end{array}$$

4. Reduce 272 liquid quarts to dry quarts.

$$\frac{272}{1} \times \frac{57\frac{3}{4}}{67\frac{1}{5}} = \frac{\overset{17}{\cancel{272}}}{1} \times \frac{\overset{11}{\cancel{231}}}{4} \times \frac{5}{\underset{16}{\cancel{336}}} = \frac{935}{4} = 233\tfrac{3}{4} \text{ qts.}$$

5. Reduce 400 dry quarts to liquid quarts.

$$\frac{67\frac{1}{5}}{57\frac{3}{4}} \times 400 = \frac{\overset{16}{\cancel{336}}}{\cancel{5}} \times \frac{4}{\underset{11}{\cancel{231}}} \times \frac{\overset{80}{\cancel{400}}}{1} = \frac{5120}{11} = 465\tfrac{5}{11} \text{ qts.}$$

6. Express a bushel in cubic feet, carrying the decimal to three places.

$$1728\overline{)2150.420}\quad 1.244$$

7. Express a cubic foot as the decimal fraction of a bushel.

$$215042\overline{)172800.0000}\quad 0.8036$$

8. Reduce $1715\frac{1}{2}$ bushels to pints.

$$\begin{array}{r} 1715\frac{1}{2} \text{ bu.} \\ \times 64 \\ \hline 109792 \text{ pts.} \end{array}$$

9. 3047 gals. to barrels.

$$\frac{3047}{31\frac{1}{2}} = \frac{3047}{1} \times \frac{2}{63} = \frac{6094}{63} = 96\tfrac{46}{63} \text{ bbl.}$$

Exercise XLI.

1. Reduce 27,587 grs. to pounds troy.

24)27587 grs.
20)1149 dwts. 11 grs.
12)57 oz. 9 dwt.
4 lbs. 9 oz.

4 lbs. 9 oz. 9 dwt. 11 grs.

2. Reduce 34,652 lbs. to long tons.

112)34652 lbs.
20)309 l. cwt. 44 lbs.
15 l. t. 9 l. cwt.

15 l. t. 9 l. cwt. 44 lbs.

3. Reduce 136,851 oz. to tons.

16)136851 oz.
100)8553 lbs. 3 oz.
20)85 cwt. 53 lbs.
4 t. 5 cwt.

4 t. 5 cwt. 53 lbs. 3 oz.

4. Reduce 864,205 grs. (troy) to pounds.

24)864205 grs.
20)36008 dwts. 13 grs.
12)1880 oz. 8 dwts.
150 lbs.

150 lbs. 8 dwts. 13 grs.

5. Reduce 864,205 grs. (apoth.) to pounds avoirdupois.

$$\begin{array}{r} 123\tfrac{3205}{7000} \text{ lbs.} \\ 7000\overline{)864205} \end{array}$$

$123\frac{3205}{7000} = 123\frac{641}{1400}$ lbs.
$= 123$ lbs. 7 oz. 5.2 drs.

6. Reduce 5 lbs. 7 oz. 6 dwts. 12 grs. to grains.

$$\begin{array}{rl} 5 & \text{lbs. 7 oz. 6 dwt. 12 grs.} \\ \times 12 & \\ \hline 60 & \\ 7 & \\ \hline 67 & \text{oz.} \\ \times 20 & \\ \hline 1340 & \\ 6 & \\ \hline 1346 & \text{dwt.} \\ \times 24 & \\ \hline 32304 & \\ 12 & \\ \hline 32316 & \text{grs.} \end{array}$$

7. Reduce 745 lbs. avoirdupois to troy weight.

$$\frac{\overset{175}{\cancel{7000}}}{\underset{144}{\cancel{5760}}} \times \frac{745}{1} \text{ lbs.}$$

$= \frac{130375}{144}$ lbs. $= 905\frac{55}{144}$ lbs.
$= 905$ lbs. 4 oz. 11 dwt. 16 grs.

8. Reduce 745 lbs. troy to avoirdupois weight.

$$\frac{\overset{144}{\cancel{5760}}}{\underset{\underset{35}{\cancel{175}}}{\cancel{7000}}} \times \frac{\overset{149}{\cancel{745}}}{1} \text{ lbs.}$$

$= \frac{21456}{35}$ lbs. $= 613\frac{1}{35}$ lbs.
$= 613\frac{1}{35}$ lbs. $= 613$ lbs. $7\frac{11}{35}$ drs.

9. Reduce 23,047,125 drs. to tons.

$$\begin{array}{rl} 16\underline{)23047125} & \text{drs.} \\ 16\underline{)1440445} & \text{oz. 5 drs.} \\ 100\underline{)90027} & \text{lbs. 13 oz.} \\ 20\underline{)900} & \text{cwt. 27 lbs.} \\ 45 & \text{t.} \end{array}$$

45 t. 27 lbs. 13 oz. 5 drs.

10. Reduce 90,252,381 drs. to tons.

$$\begin{array}{rl} 16\underline{)90252381} & \text{drs.} \\ 16\underline{)5640773} & \text{oz. 13 drs.} \\ 100\underline{)332548} & \text{lbs. 5 oz.} \\ 20\underline{)3525} & \text{cwt. 48 lbs.} \\ 176 & \text{t. 5 cwt.} \end{array}$$

176 t. 5 cwt. 48 lbs. 5 oz. 13 drs.

11. Reduce 1 pint to minims.

$$\begin{array}{rl} 1 & \text{fl. oz. xvj.} \\ 16 & \\ \hline 16 & \text{fl. drm. viij.} \\ 8 & \\ \hline 128 & \text{♏ lx.} \\ 60 & \\ \hline 7680 & \text{♏.} \end{array}$$

12. Reduce 8000 ♏ to ounces.

$$\begin{array}{rl} 60\underline{)8000} & \text{♏} \\ 8\underline{)135} & \text{♏ lx. 20 ♏.} \\ 16 & \text{fl. drm. viij. 5 ♏ lx.} \end{array}$$

16 fl. drm. viij. 5 ♏ lx. 20 ♏.

EXERCISE XLII.

1. Reduce 6 hrs. 17 min. 25 sec. to seconds.

$$\begin{array}{r l} & \text{6 hrs. 17 min. 25 sec.} \\ \times\, 60 & \\ \hline 360 & \\ 17 & \\ \hline 377 & \text{min.} \\ \times\, 60 & \\ \hline 22620 & \\ 25 & \\ \hline 22645 & \text{sec.} \end{array}$$

2. Reduce 1 yr. 13 dys. 4 min. to minutes.

$$\begin{array}{r l} & \text{1 yr. 13 d. 4 min.} \\ \times\, 365 & \\ \hline 365 & \\ 13 & \\ \hline 378 & \text{d.} \\ \times\, 24 & \\ \hline 9072 & \text{h.} \\ \times\, 60 & \\ \hline 544320 & \\ 4 & \\ \hline 544324 & \text{min.} \end{array}$$

3. Reduce 48,567 min. to days.

$$\begin{array}{l} 60)\underline{48567}\ \text{min.} \\ \quad 24)\underline{809}\ \text{hrs. 27 min.} \\ \qquad\quad 33\ \text{d. 17 hrs.} \end{array}$$

33 d. 17 hrs. 27 min.

4. Reduce 742,392 sec. to days.

$$\begin{array}{l} 60)\underline{742392}\ \text{sec.} \\ \quad 60)\underline{12373}\ \text{min. 12 sec.} \\ \qquad 24)\underline{206}\ \text{hrs. 13 min.} \\ \qquad\qquad 8\ \text{d. 14 hrs.} \end{array}$$

8 d. 14 hrs. 13 min. 12 sec.

5. Find the number of days, reckoning from noon of the one to noon of the other, between the following days in the year 1880: July 4 and December 2; February 1 and May 29; January 3 and October 15; also, between December 25, 1880, and May 25, 1881.

27 d.	28 d.	28 d.	6 d.
31 d.	31 d.	29 d.	31 d.
30 d.	30 d.	31 d.	28 d.
31 d.	29 d.	30 d.	31 d.
30 d.	118 d.	31 d.	30 d.
2 d.		30 d.	25 d.
151 d.		31 d.	151 d.
		31 d.	
		30 d.	
		15 d.	
		286 d.	

6. How many minutes are there from midnight of March 7 to midnight of June 20?

$$\begin{array}{r l} 24 & \text{d.} \\ 30 & \text{d.} \\ 31 & \text{d.} \\ 20 & \text{d.} \\ \hline 105 & \text{d.} \\ \times\, 24 & \\ \hline 2520 & \text{hrs.} \\ \times\, 60 & \\ \hline 151200 & \text{min.} \end{array}$$

7. Find the number of seconds from eight o'clock Monday morning till six o'clock the next Saturday evening.

16 hrs.
24 hrs.
24 hrs.
24 hrs.
24 hrs.
18 hrs.
130 hrs.
$\times$ 60
7800 min.
$\times$ 60
468000 sec.

8. Which of the years 1600, 1656, 1700, 1734, 1800, 1818, 1880, 1900, 1924, 2000 are leap years?

1600 (divisible by 400).
1656 " " 4).
1880 " " 4).
1924 " " 4).
2000 " " 400).

Exercise XLIII.

1. Reduce 2° 30′ 25″ to seconds.

2° 30′ 25″
$\times$ 60
120
30
150′
$\times$ 60
9000
25
9025″

2. Reduce 15° 3′ 22″ to seconds.

15° 3′ 22″.
$\times$ 60
900
3
903′
$\times$ 60
54180
22
54202″

3. Reduce 56,760″ to degrees.

60)56760″
60)946′
15° 46′

4. Reduce 212,221″ to degrees.

60)212221″
60)3537′ 1″
58° 57′
58° 57′ 1″.

5. The hour and minute hands of a watch form an angle of how many degrees at 3 o'clock? at 4 o'clock? at 6 o'clock? at 7½ o'clock? at 11 o'clock? at 12 o'clock?

$\frac{3}{12} = \frac{1}{4} = 90°$. $\frac{4}{12} = \frac{1}{3} = 120°$. $\frac{6}{12} = \frac{1}{2} = 180°$.

$\frac{1\frac{1}{2}}{12} = \frac{1}{\cancel{12}_{4}} \times \frac{\cancel{3}}{2} = \frac{1}{8} = 45°$. $\frac{1}{12} = 30°$. $\frac{0}{12} = 0°$.

6. How many geographical miles in the width of the torrid zone (46° 55′)? How many statute miles?

46° 55′
× 60
2760
55
2815′
= 2815 geog. mi.

$46° 55' = 46\frac{11}{12}° = 46.91\frac{2}{3}$.

$46.91\frac{2}{3}$
× 69.16
$3244.7566\frac{2}{3}$
$= 3244.75\frac{2}{3}$ stat. mi.

Exercise XLIV.

1. Reduce £583 6 *s.* 8 *d.* to pence.

£583 6 *s.* 8 *d.*
× 20
11660
6
11666 *s.*
× 12
139992
8
140000 *d.*

2. Reduce £79 18 *s.* $11\frac{1}{2}$ *d.* to farthings.

£79 18 *s.* $11\frac{1}{2}$ *d.*
× 20
1580
18
1598 *s.*
× 12
19176
$11\frac{1}{2}$
$19187\frac{1}{2}$ *d.*
× 4
76750 far.

3. Reduce 28,572 *d.* to pounds.

12)28572 *d.*
20)2381 *s.*
£119 1 *s.*

4. Reduce 272,191 *d.* to half-sovereigns.

12)272191 *d.*
10)22682 *s.* 7 *d.*
2268 half-sov. 2 *s.*

2268 half-sov. 2 s. 7 *d.*

5. Reduce 27,281 crowns to guineas.

27281 half-crowns.
5
21)136405 *s.*
6495 *g.* 10 *s.*

6. Reduce 1,716,114 guineas to pounds.

$$\begin{array}{r} 1716114\ g. \\ \times 21 \\ \hline 20)\underline{36038394}\ s. \\ £1801919\ 14\ s. \end{array}$$

7. Reduce 291,374 far. to pounds.

$$\begin{array}{l} 4)\underline{291374}\ \text{far.} \\ 12)\underline{72843}\ d.\ 2\ \text{far.} \\ 20)\underline{6070}\ s.\ 3\ d. \\ \quad £303\ 10\ s. \end{array}$$

£303 10 *s.* 3 *d.* 2 far.

8. Reduce 709,126*d.* to guineas.

$$\begin{array}{l} 12)\underline{709126}\ d. \\ 21)\underline{59093}\ s.\ 10\ d. \\ \quad 2813\ g.\ 20\ s. \end{array}$$

2813 *g.* 20 *s.* 10 *d.*

9. Reduce 286,347 far. to crowns.

$$\begin{array}{l} 4)\underline{286347}\ \text{far.} \\ 12)\underline{71586}\ d.\ 3\ \text{far.} \\ 5)\underline{5965}\ s.\ 6\ d. \\ \quad 1193\ \text{crowns.} \end{array}$$

1193 crowns 6 *d.* 3 far.

10. Reduce 20 francs to dollars.

$$\begin{array}{r} \$0.193 \\ \times 20 \\ \hline \$3.86 \end{array}$$

11. Reduce 20 marks to dollars.

$$\begin{array}{r} \$0.238 \\ \times 20 \\ \hline \$4.76 \end{array}$$

12. Reduce 5 roubles to dollars.

$$\begin{array}{r} \$0.734 \\ \times 5 \\ \hline \$3.67 \end{array}$$

13. Find the whole sum of money in a box containing 35 sovereigns, 27 half-sovereigns, 13 crowns, 41 half-crowns, and 85 shillings.

35 sovereigns	= 700 *s.*
27 half-sov.	= 270 *s.*
13 crowns	= 65 *s.*
41 half-crowns	= $102\frac{1}{2}$ *s.*
85 shillings	= 85 *s.*
	$1222\frac{1}{2}$ *s.*

$$\begin{array}{l} 20)\underline{1222\frac{1}{2}}\ s. \\ \quad £61\ 2\frac{1}{2}\ s. \end{array}$$

= £61 2 *s.* 6 *d.*

Exercise XLV.

1. Express 59° F. in Centigrade scale; in Réaumur's scale.

59° F. is 27° above freezing-point.

$$\frac{\overset{3}{\cancel{27}}}{1}\times\frac{5}{\cancel{9}} = 15°\ \text{C.} \qquad \frac{\overset{3}{\cancel{27}}}{1}\times\frac{4}{\cancel{9}} = 12°\ \text{R.}$$

2. Express 77° F. in Centigrade scale; in Réaumur's scale.

77° F. = 45° above freezing-point.

$$\frac{\overset{5}{\cancel{45}}}{1}\times\frac{5}{\cancel{9}}\text{ C.} = 25°\text{ C.} \qquad \frac{\overset{5}{\cancel{45}}}{1}\times\frac{4}{\cancel{9}}\text{ R.} = 20°\text{ R.}$$

3. Express 950° F. in Centigrade scale; in Réaumur's scale.

950° F. = 918° above freezing-point.

$$\frac{\overset{102}{\cancel{918}}}{1}\times\frac{5}{\cancel{9}}\text{ C.} = 510°\text{ C.} \qquad \frac{\overset{102}{\cancel{918}}}{1}\times\frac{4}{\cancel{9}}\text{ R.} = 408°\text{ R.}$$

4. Express − 40° F. in Centigrade scale; in Réaumur's scale.

− 40° F. = 72° below freezing-point.

$\frac{5}{9}$ of − 72° C. = − 40° C. $\frac{4}{9}$ of − 72° R. = − 320° R.

5. Express − 4° F. in Centigrade scale; in Réaumur's scale.

− 4° F. = 36° below freezing-point.

$\frac{5}{9}$ of − 36° C. = − 20° C. $\frac{4}{9}$ of − 36° R. = − 16° R.

6. Express 10° C. in Fahrenheit's scale; in Réaumur's scale.

10° C. = $\frac{9}{5}$ of 10° + 32° F. = 50° F. $\frac{4}{5}$ of 10° R. = 8° R.

7. Express 22° C. in Fahrenheit's scale; in Réaumur's scale.

22° C. = $\frac{9}{5}$ of 22° + 32° F. = $71\frac{3}{5}$° F. $\frac{4}{5}$ of 22° R. = $17\frac{3}{5}$° R.

8. Express − 30° C. in Fahrenheit's scale; in Réaumur's scale.

− 30° C. = $\frac{9}{5}$ × − 30° + 32° F. = − 22° F. $\frac{4}{5}$ × − 30° R. = − 24° R.

9. Express − $11\frac{3}{7}$° C. in Fahrenheit's scale; in Réaumur's scale.

− $11\frac{3}{7}$° C. = $\frac{9}{5}$ × − $11\frac{3}{7}$° + 32° F. = $11\frac{3}{7}$° F. $\frac{4}{5}$ × − $11\frac{3}{7}$° R. = − $9\frac{1}{7}$° R.

10. Express 4° R. in Fahrenheit's scale; in Centigrade scale.

4° R. = $\frac{9}{4}$ of 4° + 32° F. = 41° F. $\frac{5}{4}$ of 4° C. = 5° C.

11. Express 12° R. in Fahrenheit's scale; in Centigrade scale.

12° R. = $\frac{9}{4}$ of 12° + 32° F. = 59° F. $\frac{5}{4}$ of 12° C. = 15° C.

12. Express $-20°$ R. in Fahrenheit's scale; in Centigrade scale.

$-20°$ R. $= \frac{9}{4}$ of $-20° + 32°$ F. $= -13°$ F. $\frac{5}{4}$ of $-20°$ C. $= -25°$ C.

13. Express 4° C. in Fahrenheit's scale; in Réaumur's scale.

4° C. $= \frac{9}{5}$ of $4° + 32°$ F. $= 39\frac{1}{5}°$ F. $\frac{4}{5}$ of 4° R. $= 3\frac{1}{5}°$ R.

14. Express 0° F. in Centigrade scale; in Réaumur's scale.

0° F. = 32° below freezing-point.

$\frac{5}{9}$ of $-32°$ C. $= -17\frac{7}{9}°$ C. $\frac{4}{9}$ of $-32°$ R. $= -14\frac{2}{9}°$ R.

Exercise XLVI.

1. Add:

	hrs.	min.	sec.
	14	21	37
	17	13	32
	9	47	43
	12	53	54
	22	17	50
3 dys.	4	34	36

2. Add:

cu. yds.	cu. ft.	cu. in.
130	5	820
56	20	304
37	4	86
8	10	129
12	19	175
245	4	1514

3. Add:

£.	s.	d.
35	2	$6\frac{3}{4}$
18	5	4
27	3	10
12	0	5
92	12	1 3 far.

4. Add:

mi.	rds.	yds.	ft.	in.
6	120	3	2	2
18	15	1	1	6
3	215	2	2	3
7	95	1	1	8
35	126	$3\frac{1}{2}$	1	7
		$\frac{1}{2}$ =	1	6
35	126	4	0	1

5. Add:

A.	sq. rds.	sq. yds.	sq. ft.	sq. in.
74	21	5	4	100
23	37	13	5	83
12	106	17	8	7
41	50	23	0	24
151	55	$29\frac{3}{4}$	0	70
		$\frac{3}{4}$ =	6	108
151	55	29	7	34

6. Add 5 bu. 3 pks. 6 qts. 1 pt.; 6 bu. 2 pks. 7 qts.; 7 bu. 1 pk.

1 qt. 1 pt.; 1 pk. 7 qts.; 2 bu. 3 pks. 1 pt.

bu.	pks.	qts.	pts.
5	3	6	1
6	2	7	0
7	1	1	1
0	1	7	0
2	3	0	1
23	0	6	1

7. Add 48 t. 13 cwt. 75 lbs. 6 oz. 4 drms.; 25 t. 12 cwt. 27 lbs. 8 oz. 13 drms.; 51 t. 10 cwt. 44 lbs. 15 drms.; 80 t. 5 cwt. 6 oz.; 19 cwt. 27 lbs.; 25 lbs. 8 oz. 10 drms.; 5 t. 5 cwt. 5 oz.

t.	cwt.	lbs.	oz.	drs.
48	13	75	6	4
25	12	27	8	13
51	10	44	0	15
80	5	0	6	0
0	19	27	0	0
0	0	25	8	10
5	5	0	5	0
212	6	0	3	10

8. Add 50 gals. 3 qts. 1 pt. 3 gi.; 12 gal. 1 qt. 1 pt. 1 gi.; 5 gals. 2 qts. 1 pt. 2 gi.; 75 gal. 3 qts. 1 pt. 3 gi.; 80 gals. 3 qts. 0 pts. 1 gi.; 17 gals. 1 qt. 1 pt. 3 gi.

gals.	qts.	pts.	gi.
50	3	1	3
12	1	1	1
5	2	1	2
75	3	1	3
80	3	0	1
17	1	1	3
243	1	0	1

9. Add 13 lbs. 4 oz. 8 dwt. 6 grs.; 25 lbs. 8 oz. 13 dwt. 20 grs.; 8 lbs. 11 oz. 14 grs.; 20 lbs. 16 dwt. 8 grs.; 15 lbs. 9 oz. 12 dwt.; 4 oz. 3 dwt.

lbs.	oz.	dwt.	grs.
13	4	8	6
25	8	13	20
8	11	0	14
20	0	16	8
15	9	12	0
0	4	3	0
84	2	14	0

10. Add 4 gals. 3 qts. 1 pt.; 3 gals. 2 qts. $1\frac{1}{2}$ pts.; 12 gals. 3 qts.; 14 gals. $1\frac{1}{2}$ pts.; 5 gals. 2 qts. 1 pt.

gals.	qts.	pts.
4	3	1
3	2	$1\frac{1}{2}$
12	3	0
14	0	$1\frac{1}{2}$
5	2	1
41	0	1

11. Add 60° 50′ 50″; 20° 41′ 52″; 30° 25′ 20″; 20° 32′ 43″.

°	′	″
60	50	50
20	41	52
30	25	20
20	32	43
132	30	45

Exercise XLVII.

1. Subtract 23 lbs. 8 oz. 19 dwt. 10 grs. from 58 lbs. 6 oz. 17 dwt. 21 grs.

lbs.	oz.	dwt.	grs.
58	6	17	21
23	8	19	10
34	9	18	11

2. Subtract 5 bu. 1 pk. 6 qts. 1 pt. from 5 bu. 3 pks. 3 qts.

bu.	pks.	qts.	pts.
5	3	3	0
5	1	6	1
	1	4	1

3. Subtract 32 cu. yds. 13 cu. ft. 1600 cu. in. from 39 cu. yds. 17 cu. ft. 1400 cu. in.

cu. yds.	cu. ft.	cu. in.
39	17	1400
32	13	1600
7	3	1528

4. Subtract £92 15 *s.* $1\frac{1}{4}$ *d.* from £120 13 *s.* 4 *d.*

£.	s.	d.
120	13	4
92	15	$1\frac{1}{4}$
27	18	2 3 far.

5. Subtract 22 gals. 3 qts. 1 pt. from 30 gals. 2 qts.

gals.	qts.	pts.
30	2	0
22	3	1
7	2	1.

6. Subtract 17 t. 7 cwt. 17 lbs. 6 oz. 10 drs. from 25 t. 13 cwt. 15 lbs. 12 oz. 5 drs.

t.	cwt.	lbs.	oz.	drs.
25	13	15	12	5
17	7	17	6	10
8	5	98	5	11

7. Subtract 13 A. 150 sq. rds. 98 sq. ft. 10 sq. in. from 20 A.

A.	sq. rds.	sq. ft.	sq. in.
20	0	0	0
13	150	98	10
6	9	$173\frac{1}{4}$	134
		$\frac{1}{4} =$	36
6	9	174	26

8. Subtract 58° 33′ 36″ from 90° 11′ 21″.

°	′	″
90	11	21
58	33	36
31	37	45

9. Subtract 2 yrs. 213 dys. 17 hrs. from 3 yrs. 147 dys. 14 hrs.

yrs.	dys.	hrs.
3	147	14
2	213	17
	298	21

10. Subtract 3 mi. 217 rds. 4 yds. 1 ft. 3 in. from 4 mi. 100 rds. 3 yds.

mi.	rds.	yds.	ft.	in.
4	100	3	0	0
3	217	4	1	3
	202	$3\frac{1}{2}$	1	9
		$\frac{1}{2} =$	1	6
	202	4	0	3

EXERCISE XLVIII.

1. Multiply £31 2 *s.* 6½ *d.* by 8.

£.	s.	d.
31	2	6½
		8
249	0	4

2. Multiply 19 gals. 3 qts. 1 pt. by 70.

gals.	qts.	pts.
19	3	1
		70
1391	1	0

3. Multiply 3 lbs. 4 oz. 8 dwt. 10 grs. by 10.

lbs.	oz.	dwt.	grs.
3	4	8	10
			10
33	8	4	4

4. Multiply 5 t. 10 cwt. 67 lbs. by 10.

t.	cwt.	lbs.
5	10	67
		10
55	6	70

5. Multiply 43 bu. 2 pks. by 63

bu.	pks.
43	2
	7
304	2
	9
2740	2

6. Multiply 15 wks. 3 dys. 5 hrs. 12 min. by 7.

wks.	dys.	hrs.	min.
15	3	5	12
			7
108	1	12	24

7. Multiply 5 cu. yds. 16 cu. ft. 371 cu. in. by 6.

cu. yds.	cu. ft.	cu. in.
5	10	371
		6
32	7	498

8. Multiply 27 A. 76 sq rds. 22 sq. yds. 5 sq. ft. by 90.

sq. mi.	A.	sq. rds.	sq. yds.	sq. ft.	sq. in.
	27	76	22	5	
				9	
	247	50	21½	0	
			½=4½		
	247	50	21	4½	
				10	
3	553	27	3¼	0	
			¼=2		36
3	553	27	3	2	36

9. Multiply 32 rds. 3 yds. 1 ft. by 57.

$$\begin{array}{r} 57 \\ \times 1 \\ \hline 3)\underline{57} \\ 19 \text{ yds.} \end{array} \qquad \begin{array}{r} 57 \\ \times 3 \\ \hline 171 \\ 19 \\ \hline 5\tfrac{1}{2})\,190 \\ 2 \\ \hline 11)\,380 \\ \hline 34 \ldots \end{array} \qquad \begin{array}{r} 57 \\ \times 32 \\ \hline 1824 \\ 34 \\ \hline 320)\,1858 \\ \hline 5 \text{ mi. } 258 \text{ rds.} \end{array}$$

34 . . . 6 half-yds. = 3 yds.

5 mi. 258 rds. 3 yds. *Ans.*

10. Multiply 34 dys. 10 hrs. 13 min. 12 sec. by 108.

$$\begin{array}{r} 108 \\ \times 12 \\ \hline 60\left\{\begin{array}{r} 6 \\ 10 \end{array}\right. \begin{array}{|r} 1296 \\ \hline 216 \end{array} \\ \hline 21 \ldots \tfrac{6}{10} \text{ min.} = 36 \text{ sec.} \end{array} \qquad \begin{array}{r} 108 \\ \times 13 \\ \hline 1404 \\ 21 \\ \hline 60)\,1425 \\ \hline 23 \ldots 45 \text{ min.} \end{array}$$

$$\begin{array}{r} 108 \\ \times 10 \\ \hline 1080 \\ 23 \\ \hline 24)\,1103 \\ \hline 45 \ldots 23 \text{ hrs.} \end{array} \qquad \begin{array}{r} 108 \\ \times 34 \\ \hline 3672 \\ 45 \\ \hline 365)\,3717 \\ \hline 10 \ldots 67 \text{ dys.} \end{array}$$

10 yrs. 67 dys. 23 hrs. 45 min. 36 sec. *Ans.*

11. Multiply 5 mi. 126 rds. 19 yds. 6 in. by 7125.

$$\begin{array}{r} 7125 \\ \times 6 \\ \hline 12\,\big|\,42750 \\ \hline 3\,\big|\,3562 \ldots 6 \text{ in.} \\ \hline 1187 \ldots 1 \text{ ft.} \end{array} \qquad \begin{array}{r} 7125 \\ \times 19 \\ \hline 135375 \\ 1187 \\ \hline 5\tfrac{1}{2})\,136562 \\ 2 \\ \hline 11)\,273124 \\ \hline 24829 \ldots \tfrac{5}{2} = 2\tfrac{1}{2} \text{ yds.} \end{array}$$

7125
× 126
320)897750
2883 . . . 19 rds.

7125
× 5
35625
2883
38508 mi.

38,508 mi. 19 rds. 2½ yds. 1 ft. 6 in. = 38,508 mi. 19 rds. 3 yds. *Ans.*

12. Multiply 11 ℥ 5 ʒ 2 ℈ 11 grs. by 2197.

2197
× 11
20)24167
1208 . . . 7 grs.

2197
× 2
4394
1208
3)5602
1867 . . . 1 ℈.

2197
× 5
10985
1867
8)12852
1606 . . . 4 ʒ.

2197
× 11
24167
1606
12)25773
2147 lbs. 9 ℥.

2147 lbs. 9 ℥ 4 ʒ 1 ℈ 7 grs. *Ans.*

Exercise XLIX.

1. Divide 54 mi. 124 rds. 1 yd. 2 ft. 6 in. by 33.

mi. rds. yd. ft. in.
33)54 124 1 2 6(1
33
21
× 320
6720
124
33)6844(207
66
244
231
13
× 5½
72½

33)72½(2
66
6½
× 3
21½
× 12
258
6
33)264(8
264

1 mi. 207 rds. 2 yds. 8 in. *Ans.*

2. Divide 5 cu. yds. 1 cu. ft. 84 cu. in. by 1716 cu. in.

```
         cu. yds.  cu. ft.  cu. in.
             5        1       84
          × 27
           135
             1
           136
        × 1728
        235008
            84
  1716)235092(137.  Ans.
```

3. Divide 8426 wks. 6 dys. 21 hrs. 10 min. 21 sec. by 1029.

```
         wks.   dys.  hrs.  min.  sec.
  1029) 8426     6     21    10    21(8
        8232
         194
         × 7(+6)
  1029) 1364(1
        1029
         335
        × 24
        8040
          21
  1029) 8061(7
        7203
         858
        × 60
  1029)51490(50
        5145
          40
          60
        2400
          21
  1029) 2421(2 363/1029 = 2 121/343.
        2058
         363
```

8 wks. 1 dy. 7 hrs. 50 min. $2\frac{121}{343}$ sec. *Ans.*

4. Divide £394 2 *s.* $10\frac{1}{2}$ *d.* by £5 2 *s.* $4\frac{1}{2}$ *d.*

£394 2 *s.* $10\frac{1}{2}$ *d.* = 378,378 far.
£5 2 *s.* $4\frac{1}{2}$ *d.* = 4914 far.

```
              77.  Ans.
  4914)378378
```

5. Divide 22 wks. 2 dys. by 11 hrs. 31 min. 12 sec.

22 wks. 2 dys. = 13,478,400 sec.
11 hrs. 31 min. 12 sec. = 41,472 sec.

```
                 325.  Ans.
  41472)13478400
```

6. Divide 74,128 sq. mi. 517 A. 80 sq. rds. by 10,000.

```
          sq. mi.   A.   sq. rds.
  10000) 74128    517    80(7
         70000
          4128
         × 640
       2641920
           517
  10000)2642437(264
        2640000
           2437
          × 160
         389920
             80
  10000) 390000(39
         390000
```

7 sq. mi. 264 A. 39 sq. rds. *Ans.*

7. Divide 38° 37′ 42″ by 5° 31′ 6″.

38° 37′ 42″ = 139,062″.
5° 31′ 6″ = 19,866″.

```
  19866)139062(7.  Ans.
        139062
```

Exercise L.

1. Find the value of $\frac{4}{5}$ of a mile.

$\frac{4}{5}$ mi. = $\frac{4}{5}$ of 320 rds. = 256 rds. *Ans.*

2. Find the value of $\frac{3}{16}$ of an acre.

$\frac{3}{16}$ A. = $\frac{3}{16}$ of 160 sq. rds.
= 30 sq. rds. *Ans.*

3. Find the value of $\frac{5}{8}$ of a hundredweight.

$\frac{5}{8}$ cwt. = $\frac{5}{8}$ of 100 lbs. = $62\frac{1}{2}$ lbs.
$\frac{1}{2}$ lb. = $\frac{1}{2}$ of 16 oz. = 8 oz.
62 lbs. 8 oz. *Ans.*

4. Find the value of $\frac{2}{3}$ of a pound sterling.

£$\frac{2}{3}$ = $\frac{2}{3}$ of 20 *s.* = $13\frac{1}{3}$ *s.*
$\frac{1}{3}$ *s.* = $\frac{1}{3}$ of 12 *d.* = 4 *d.*
13 *s.* 4 *d.* *Ans.*

5. Find the value of $\frac{9}{11}$ of a mile.

$\frac{9}{11}$ mi. = $\frac{9}{11}$ of 320 rds. = $261\frac{9}{11}$ rds.
$\frac{9}{11}$ rds. = $\frac{9}{11}$ of $5\frac{1}{2}$ yds. = $4\frac{1}{2}$ yds.
$\frac{1}{2}$ yd. = $\frac{1}{2}$ of 3 ft. = $1\frac{1}{2}$ ft.
$\frac{1}{2}$ ft. = $\frac{1}{2}$ of 12 in. = 6 in.

261 rds. 4 yds. 1 ft. 6 in. *Ans.*

6. Find the value of $\frac{7}{11}$ of an acre.

$\frac{7}{11}$ A. = $\frac{7}{11}$ of 160 sq. rds. = $101\frac{9}{11}$ sq. rds.
$\frac{9}{11}$ sq. rds. = $\frac{9}{11}$ of $30\frac{1}{4}$ sq. yds. = $24\frac{3}{4}$ sq. yds.
$\frac{3}{4}$ sq. yds. = $\frac{3}{4}$ of 9 sq. ft. = $6\frac{3}{4}$ sq. ft.
$\frac{3}{4}$ sq. ft. = $\frac{3}{4}$ of 144 sq. in. = 108 sq. in.

101 sq. rds. 24 sq. yds. 6 sq. ft. 108 sq. in. *Ans.*

7. Find the value of $\frac{4}{9}$ of a degree.

$\frac{4}{9}°$ = $\frac{4}{9}$ of 60′ = $26\frac{2}{3}$′.
$\frac{2}{3}$′ = $\frac{2}{3}$ of 60″ = 40″.

26′ 40″. *Ans.*

8. Find the value of $\frac{1}{3}$ of a year.

$\frac{1}{3}$ yr. = $\frac{1}{3}$ of 365 dys. = $121\frac{2}{3}$ dys.
$\frac{2}{3}$ dy. = $\frac{2}{3}$ of 24 hrs. = 16 hrs.

121 dys. 16 hrs. *Ans.*

9. Find the value of 0.15625 of a bushel.

0.15625 bu.
$\times 4$
0.625 pks.
$\times 8$
5 qts. *Ans.*

10. Find the value of 0.625 of a gallon.

0.625 gal.
$\times 4$
2.5 qts.
$\times 2$
1 pt.

2 qts. 1 pt. *Ans.*

11. Find the value of 0.875 of a leap-year.

0.875
$\times 366$
320.25 dys.
$\times 24$
6 hrs.
320 dys. 6 hrs.

12. Find the value of 0.325 of a pound troy.

0.325 lbs.
$\times 12$
3.9 oz.
$\times 20$
18 dwt.
3 oz. 18 dwt. *Ans.*

Exercise LI.

1. Find the value of $6\frac{2}{5}$ of 3 A. $101\frac{1}{3}$ sq. rds.

	A.	sq. rds.	
	3	$101\frac{1}{3}$	
		$\times 2$	
5)	6	$202\frac{2}{3}$	
	1	72	16 sq. yds. 1 sq. ft. $28\frac{4}{5}$ sq. in.

A.	sq. rds.	sq. yds.	sq. ft.	sq. in.
3	$101\frac{1}{3}$			
	$\times 6$			
21	128			
1	72	16	1	$28\frac{4}{5}$
23	40	16	1	$28\frac{4}{5}$

2. Find the value of $1\frac{3}{7}$ of 7 hrs. 21 min. 27 sec.

	hrs.	min.	sec.
	7	21	27
			$\times 3$
7)	22	4	21
	3	9	$11\frac{4}{7}$
	7	21	27
	10	30	$38\frac{4}{7}$

3. Find the value of 10.0175 of 1 dy. 13 hrs.

10.0175
$\times 37$ hrs.
370.6475
$\times 60$
38.85 min.
$\times 60$
51 sec.

370 hrs. 38 min. 51 sec. = 15 dys. 10 hrs. 38 min. 51 sec. *Ans.*

4. Find the value of $17\frac{7}{12}$ of 10 yds. 2 ft. $3\frac{1}{5}$ in.

	yds.	ft.	in.
	10	2	$3\frac{1}{5}$
			$\times 7$
12)	75	0	$10\frac{2}{5}$
	6	0	$9\frac{13}{15}$

yds.	ft.	in.
10	2	$3\frac{1}{5}$
		$\times 17$
182	2	$6\frac{2}{5}$
6	0	$9\frac{13}{15}$
189	0	$4\frac{4}{15}$

34 rds. 2 yds. 0 ft. $4\frac{4}{15}$ in. *Ans.*

5. Find the value of 0.01284 of 14 mi.

0.01284
$\times$ 14 mi.
0.17976 mi.
$\times$ 320
57.5232 rds.
$\times 5\frac{1}{2}$
2.8776 yds.
$\times$ 3
2.6328 ft.
$\times$ 12
7.5936 in.

57 rds. 2 yds. 2 ft. 7.5936 in. *Ans.*

6. Find the value of 0.42776 of 12 t. 10 cwt.

10 cwt. = $\frac{1}{2}$ t.

0.42776
$\times 12\frac{1}{2}$ t.
5.347 t.
$\times$ 20
6.94 cwt.
$\times$ 100
94 lbs.

5 t. 6 cwt. 94 lbs. *Ans.*

7. Find the value of $\frac{2}{5}$ of 1 lb. + $3\frac{2}{9}$ oz. + $5\frac{2}{3}$ dwt.

$\frac{2}{5}$ lb. = $\frac{2}{5}$ of 12 oz. = $4\frac{4}{5}$ oz.
$4\frac{4}{5}$ oz. + $3\frac{2}{9}$ oz. = $8\frac{1}{45}$ oz.
$\frac{1}{45}$ oz. = $\frac{1}{45}$ of 20 dwt. = $\frac{4}{9}$ dwt.
$\frac{4}{9}$ dwt. + $5\frac{2}{3}$ dwt. = $6\frac{1}{9}$ dwt.
$\frac{1}{9}$ dwt. = $\frac{1}{9}$ of 24 grs. = $2\frac{2}{3}$ grs.

8 oz. 6 dwt. $2\frac{2}{3}$ grs. *Ans.*

8. Find the value of 0.35 of 4 lbs. 5 oz. 6 dwt. 16 grs.

$0.35 = \frac{35}{100} = \frac{7}{20}$.

	lbs.	oz.	dwt.	grs.
	4	5	6	16
				$\times 7$
20)	31	1	6	16
	1	6	13	8 *Ans.*

9. Find the value of 3.726 mi. − 33.57 rds.

3.726 mi.
$\times$ 320
1192.32 rds.
33.57
1158.75 rds.

= 3 mi. 198 rds. 4 yds. $4\frac{1}{2}$ in. *Ans.*

10. Find the value of $\frac{3}{73}$ of a year + $\frac{9}{56}$ of a week + $\frac{7}{12}$ of an hour.

$\frac{3}{73}$ yr. = $\frac{3}{73}$ of 365 dys. = 15 dys.
$\frac{9}{56}$ wk. = $\frac{9}{56}$ of 7 dys. = $1\frac{1}{8}$ dys.
$\frac{1}{8}$ dy. = $\frac{1}{8}$ of 24 hrs. = 3 hrs.
$\frac{7}{12}$ hr. = $\frac{7}{12}$ of 60 min. = 35 min.

15 dys.		
1 dy.	3 hrs.	
		35 min.
16 dys.	3 hrs.	35 min.

= 2 wks. 2 dys. 3 hrs. 35 min. *Ans.*

11. Find the value of 5.268 of 2 dys. + 2.829 of 16 hrs. + 0.9528 of 25 min.

5.268	2.829	0.9528
$\times$ 2 dys.	$\times$ 16 hrs.	$\times$ 25 min.
10.536 dys.	45.264 hrs.	23.82 min.
$\times$ 24	12.864 hrs.	7.68 min.
12.864 hrs.	58.128 hrs.	31.5 min.
	$\times$ 60	$\times$ 60
	7.68 min.	30 sec.

10 dys. 58 hrs. 31 min. 30 sec. = 12 dys. 10 hrs. 31 min. 30 sec. *Ans.*

12. Find the value of $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of 40 rds. + $\frac{3}{8}$ of a yd.

$\frac{3}{16}$ mi. = $\frac{3}{16}$ of 320 rds.	= 60 rds.
$\frac{2}{3}$ of 40 rds.	= $26\frac{2}{3}$ rds.
60 rds. + $26\frac{2}{3}$ rds.	= $86\frac{2}{3}$ rds.
$\frac{2}{3}$ rds. = $\frac{2}{3}$ of $5\frac{1}{2}$ yds.	= $3\frac{2}{3}$ yds.
$\frac{2}{3}$ yd. + $\frac{3}{8}$ yd.	= $1\frac{1}{24}$ yds.
$1\frac{1}{24}$ yds. + 3 yds.	= $4\frac{1}{24}$ yds.
$\frac{1}{24}$ yd. = $\frac{1}{24}$ of 36 in.	= $1\frac{1}{2}$ in.

86 rds. 4 yds. $1\frac{1}{2}$ in. *Ans.*

13. Find the value of $\frac{3}{4}$ of 2 cwt. 84 lbs. + $\frac{3}{7}$ of 5 cwt. 98 lbs. + $\frac{2}{5}$ of $7\frac{1}{2}$ lbs.

2 cwt. 84 lbs.	5 cwt. 98 lbs.
$\times$ 100	$\times$ 100
4)284 lbs.	598 lbs.
71	$\times$ 3
$\times$ 3	7 | 1794
100)213 lbs.	100 | $256\frac{2}{7}$ lbs.
2 cwt. 13 lbs.	2 cwt. 56 lbs. 4 oz. $9\frac{1}{7}$ drs.

$\frac{2}{5}$ of $7\frac{1}{2}$ lbs. = 3 lbs.

2 cwt. 13 lbs.
2 cwt. 56 lbs. 4 oz. $9\frac{1}{7}$ drs.
3 lbs.

4 cwt. 72 lbs. 4 oz. $9\frac{1}{7}$ drs. *Ans.*

EXERCISE LII.

1. Express a pound avoirdupois as the fraction of a pound troy.

1 lb. avoird. = 7000 troy grs.
1 lb. troy = 5760 troy grs.

$$\frac{7000}{5760} = \frac{175}{144}. \ \textit{Ans.}$$

2. Express an ounce avoirdupois as the fraction of an ounce troy.

$$16\overline{)7000}\quad 437\tfrac{1}{2} \text{ grs.}$$

1 oz. troy = 480 grs.

$$\frac{437\frac{1}{2}}{480} = \frac{875}{960} = \frac{175}{192}. \ \textit{Ans.}$$

3. Express 363 sq. yds. as the fraction of an acre.

$$\begin{array}{r} 1 \text{ A.} \\ \times 160 \\ \hline 160 \text{ sq. rds.} \\ \times 30\frac{1}{4} \\ \hline 4840 \text{ sq. yds.} \end{array}$$

$$\frac{363}{4840} = \frac{3}{40}. \ \textit{Ans.}$$

4. Express $\frac{3}{5}$ of £2 1*s.* 3*d.* + $\frac{5}{11}$ of £1 4*s.* 9*d.* as the fraction of £2 14*s.*

$$\begin{array}{rrr} £ & s. & d. \\ 2 & 1 & 3 \\ \times 20 & & \\ \hline 41\ s. & & \\ \times 12 & & \\ \hline 5)\,495\ d. & & \\ \hline 99 & & \\ \times 3 & & \\ \hline 297\ d. & & \end{array}
\qquad
\begin{array}{rrr} £ & s. & d. \\ 1 & 4 & 9 \\ \times 20 & & \\ \hline 24\ s. & & \\ \times 12 & & \\ \hline 11)\,297\ d. & & \\ \hline 27 & & \\ \times 5 & & \\ \hline 135\ d. & & \end{array}
\qquad
\begin{array}{rr} £ & s. \\ 2 & 14 \\ \times 20 & \\ \hline 54\ s. & \\ \times 12 & \\ \hline 648\ d. & \end{array}$$

$$\frac{297 + 135}{672} = \frac{432}{648} = \frac{2}{3}. \ \textit{Ans.}$$

5. Express 2 mi. 138 rds. 1 yd. as the fraction of 3 mi. 265 rds. 3 yds. 1 ft. 6 in.

$$\begin{array}{r l}
 & \text{2 mi. 138 rds. 1 yd.} \\
\times & \underline{320} \\
 & 640 \\
 & \underline{138} \\
 & 778 \text{ rds.} \\
 & \underline{\times 5\frac{1}{2}} \\
 & 4280 \text{ yds.} \\
 & \underline{\times 36} \\
 & 154080 \text{ in.}
\end{array}
\qquad
\begin{array}{r l}
 & \text{3 mi. 265 rds. 3 yds. 1 ft. 6 in.} \\
\times & \underline{320} \\
 & 960 \\
 & \underline{265} \\
 & 1225 \text{ rds.} \\
 & \underline{\times 5\frac{1}{2}} \\
 & 6740\frac{1}{2} \text{ yds.} \\
 & \underline{\times 3} \\
 & 20222\frac{1}{2} \text{ ft.} \\
 & \underline{\times 12} \\
 & 242676 \text{ in.}
\end{array}$$

$$\frac{154080}{242676}=\frac{40}{63}. \quad \textit{Ans.}$$

6. Express $\frac{2}{7}$ of 560 lbs. as the fraction of 5 long tons.

$$\tfrac{2}{7} \text{ of 560 lbs.} = 160 \text{ lbs.}$$

$$\begin{array}{r l}
 & 2240 \text{ lbs.} \\
 & \underline{\times 5} \\
 & 11200 \text{ lbs.}
\end{array}
\qquad
\frac{160}{11200}=\frac{1}{70}. \quad \textit{Ans.}$$

7. Express $\frac{2}{3}$ of 200 rds. as the fraction of 4 miles.

$$\tfrac{2}{3} \text{ of 200 rds.} = 133\tfrac{1}{3} \text{ rds.}$$

$$\begin{array}{r l}
 & 320 \text{ rds.} \\
 & \underline{\times 4} \\
 & 1280 \text{ rds.}
\end{array}
\qquad
\frac{133\frac{1}{3}}{1280}=\frac{1}{\underset{16}{\cancel{1280}}}\times\frac{\overset{5}{\cancel{400}}}{3}=\frac{5}{48}. \quad \textit{Ans.}$$

8. Express $\frac{10}{27}$ of 2 dys. 2 hrs. 24 min. as the fraction of 2 wks. 1 d.

$$\begin{array}{r l}
 & \text{2 dys. 2 hrs. 24 min.} \\
\times & \underline{24} \\
 & 50 \text{ hrs.} \\
 & \underline{\times 60} \\
27) & \underline{3024} \text{ min.} \\
 & 112 \\
 & \underline{\times 10} \\
 & 1120 \text{ min.}
\end{array}
\qquad
\begin{array}{r l}
 & \text{2 wks. 1 dy.} \\
 & \underline{\times 7} \\
 & 15 \text{ dys.} \\
 & \underline{\times 24} \\
 & 360 \text{ hrs.} \\
 & \underline{\times 60} \\
 & 21600 \text{ min.}
\end{array}$$

$$\frac{1120}{21600}=\frac{7}{135}. \quad \textit{Ans.}$$

9. Express $\frac{4}{5}$ of the difference between 3 yds. 2 ft. 11 in. and 10 yds. 7 in. as the fraction of 8 yds.

$$\begin{array}{rrr} \text{yds.} & \text{ft.} & \text{in.} \\ 10 & 0 & 7 \\ 3 & 2 & 11 \\ \hline 6 & 0 & 8 \end{array}$$

$$\begin{array}{l} \times 36 \\ \hline 224 \text{ in.} \\ \times 4 \\ \hline 5)\underline{896} \\ 179\tfrac{1}{5} \text{ in.} \end{array} \qquad \begin{array}{l} 8 \text{ yds.} \\ \times 36 \\ \hline 288 \text{ in.} \end{array}$$

$$\frac{179\frac{1}{5}}{288} = \frac{1}{\cancel{288}_{9}} \times \frac{\cancel{896}^{28}}{5} = \frac{28}{45}. \textit{ Ans.}$$

10. Express $\frac{10}{21}$ of the difference between $\frac{5}{8}$ of 7 hrs. and $\frac{7}{25}$ of 15 min. as the fraction of 12 hrs. 18 min.

7 hrs. = 420 min.

$$\frac{5}{\cancel{8}_{2}} \times \frac{\cancel{420}^{105}}{\cancel{1}} = \frac{525}{2}.$$

12 hrs. 18 min.

$$\begin{array}{l} 60 \\ \hline 738 \text{ min.} \end{array}$$

$$\frac{7}{\cancel{25}_{5}} \times \frac{\cancel{15}^{3}}{1} = \frac{21}{5}.$$

$$\frac{525}{2} \times \frac{21}{5} = \frac{2583}{10}.$$

$$\frac{\cancel{10}}{\cancel{21}} \times \frac{\cancel{2583}^{123}}{\cancel{10}} = 123 \text{ min.}$$

$$\frac{123}{738} = \frac{1}{6}. \textit{ Ans.}$$

11. Express $\frac{2}{5}$ pt. as the fraction of a gallon.

1 gal. = 8 pts.

$$\frac{\frac{2}{5}}{8} = \frac{1}{\cancel{8}_{4}} \times \frac{\cancel{2}}{5} = \frac{1}{20}. \textit{ Ans.}$$

12. What part of 4 lbs. 1 oz. 8 dwt. 15 grs. is 1 lb. 1 oz. 9 dwt. 15 grs.

4 lbs. 1 oz. 8 dwt. 15 grs.:

15 grs. $= \frac{15}{24}$ dwt. $= \frac{5}{8}$ dwt.

$8\frac{5}{8}$ dwt. $= \dfrac{8\frac{5}{8}}{20}$ oz. $= \frac{69}{160}$ oz.

$1\frac{69}{160}$ oz. $= \dfrac{1\frac{69}{160}}{12}$ lbs. $= \frac{229}{1920}$ lbs.

$4\frac{229}{1920}$ lbs.

1 lb. 1 oz. 9 dwt. 15 grs.:

15 grs. $= \frac{15}{24}$ dwt. $= \frac{5}{8}$ dwt.

$9\frac{5}{8}$ dwt. $= \dfrac{9\frac{5}{8}}{20}$ oz. $= \frac{77}{160}$ oz.

$1\frac{77}{160}$ oz. $= \dfrac{1\frac{77}{160}}{12}$ lbs. $= \frac{79}{640}$ lbs.

$1\frac{79}{640}$ lbs.

$$\frac{1\frac{79}{640}}{4\frac{229}{1920}} = \frac{\overset{3}{\cancel{1920}}}{\underset{11}{\cancel{7909}}} \times \frac{\cancel{719}}{\cancel{640}} = \frac{3}{11}. \textit{ Ans.}$$

13. What part of 2 mi. is $\frac{2}{3}$ of 6 rds. 3 yds. 2 in.?

6 rds. 3 yds. 2 in.:

2 in. $= \frac{2}{36}$ yd. $= \frac{1}{18}$ yd.

$3\frac{1}{18}$ yds. $= \dfrac{3\frac{1}{18}}{5\frac{1}{2}}$ rds. $= \frac{5}{9}$ rd.

$6\frac{5}{9}$ rds.

$\frac{2}{3}$ of $6\frac{5}{9}$ rds. $= \frac{118}{27}$ rds.

$\frac{118}{27}$ rds. $= \dfrac{\frac{118}{27}}{320}$ mi. $= \frac{59}{4320}$ mi.

$$\frac{\frac{59}{4320}}{2} = \frac{1}{2} \text{ of } \frac{59}{4320} = \frac{59}{8640}. \textit{ Ans.}$$

14. What part of a bushel is 1 pk. 2 qts. 1 pt.?

1 pk. 2 qts. 1 pt.:

1 pt. $= \frac{1}{2}$ qt.

$2\frac{1}{2}$ qts. $= \dfrac{2\frac{1}{2}}{8}$ pks. $= \frac{5}{16}$ pks.

$1\frac{5}{16}$ pks. $= \dfrac{1\frac{5}{16}}{4}$ bu. $= \frac{21}{64}$ bu. *Ans.*

15. What part of 20 A. are 19 A. 3.5 sq. ch.

19. A. 3.5 sq. ch.:

3.5 sq. ch. $= \dfrac{3.5}{10}$ A. $= \frac{7}{20}$ A.

$$\frac{19\frac{7}{20}}{20} = \frac{387}{20} \times \frac{1}{20} = \frac{387}{400}. \textit{ Ans.}$$

16. What part of 5 tons are 3 t. 240 lbs.?

3 t. 240 lbs.:

$$240 \text{ lbs.} = \tfrac{240}{2000} \text{ t.} = \tfrac{3}{25} \text{ t.}$$

$$3\tfrac{3}{25} \text{ t.}$$

$$\frac{3\frac{3}{25}}{5} = \frac{1}{5} \times \frac{78}{25} = \frac{78}{125}. \textit{ Ans.}$$

17. 38 sq. rds. 194 sq. ft. 108 sq. in = what part of an acre?

38 sq. rds. 194 sq. ft. 108 sq. in.:

$$108 \text{ sq. in.} = \tfrac{108}{144} \text{ sq. ft.} \quad = \tfrac{3}{4} \text{ sq. ft.}$$

$$194\tfrac{3}{4} \text{ sq. ft.} = \frac{194\frac{3}{4}}{272\frac{1}{4}} \text{ sq. rds.} = \tfrac{779}{1089} \text{ sq. rds.}$$

$$38\tfrac{779}{1089} \text{ sq. rds.} = \frac{38\frac{779}{1089}}{160} \text{ A.} \quad = \frac{42161}{174240} \text{ A. } \textit{Ans.}$$

Exercise LIII.

1. Express 16 *s.* $3\frac{3}{4}$ *d.* as the decimal of a pound.

$$\begin{array}{r|l} 12 & 3.75\ d. \\ 20 & \underline{16.3125\ s.} \\ & £0.815625.\ \textit{Ans.} \end{array}$$

2. Express 233 rds. 9 ft. 10.8 in. as the decimal of a mile.

$$\begin{array}{r|rl} 12 & 10.8 & \text{in.} \\ 3 & 9.9 & \text{ft.} \\ 5\frac{1}{2} & 3.3 & \text{yds.} \\ 320 & \underline{233.6} & \text{rds.} \\ & 0.73 & \text{mi. } \textit{Ans.} \end{array}$$

3. Express 71 sq. rds. 54 sq. ft. 64.8 sq. in. as the decimal of an acre.

$$\begin{array}{r|ll} 144 & 64.8 & \text{sq. in.} \\ 9 & 54.45 & \text{sq. ft.} \\ 30\frac{1}{4} & 6.05 & \text{sq. yds.} \\ 160 & \underline{71.20} & \text{sq. rds.} \\ & 0.445 & \text{A. } \textit{Ans.} \end{array}$$

4. Express 15 hrs. 14 min. 6 sec. as the decimal of 2 days.

$$\begin{array}{r|rl} 60 & 6.000 & \text{sec.} \\ 60 & 14.100 & \text{min.} \\ 24 & \underline{15.235} & \text{hrs.} \\ & 0.6348 & \text{dys.} \end{array}$$

$$\frac{0.6348}{2} = 0.3174. \textit{ Ans.}$$

5. Express 38 sq. rds. 21 sq. yds. 5 sq. ft. 108 sq. in. as the decimal of an acre.

$$\begin{array}{r|rl} 144 & 108.000 & \text{sq. in.} \\ 9 & 5.750 & \text{sq. ft.} \\ 30\frac{1}{4} & 21.638 & \text{sq. yds.} \\ 160 & \underline{38.715} & \text{sq. rds.} \\ & 0.242 & \text{A. } \textit{Ans.} \end{array}$$

6. Express 3 mi. 242 rds. 2 yds. 2 ft. 3 in. as the decimal of 7 mi. 160 rds.

$$\begin{array}{r|l}
12 & 3.00 \text{ in.} \\
3 & 2.25 \text{ ft.} \\
5\frac{1}{2} & 2.75 \text{ yds.} \\
320 & 242.5 \text{ rds.} \\
\hline
 & 3.7578 \text{ mi.}
\end{array}
\qquad
\begin{array}{r|l}
320 & 160 \\
\hline
 & 7.5 \text{ mi.}
\end{array}$$

$$\frac{3.7578}{7.5} = 0.501. \textit{ Ans.}$$

7. Express 5 hrs. 13 min. 30 sec. as the decimal of a week.

$$\begin{array}{r|l}
60 & 30.000 \text{ sec.} \\
60 & 13.500 \text{ min.} \\
24 & 5.225 \text{ hrs.} \\
7 & 0.2177 \text{ dy.} \\
\hline
 & 0.0311 \text{ wk. } \textit{Ans.}
\end{array}$$

8. Express 27° 14′ 45″ as the decimal of 90°.

$$\begin{array}{r|l}
60 & 45.00'' \\
60 & 14.75' \\
\hline
 & 27.246°
\end{array}$$

$$\frac{27.246}{90} = 0.303. \textit{ Ans.}$$

9. Express 54 dys. 2 hrs. 40 min. as the decimal of 365¼ dys.

$$\begin{array}{r|l}
60 & 40.0 \text{ min.} \\
24 & 2.\dot{6} \text{ hrs.} \\
\hline
 & 54.\dot{1} \text{ dys.}
\end{array}$$

$$\frac{54.\dot{1}}{365\frac{1}{4}} = 0.\dot{1}4\dot{8}. \textit{ Ans.}$$

10. Express 2 lbs. avoirdupois as the decimal of 10 lbs troy.

2 lbs. av. = 14,000 grs. troy.
10 lbs. troy = 57,600 grs. troy.

$$\frac{14000}{57600} = 0.243. \textit{ Ans.}$$

11. Express 44,920.9025 hrs. as the decimal of a year.

1 yr. = 8760 hrs.

$$\frac{44920.9025}{8760} = 5.128. \textit{ Ans.}$$

12. Express 1 drm. avoirdupois as the decimal of 1 dwt. troy.

$$\begin{aligned}
1 \text{ drm. avoird.} &= \tfrac{1}{256} \text{ of } 7000 \text{ troy grs.} \\
&= 27.344 \text{ troy grs.} \\
1 \text{ dwt.} &= 24 \text{ troy grs.}
\end{aligned}$$

$$\frac{27.344}{24.} = 1.139. \textit{ Ans.}$$

13. Express 10 milligrams as the decimal of a grain, if a kilogram equals 2 lbs. 8 oz. 3 dwt. 1 gr.

$$\begin{aligned}
2 \text{ lbs. } 8 \text{ oz. } 3 \text{ dwt. } 1 \text{ gr.} &= 15{,}433 \text{ grs.} \\
1 \text{ kg.} &= 100{,}000 \times 10 \text{ mg.}
\end{aligned}$$

$$\frac{15433}{100000} = 0.15433. \textit{ Ans.}$$

14. Express 14.52 sq. yds. as the decimal of a square chain.

1 sq. ch. = 16 sq. rds.
= 484 sq. yds.

$\frac{14.52}{484} = 0.03$. *Ans.*

15. Express 8 cwt. 77 lbs. 9.6 oz. as the decimal of a ton.

16	9.600	oz.
100	77.600	lbs.
20	8.776	cwt.
	0.4388	t. *Ans.*

Exercise LIV.

1. Find the difference in longitude between two places, if the difference in time be 1 hr. 15 min.

1 hr. 15 min. = 75 min. = $\frac{1}{4}(75°) = 18° \ 45'$. *Ans.*

2. Find the difference in longitude between two places, if the difference in time be 2 hrs. 11 min.

2 hrs. 11 min. = 131 min. = $\frac{1}{4}(131°) = 32° \ 45'$. *Ans.*

3. Find the difference in longitude between two places, if the difference in time be 5 hrs. 10 min. 10 sec.

5 hrs. 10 min. 10 sec. = 310 min. 10 sec. = $\frac{1}{4}(310° \ 10')$
= $77° \ 32' \ 30''$. *Ans.*

4. Find the difference in longitude between two places, if the difference in time be 3 hrs. 25 min. 35 sec,

3 hrs. 25 min. 35 sec = 205 min. 35 sec. = $\frac{1}{4}(205° \ 35')$
= $51° \ 23' \ 45''$. *Ans.*

5. Find the difference in longitude between two places, if the difference in time be 6 hrs. 12 min. 30 sec.

6 hrs. 12 min. 30 sec. = 372 min. 30 sec. = $\frac{1}{4}(372° \ 30')$
= $93° \ 7' \ 30''$. *Ans.*

6. Find the difference in longitude between two places, if the difference in time be 4 hrs. 8 min. 12 sec.

4 hrs. 8 min. 12 sec. = 248 min. 12 sec. = $\frac{1}{4}(248° \ 12') = 62° \ 3'$. *Ans.*

7. Find the difference in longitude between two places, if the difference in time be 18 hrs. 10 min.

18 hrs. 10 min. = 1090 min. = $\frac{1}{4}(1090°) = 272° \ 30'$. *Ans.*

8. Find the difference in longitude between two places, if the difference in time be 15 hrs. 15 min. 15 sec.

$$15 \text{ hrs. } 15 \text{ min. } 15 \text{ sec.} = 915 \text{ min. } 15 \text{ sec.} = \tfrac{1}{4}(915^\circ\ 15') = 228^\circ\ 48'\ 45''. \textit{ Ans.}$$

9. Find the difference in time between two places, if the difference in longitude be 9° 20′.

$$9^\circ\ 20' = 4 \times (9 \text{ min. } 20 \text{ sec.}) = 37 \text{ min. } 20 \text{ sec. } \textit{Ans.}$$

10. Find the difference in time between two places, if the difference in longitude be 70° 30′.

$$70^\circ\ 30' = 4 \times (70 \text{ min. } 30 \text{ sec.}) = 4 \text{ hrs. } 42 \text{ min. } \textit{Ans.}$$

11. Find the difference in time between two places, if the difference in longitude be 56° 36′ 12″.

$$56^\circ\ 36'\ 12'' = 56^\circ\ 36.2' = 4 \times (56 \text{ min. } 36.2 \text{ sec.}) = 3 \text{ hrs. } 46 \text{ min. } 24.8 \text{ sec. } \textit{Ans.}$$

12. Find the difference in time between two places, if the difference in longitude be 108° 32′ 36″.

$$108^\circ\ 32'\ 36'' = 108^\circ\ 32.6' = 4 \times (108 \text{ min. } 32.6 \text{ sec.}) = 7 \text{ hrs. } 14 \text{ min. } 10.4. \text{ sec. } \textit{Ans.}$$

13. Find the difference in time between two places, if the difference in longitude be 120° 14′ 30″.

$$120^\circ\ 14'\ 30'' = 120^\circ\ 14.5' = 4 \times (120 \text{ min. } 14.5 \text{ sec.}) = 8 \text{ hrs. } 58 \text{ sec. } \textit{Ans.}$$

14. Find the difference in time between two places, if the difference in longitude be 100° 45′ 54″.

$$100^\circ\ 45'\ 54'' = 100^\circ\ 45.9' = 4 \times (100 \text{ min. } 45.9 \text{ sec.}) = 6 \text{ hrs. } 46 \text{ min. } 3.6 \text{ sec. } \textit{Ans.}$$

15. Find the difference in time between two places, if the difference in longitude be 2° 2′ 2″.

$$2^\circ\ 2'\ 2'' = 2^\circ\ 2\tfrac{1}{30}' = 4 \times (2 \text{ min. } 2\tfrac{1}{30} \text{ sec.}) = 8 \text{ min. } 8\tfrac{2}{15} \text{ sec. } \textit{Ans.}$$

16. Find the difference in time between two places, if the difference in longitude be 75° 10′.

$$75^\circ\ 10' = 4 \times (75 \text{ min. } 10 \text{ sec.}) = 5 \text{ hrs. } 40 \text{ sec. } \textit{Ans.}$$

EXERCISE LV.

The longitude of some public building in:

(1) Berlin is 13° 23′ 43″ E.
(2) Rome, 12° 27′ 14″ E.
(3) Constantinople, 28° 59′ E.
(4) Pekin, 116° 23′ 45″ E.
(5) San Francisco, 122° 26′ 15″ W.
(6) St. Louis, 90° 15′ 15″ W.
(7) Jerusalem, 35° 32′ E.
(8) Bombay, 72° 54′ E.
(9) Calcutta, 88° 19′ 2″ E.
(10) Chicago, 87° 35′ W.
(11) New York, 74° 0′ 3″ W.
(12) Montreal, 73° 25′ W.

1. When it is noon at Greenwich, what is the clock-time at each of the above places?

(1)

$13° \; 23' \; 43''$
$= 13° \; 23\tfrac{43}{60}'$
$= 4 \times (13$ min. $23\tfrac{43}{60}$ sec.)
$= 53$ min. $34\tfrac{13}{15}$ sec. P.M.
$= 12$ hrs. 53 min. $34\tfrac{13}{15}$ sec. P.M. *Ans.*

(2)

$12° \; 27' \; 14''$
$= 12° \; 27\tfrac{7}{30}'$
$= 4 \times (12$ min. $27\tfrac{7}{30}$ sec.)
$= 49$ min. $48\tfrac{14}{15}$ sec. P.M.
$= 12$ hrs. 49 min. $48\tfrac{14}{15}$ sec. P.M. *Ans.*

(3)

$28° \; 59'$
$= 4 \times (28$ min. 59 sec.)
$= 1$ hr. 55 min. 56 sec. P.M. *Ans.*

(4)

$116° \; 23' \; 45''$
$= 116° \; 23\tfrac{3}{4}'$
$= 4 \times (116$ min. $23\tfrac{3}{4}$ sec.)
$= 7$ hrs. 45 min. 35 sec. P.M. *Ans.*

(5)

$122° \; 26' \; 15''$
$= 122° \; 26\tfrac{1}{4}'$
$= 4 \times (122$ min. $26\tfrac{1}{4}$ sec.)
$= 8$ hrs. 9 min. 45 sec.

12 hrs.
8 hrs. 9 min. 45 sec.
3 hrs. 50 min. 15 sec. A.M. *Ans.*

(6)

$90° \; 15' \; 15''$
$= 90° \; 15\tfrac{1}{4}'$
$= 4 \times (90$ min. $15\tfrac{1}{4}$ sec.)
$= 6$ hrs. 1 min. 1 sec.

12 hrs. M.
6 hrs. 1 min. 1 sec.
5 hrs. 58 min. 59 sec. A.M. *Ans.*

(7)

$35° \; 32'$
$= 4 \times (35$ min. 32 sec.)
$= 2$ hrs. 22 min. 8 sec. P.M. *Ans.*

(8)

$72° \; 54'$
$= 4 \times (72$ min. 54 sec.)
$= 4$ hrs. 51 min. 36 sec. P.M. *Ans.*

(9)

$88° \ 19' \ 2''$
$= 88° \ 19\frac{1}{30}'$
$= 4 \times (88$ min. $19\frac{1}{30}$ sec.)
$= 5$ hrs. 53 min. $16\frac{2}{15}$ sec. *Ans.*

(10)

$87° \ 35'$
$= 4 \times (87$ min. 35 sec.)
$= 5$ hrs. 50 min. 20 sec.

12 hrs. M.
5 hrs. 50 min. 20 sec.
6 hrs. 9 min. 40 sec. A.M. *Ans.*

(11)

$74° \ 0' \ 3''$
$= 74° \ \frac{1}{20}'$
$= 4 \times (74$ min. $\frac{1}{20}$ sec.)
$= 4$ hrs. 56 min. $\frac{1}{5}$ sec.
12 hrs. M.
4 hrs. 56 min. $\frac{1}{5}$ sec.
7 hrs. 3 min. $59\frac{4}{5}$ sec. A.M. *Ans.*

(12)

$73° \ 25'$
$= 4 \times (73$ min. 25 sec.)
$= 4$ hrs. 53 min. 40 sec.

12 hrs. M.
4 hrs. 53 min. 40 sec.
7 hrs. 6 min. 20 sec. A.M. *Ans.*

2. When it is half-past four P.M. at Chicago, what is the clock-time at each of the above places?

(1)

$87° \ 35'$ W.
$13° \ 23' \ 43''$ E.
$100° \ 58' \ 43''$

$= 100° \ 58\frac{43}{60}'$
$= 4 \times (100$ min. $58\frac{43}{60}$ sec.
$= 6$ hrs. 43 min. $54\frac{13}{15}$ sec.
4 hrs. 30 min. P.M.
11 hrs. 13 min. $54\frac{13}{15}$ sec. P.M.
Ans.

(2)

$87° \ 35'$ W.
$12° \ 27' \ 14''$ E.
$100° \ 2' \ 14''$

$= 100° \ 2\frac{7}{30}'$
$= 4 \times (100$ min. $2\frac{7}{30}$ sec.)
$= 6$ hrs. 40 min. $8\frac{14}{15}$ sec.
4 hrs. 30 min. P.M.
11 hrs. 10 min. $8\frac{14}{15}$ sec. P.M.
Ans.

(3)

$87° \ 35'$ W.
$28° \ 59'$ E.
$116° \ 34'$

$= 4 \times (116$ min. 34 sec.)
$= 7$ hrs. 46 min. 16 sec.
4 hrs. 30 min. P.M.
12 hrs. 16 min. 16 sec. A.M. *Ans.*

(4)

$87° \ 35'$ W.
$116° \ 23' \ 45''$ E.
$203° \ 58' \ 45''$

$360°$
$203° \ 58' \ 45''$
$156° \ 1' \ 15''$

$= 156° \ 1\frac{1}{4}'$
$= \ 4 \times (156$ min. $1\frac{1}{4}$ sec.)
$= 10$ hrs. 24 min. 5 sec.
4 hrs. 30 min. P.M.
10 hrs. 24 min. 5 sec.
6 hrs. 5 min. 55 sec. A.M.
Ans.

(5)

$$\begin{array}{r} 122^\circ\ 26\tfrac{1}{4}'\ \text{W.} \\ \underline{87^\circ\ 35'\ \text{W.}} \\ 34^\circ\ 51\tfrac{1}{4}' \end{array}$$

= 4 × (34 min. $51\frac{1}{4}$ sec.)
= 2 hrs. 19 min. 25 sec.

4 hrs. 30 min. P.M.
2 hrs. 19 min. 25 sec.

2 hrs. 10 min. 35 sec. P.M. *Ans.*

(6)

$$\begin{array}{r} 90^\circ\ 15\tfrac{1}{4}'\ \text{W.} \\ \underline{87^\circ\ 35'\ \text{W.}} \\ 2^\circ\ 40\tfrac{1}{4}' \end{array}$$

= 4 × (2 min. $40\frac{1}{4}$ sec.)
= 10 min. 41 sec.

4 hrs. 30 min. P.M.
10 min. 41 sec.

4 hrs. 19 min. 19 sec. P.M. *Ans.*

(7)

$$\begin{array}{r} 87^\circ\ 35'\ \text{W.} \\ \underline{35^\circ\ 32'\ \text{E.}} \\ 123^\circ\ \ 7' \end{array}$$

= 4 × (123 min. 7 sec.)
= 8 hrs. 12 min. 28 sec.
4 hrs. 30 min. P.M.

12 hrs. 42 min. 28 sec. A.M. *Ans.*

(8)

$$\begin{array}{r} 72^\circ\ 54'\ \text{E.} \\ \underline{87^\circ\ 35'\ \text{W.}} \\ 160^\circ\ 29' \end{array}$$

= 4 × (160 min. 29 sec.)
= 10 hrs. 41 min. 56 sec.
4 hrs. 30 min. P.M.

15 hrs. 11 min. 56 sec.
= 3 hrs. 11 min. 56 sec. A.M. *Ans.*

(9)

$$\begin{array}{r} 88^\circ\ 19'\ 2''\ \text{E.} \\ \underline{87^\circ\ 35'\quad\ \text{W.}} \\ 175^\circ\ 54'\ 2'' \end{array}$$

= 175° $54\frac{1}{30}$′.
= 4 × (175 min. $54\frac{1}{30}$ sec.)
= 11 hrs. 43 min. $36\frac{2}{15}$ sec.
4 hrs. 30 min. P.M.

16 hrs. 13 min. $36\frac{2}{15}$ sec.
= 4 hrs. 13 min. $36\frac{2}{15}$ sec. A.M. *Ans.*

(10)

(4 hrs. 30 min. P.M.). *Ans.*

(11)

$$\begin{array}{r} 87^\circ\ 35'\ \text{W.} \\ \underline{74^\circ\ \ 0'\ \ 3''} \\ 13^\circ\ 34'\ 57'' \end{array}$$

= 13° $34\frac{19}{20}$′
= 4 × (13 min. $34\frac{19}{20}$ sec.)
= 54 min. $19\frac{4}{5}$ sec.
4 hrs. 30 min. P.M.

5 hrs. 24 min. $19\frac{4}{5}$ sec. P.M. *Ans.*

(12)

$$\begin{array}{r} 87^\circ\ 35'\ \text{W.} \\ \underline{73^\circ\ 25'\ \text{W.}} \\ 14^\circ\ 10' \end{array}$$

= 4 × (14 min. 10 sec.)
= 56 min. 40 sec.
4 hrs. 30 min. P.M.

5 hrs. 26 min. 40 sec. P.M. *Ans.*

3. When it is eight o'clock A.M. at Constantinople, what is the clock-time at each of the above places?

(1)

$$\begin{array}{r} 28^\circ\ 59'\ \text{E.} \\ 13^\circ\ 23'\ 43''\ \text{E.} \\ \hline 15^\circ\ 35'\ 17'' \end{array}$$

$= 15^\circ\ 35\tfrac{17}{60}'$
$= 4 \times (15$ min. $35\tfrac{17}{60}$ sec.)
$= 1$ hr. 2 min. $21\tfrac{2}{15}$ sec.

8 hrs. A.M.
1 hr. 2 min. $21\tfrac{2}{15}$ sec.

6 hrs. 57 min. $38\tfrac{13}{15}$ sec. A.M. *Ans.*

(2)

$$\begin{array}{r} 28^\circ\ 59'\ \text{E.} \\ 12^\circ\ 27'\ 14''\ \text{E.} \\ \hline 16^\circ\ 31'\ 46'' \end{array}$$

$= 16^\circ\ 31\tfrac{23}{30}'$
$= 4 \times (16$ min. $31\tfrac{23}{30}$ sec.)
$= 1$ hr. 6 min. $7\tfrac{1}{15}$ sec.

8 hrs. A.M.
1 hr. 6 min. $7\tfrac{1}{15}$ sec.

6 hrs. 53 min. $52\tfrac{14}{15}$ sec. A.M. *Ans.*

(3)

(8 hrs. A.M.). *Ans.*

(4)

$$\begin{array}{r} 116^\circ\ 23\tfrac{3}{4}'\ \text{E.} \\ 28^\circ\ 59'\ \text{E.} \\ \hline 87^\circ\ 24\tfrac{3}{4}' \end{array}$$

$= 4 \times (87$ min. $24\tfrac{3}{4}$ sec.)
$= 5$ hrs. 49 min. 39 sec.
8 hrs. A.M.

1 hr. 49 min. 39 sec. P.M. *Ans.*

(5)

$$\begin{array}{r} 28^\circ\ 59'\ \text{E.} \\ 122^\circ\ 26\tfrac{1}{4}'\ \text{W.} \\ \hline 151^\circ\ 25\tfrac{1}{4}' \end{array}$$

$= 4 \times (151$ min. $25\tfrac{1}{4}$ sec.)
$= 10$ hrs. 5 min. 41 sec.

8 hrs. A.M.
10 hrs. 5 min. 41 sec.

9 hrs. 54 min. 19 sec. P.M. *Ans.*

(6)

$$\begin{array}{r} 28^\circ\ 59'\ \text{E.} \\ 90^\circ\ 15'\ 15''\ \text{W.} \\ \hline 119^\circ\ 14'\ 15'' \end{array}$$

$= 119^\circ\ 14\tfrac{1}{4}'$
$= 4 \times (119$ min. $14\tfrac{1}{4}$ sec.)
$= 7$ hrs. 56 min. 57 sec.

8 hrs. A.M.
7 hrs. 56 min. 57 sec.

3 min. 3 sec. A.M.
$= 12$ hrs. 3 min. 3 sec. A.M. *Ans.*

(7)

$$\begin{array}{r} 35^\circ\ 32'\ \text{E.} \\ 28^\circ\ 59'\ \text{E.} \\ \hline 6^\circ\ 33' \end{array}$$

$= 4 \times (6$ min. 33 sec.)
$=$ 26 min. 12 sec.
8 hrs. A.M.

8 hrs. 26 min. 12 sec. A.M. *Ans.*

(8)

72° 54′ E.
28° 59′ E.

43° 55′

= 4 × (43 min. 55 sec.)
= 2 hrs. 55 min. 40 sec.
8 hrs. A.M.

10 hrs. 55 min. 40 sec. A.M.
Ans.

(10)

87° 35′ W.
28° 59′ E.

116° 34′

= 4 × (116 min. 34 sec.)
= 7 hrs. 46 min. 16 sec.
8 hrs. A.M.
7 hrs. 46 min. 16 sec.

13 min. 44 sec. A.M.
= 12 hrs. 13 min. 44 sec. A.M.
Ans.

(9)

88° 19′ 2″ E.
28° 59′ E.

59° 20′ 2″

= 59° $20\frac{1}{30}$′
= 4 × (59 min. $20\frac{1}{30}$ sec.)
= 3 hrs. 57 min. $20\frac{2}{15}$ sec.
8 hrs. A.M.

11 hrs. 57 min. $20\frac{2}{15}$ sec. A.M.
Ans.

(11)

74° 0′ 3″ W.
28° 59′ E.

102° 59′ 3″

= 102° $59\frac{1}{20}$′
= 4 × (102 min. $59\frac{1}{20}$ sec.)
= 6 hrs. 51 min. $56\frac{1}{5}$ sec.
8 hrs. A.M.
6 hrs. 51 min. $56\frac{1}{5}$ sec.

1 hr. 8 min. $3\frac{4}{5}$ sec. A.M. *Ans.*

(12)

73° 25′ W.
28° 59′ E.

102° 24′

= 4 × (102 min. 24 sec.)
= 6 hrs. 49 min. 36 sec.
8 hrs. A.M.
6 hrs. 49 min. 36 sec.

1 hr. 10 min. 24 sec. A.M. *Ans.*

Exercise LVI.

When it is noon at Greenwich the time at

(1) Boston, Mass., is 7 hrs. 15 min. 46 sec. A.M.
(2) Augusta, Me., 7 hrs. 20 min. 40 sec. A.M.
(3) Columbia, S.C., 6 hrs. 35 min. 32 sec. A.M.
(4) Little Rock, Ark., 5 hrs. 51 min. 12 sec. A.M.
(5) Salt Lake, 4 hrs. 30 min. A.M.
(6) Albany, N.Y., 7 hrs. 5 min. 1 sec. A.M.
(7) Columbus, O., 6 hrs. 27 min. 48 sec. A.M.
(8) Harrisburg, Penn., 6 hrs. 52 min. 40 sec. A.M.
(9) New Orleans, La., 6 hrs. A.M.
(10) Springfield, Ill., 6 hrs. 1 min. 48 sec. A.M.
(11) Washington, D.C., 6 hrs. 51 min. 44 sec. A.M.

1. What is the longitude of each of the above places?

(1)

hrs.	min.	sec.
12	0	0
7	5	46
4	44	14

= 284 min. 14 sec.
= $\frac{1}{4}$ of 284° 14′
= 71° 3′ 30″ W. *Ans.*

(2)

hrs.	min.	sec.
12	0	0
7	20	40
4	39	20

= 279 min. 20 sec.
= $\frac{1}{4}$ of 279° 20′
= 69° 50′ W. *Ans*

(3)

hrs.	min.	sec.
12	0	0
6	35	32
5	24	28

= 324 min. 28 sec.
= $\frac{1}{4}$ of 324° 28′
= 81° 7′ W. *Ans.*

(4)

hrs.	min.	sec.
12	0	0
5	51	12
6	8	48

= 368 min. 48 sec.
= $\frac{1}{4}$ of 368° 48′
= 92° 12′ W. *Ans.*

(5)

hrs.	min.
12	0
4	30
7	30

= 450 min.
= $\frac{1}{4}$ of 450°
= 112° 30′ W. *Ans.*

(6)

hrs.	min.	sec.
12	0	0
7	5	1
4	54	59

= 294 min. 59 sec.
= $\frac{1}{4}$ of 294° 59′
= 73° 44′ 45″ W. *Ans.*

(7)

hrs.	min.	sec.
12	0	0
6	27	48
5	32	12

= 332 min. 12 sec.
= $\frac{1}{4}$ of 332° 12′
= 83° 3′ W. *Ans.*

(9)

hrs.
12
6
6

= 360 min.
= $\frac{1}{4}$ of 360°
= 90 ° W. *Ans.*

(8)

hrs.	min.	sec.
12	0	0
6	52	40
5	7	20

= 307 min. 20 sec.
= $\frac{1}{4}$ of 307° 20′
= 76° 50′ W. *Ans.*

(10)

hrs.	min.	sec.
12	0	0
6	1	48
5	58	12

= 358 min. 12 sec.
= $\frac{1}{4}$ of 358° 12′
= 89° 33′ W. *Ans.*

(11)

hrs.	min.	sec.
12	0	0
6	51	44
5	8	16

= 308 min. 16 sec.
= $\frac{1}{4}$ of 308° 16′
= 77° 4′ W. *Ans.*

EXERCISE LVII.

1. Reduce 7 gals. 3 qts. 1 pt. to gallons and decimal of a gallon.

2 | 1.000 pt.
4 | 3.500 qts.
7.875 gal. *Ans.*

2. Reduce £4.375 to pounds, shillings, and pence.

£4.375
20
7.5 *s.*
12
6 *d*
£4 7 *s.* 6 *d.* *Ans.*

3. Reduce 7.6875 gals. to gallons, quarts, and pints.

$$\begin{array}{r} 7.6875 \text{ gal.} \\ 4 \\ \hline 2.75 \text{ qts.} \\ 2 \\ \hline 1.5 \end{array}$$

7 gals. 2 qts. 1.5 pts. *Ans.*

4. Reduce to pounds, shillings, and pence \$5.875; \$7.38; \$17.85; \$21.75; if \$4.85 be equal to a pound.

(1)

$$485\overline{)587.5}\quad \pounds 1.2\tfrac{11}{97}$$

$0.2\frac{11}{97} = \frac{41}{194}$.

$\frac{41}{194}$ of 20 *s.* $= 4\frac{22}{97}$ *s.*

$\frac{22}{97}$ of 12 *d.* $= 2\frac{70}{97}$ *d.*

£1 4 *s.* $2\frac{70}{97}$ *d.* *Ans.*

(2)

$$485\overline{)738}\quad \pounds 1\tfrac{253}{485}$$

$\frac{253}{485}$ of 20 *s.* $= 10\frac{42}{97}$ *s.*

$\frac{42}{97}$ of 12 *d.* $= 5\frac{19}{97}$ *d.*

£1 10 *s.* $5\frac{19}{97}$ *d.* *Ans.*

(3)

$$485\overline{)1785}\quad \pounds 3\tfrac{330}{485}$$

$\frac{330}{485} = \frac{66}{97}$.

$\frac{66}{97}$ of 20 *s.* $= 13\frac{59}{97}$ *s.*

$\frac{59}{97}$ of 12 *d.* $= 7\frac{29}{97}$ *d.*

£3 13 *s.* $7\frac{29}{97}$ *d.* *Ans.*

(4)

$$485\overline{)2175}\quad \pounds 4\tfrac{235}{485}$$

$\frac{235}{485} = \frac{47}{97}$.

$\frac{47}{97}$ of 20 *s.* $= 9\frac{67}{97}$ *s.*

$\frac{67}{97}$ of 12 *d.* $= 8\frac{28}{97}$ *d.*

£4 9 *s.* $8\frac{28}{97}$ *d.* *Ans.*

5. How many square yards in 3.7156 acres?

$$\begin{array}{r} 3.7156 \text{ A.} \\ \times 160 \\ \hline 594.496 \text{ sq. rds.} \\ \times 30\tfrac{1}{4} \\ \hline 17983.504 \text{ sq. yds.} \end{array}$$

Ans.

6. If 2 qts. of linseed oil be mixed with $\frac{1}{2}$ pt. spirits of turpentine, what fraction of the mixture is turpentine? How much turpentine in one pint of the mixture?

2 qts. = 4 pts.

4 pts. + $\frac{1}{2}$ pt. = $4\frac{1}{2}$ pts.

$$\frac{\frac{1}{2}}{4\frac{1}{2}} = \frac{2}{9} \times \frac{1}{2} = \frac{1}{9} \qquad (1)$$

$\frac{1}{9}$ of 1 pt. = $\frac{1}{9}$ pt. (2)

7. Reduce 5.1732 mi. to yards, feet, and inches.

$$\begin{array}{r} 5.1732 \text{ mi.} \\ \times 1760 \\ \hline 9104.832 \text{ yds.} \\ \times 3 \\ \hline 2.496 \text{ ft.} \\ \times 12 \\ \hline 5.952 \text{ in.} \end{array}$$

9104 yds. 2 ft. 5.952 in. *Ans.*

8. If a man walk 88 mi. in 26 hrs., how many feet does he walk each second?

$$\frac{\overset{22}{\cancel{5280}}}{1}\times\frac{\overset{44}{\cancel{88}}}{1}\times\frac{1}{\underset{15}{\cancel{3600}}}\times\frac{1}{\underset{13}{\cancel{26}}}=\frac{968}{195}\text{ ft.}$$

$= 4\frac{188}{195}$ ft. *Ans.*

9. Of a mixture of sand and lime 0.27 of the weight is lime. How many ounces of lime in a pound of the mixture? How many troy grains of lime in an avoirdupois pound of the mixture?

$$\begin{array}{r} 16 \text{ oz.} \\ \times 0.27 \\ \hline 4.32 \text{ oz. } \textit{Ans.} \end{array} \qquad \begin{array}{r} 0.27 \\ \times 7000 \\ \hline 1890 \text{ troy grs.} \end{array}$$

10. A gill of water is put into a quart measure, and the measure filled with milk. What part of the mixture is water?

8 gi. = 1 qt.

$\therefore$ 1 gi. = $\frac{1}{8}\times 1$ qt. = $\frac{1}{8}$ qt.

$\therefore$ $\frac{1}{8}$ is water.

11. Reduce 555 ft. to the decimal of a mile.

$$\begin{array}{r} 0.10511\dot{3}\dot{6} \text{ mi. } \textit{Ans.} \\ 528\overline{)55.5000000} \end{array}$$

12. Reduce 1 mi. 13 rds. 2 yds. 2 ft. 6 in. to inches.

$$\begin{array}{r} 1 \text{ mi.} \\ \times 320 \\ \hline 320 \\ 13 \\ \hline 333 \text{ rds.} \\ \times 5\frac{1}{2} \\ \hline 1833\frac{1}{2} \text{ yds.} \\ \times 3 \\ \hline 5502\frac{1}{2} \text{ ft.} \\ \times 12 \\ \hline 66036 \text{ in. } \textit{Ans.} \end{array}$$

13. How many cubic inches in $2\frac{1}{2}$ cubic feet?

$$\begin{array}{r} 1728 \text{ cu. in.} \\ \times 2\frac{1}{2} \\ \hline 4320 \text{ cu. in. } \textit{Ans.} \end{array}$$

14. How many pounds avoirdupois does a cubic yard of water weigh if a cubic foot weigh 1000 ounces?

$$\begin{array}{r} 27 \\ \times 1000 \text{ oz.} \\ \hline 16\overline{)27000} \text{ oz.} \\ 1687\frac{1}{2} \text{ lbs. } \textit{Ans.} \end{array}$$

15. Express the weight of a cubic yard of water as the decimal of a ton.

$$\frac{16875}{20000} = \frac{675}{800} = \frac{27}{32}.$$

$$\begin{array}{r} 0.84375 \text{ t. } \textit{Ans.} \\ 32\overline{)27.00000} \end{array}$$

16. What is the weight of 7 bu. $3\frac{1}{2}$ pks. of potatoes?

$$3\tfrac{1}{2} \text{ pks.} = \frac{3\frac{1}{2}}{4} \text{ bu.} = \tfrac{7}{8} \text{ bu.}$$

$$\begin{array}{r} 60 \text{ lbs.} \\ \times\, 7\frac{7}{8} \\ \hline 472\frac{1}{2} \text{ lbs. } \textit{Ans.} \end{array}$$

17. A farmer sowed 5 bu. 1 pk. 1 qt. of seed, and harvested from it 103 bu. 3 pks. 5 qts. How much did he raise from a bushel of seed?

5 bu. 1 pk. 1 qt.:

$$1 \text{ qt.} = \tfrac{1}{8} \text{ pk.}$$

$$1\tfrac{1}{8} \text{ pk.} = \frac{1\frac{1}{8}}{4} \text{ bu.} = \tfrac{9}{32} \text{ bu.}$$

$$5\tfrac{9}{32} \text{ bu.}$$

103 bu. 3 pks. 5 qts.:

$$5 \text{ qts.} = \tfrac{5}{8} \text{ pk.}$$

$$3\tfrac{5}{8} \text{ pks.} = \frac{3\frac{5}{8}}{4} \text{ bu.} = \tfrac{29}{32} \text{ bu.}$$

$$103\tfrac{29}{32} \text{ bu.}$$

$$\frac{103\frac{29}{32}}{5\frac{9}{32}} = \frac{3325}{169} = 19\tfrac{114}{169} \text{ bu.}$$

$$\tfrac{114}{169} \text{ of 4 pks.} = 2\tfrac{118}{169} \text{ pks.}$$

$$\tfrac{118}{169} \text{ of 8 qts.} = 5\tfrac{99}{169} \text{ qts.}$$

19 bu. 2 pks. 5.6 qts. *Ans.*

18. How many bushels in 5 t. of oats?

$$\begin{array}{r} 2000 \text{ lbs.} \\ 5 \\ \hline 10000 \text{ lbs.} \end{array}$$

$$\begin{array}{r} 312\frac{1}{2} \text{ bu. } \textit{Ans.} \\ 32\overline{)10000} \end{array}$$

19. How many bottles, each holding 1 pt. 3 gi., can be filled from a barrel of cider?

1 pt. 3 gi.:

$$3 \text{ gi.} = \tfrac{3}{4} \text{ pt.}$$

$$1\tfrac{3}{4} \text{ pt.} = \frac{1\frac{3}{4}}{8} \text{ gal.} = \tfrac{7}{32} \text{ gal.}$$

$$\frac{31\frac{1}{2}}{\frac{7}{32}} = \frac{\overset{16}{\cancel{32}}}{\cancel{7}} \times \frac{\overset{9}{\cancel{63}}}{\cancel{2}} = 144. \ \textit{Ans.}$$

20. If a steamer make 13 mi. 6 rds. an hour, how far will she go between 6 A.M. and 6 P.M.? How many hours will she require to make 113 miles?

$$\begin{array}{rr} \text{mi.} & \text{rds.} \\ 13 & 6 \\ & 12 \\ \hline 156 & 72. \ \textit{Ans.} \end{array}$$

13 mi. 6 rds.:

$$6 \text{ rds.} = \tfrac{3}{160} \text{ mi.}$$

$$13\tfrac{3}{160} \text{ mi.}$$

$$\frac{113}{13\frac{3}{160}} = \frac{160}{2083} \times \frac{113}{1}$$

$$= \frac{18080}{2083} = 8\tfrac{1416}{2083} \text{ hrs.}$$

21. If a locomotive run at the rate of 111 rds. a minute, how many hours will it require to run from Boston to Buffalo, 498 miles?

$$\begin{array}{r} 498 \text{ mi.} \\ \times 320 \\ \hline 159360 \text{ rds.} \end{array}$$

$$111\overline{)159360}\quad 1435\tfrac{25}{37}$$

$$6\underline{)143.5\tfrac{25}{37}}$$
24 nearly. *Ans.*

22. What is the cost of 12 A. 146 sq. rds. land at $16.25 an acre?

146 sq. rds. $= \frac{73}{80}$ A.

$$12\tfrac{73}{80} \times \$16\tfrac{1}{4} = \frac{1033}{\overset{}{\cancel{80}}_{16}} \times \frac{\$\cancel{65}^{13}}{4}$$

$$= \frac{\$13429}{64} = \$209.83. \textit{ Ans.}$$

23. What is the cost of 8 t. 3 cwt. 27 lbs. of coal at $5.75 a ton?

100	27 lbs.
20	3.27 cwt.
	8.1635 t.

$$\begin{array}{r} 8.1635 \text{ t.} \\ \times \$5\tfrac{3}{4} \\ \hline \$46.94. \end{array}$$ *Ans.*

24. What is the cost of 7 t. 1560 lbs. of hay at $15.50 a ton?

1560 lbs. $= \frac{1560}{2000}$ t. $= \frac{78}{100}$ t.

$$\begin{array}{r} 7.78 \\ \times \$15\tfrac{1}{2} \\ \hline \$120.59. \end{array}$$ *Ans.*

25. What is the cost of a carload of wheat weighing 20,000 lbs., at $1.05 a bushel?

$$\begin{array}{r} 6\underline{)2000} \\ 333\tfrac{1}{3} \\ \times \$1.05 \\ \hline \$350. \end{array}$$ *Ans.*

26. Reduce 5 rds. 4 yds. $2\frac{1}{2}$ ft. to the decimal of a mile.

3	2.5	ft.
$5\frac{1}{2}$	$4.8\dot{3}$	yds.
320	$5.8\dot{7}$	rds.
	0.0184 mi. *Ans.*	

27. Reduce 9 sq. ch. 11.25 sq. rds. to the decimal of an acre.

16	11.25
10	9.703125
	0.9703125 A. *Ans.*

28. Reduce 0.09375 bu. to quarts.

$$\begin{array}{r} 0.09375 \text{ bu.} \\ \times 32 \\ \hline 3 \text{ qts.} \end{array}$$ *Ans.*

29. Reduce 7560 chains to miles.

$$\begin{array}{r} 7560 \text{ ch.} \\ 4 \\ \hline 30240 \text{ rds.} \end{array}$$

$$32\overline{)3024.0}\quad 94.5 \text{ mi.}$$ *Ans.*

30. How many gross are 2000 pens?

$13\frac{8}{9}$. *Ans.*

$144\overline{)2000}$

31. Find the cost of 27.248 A., at $93.75 an acre.

27.248
× $$93\frac{3}{4}$

$2554.50. *Ans.*

32. Which is the greater, 2.8 of 3 ft. 11 in. or 3.11 of 2 ft. 8 in., and by how much?

3 ft. 11 in.	2 ft. 8 in.
× 12	× 12
47 in.	32 in.
× 2.8	× 3.11
131.6 in.	99.52 in.
99.52	

$12\overline{)32.08}$ in.
2 ft. 8.08 in.

The former by 2 ft. 8.08 in. *Ans.*

33. Reduce 171 lbs. 6 oz. troy to the decimal of a ton avoirdupois.

$7\overline{)0.1715}$
0.0245
× 5760

141.12 lbs. avoird.

100 | 141.12
20 | 1.4112
0.07056 t. *Ans.*

34. Express 14.52 sq. yds. as the decimal of a square chain.

$30\frac{1}{4}\overline{)14.52}$
4
$121\overline{)58.08}$(0.48 sq. rd.
0.03 sq. ch. *Ans*
$16\overline{)0.48}$

35. If a sovereign be equal to 25.22 francs, or to $4.85, what decimal of a dollar is a franc?

$0.192. *Ans.*

$2522\overline{)485.000}$

36. Express 2.805 florins — 1.89 half-crowns as the decimal of £0.472.

2.805	1.89
2 *s.*	× 2.5 *s.*
5.61 *s.*	4.725 *s.*
4.725	

$20\overline{)0.885}$ *s.*
0.04425

0.09375. *Ans.*

$472\overline{)44.25000}$

37. If 0.327 of some work be done in 3 hrs. 38 min., how long will the whole work require?

3 hrs. 38 min.
60

218 min.

$666.\dot{6}$

$327\overline{)218000.0}$

$0.\dot{6} = \frac{6}{9} = \frac{2}{3}$.

$666.\dot{6}$ min. $= 666\frac{2}{3}$ min.

$= 11$ hrs. $6\frac{2}{3}$ min.

$= 11$ hrs. 6 min. 40 sec. *Ans.*

38. A can run a mile in 7.68 min.; B can run at the rate of 7.68 mi. an hour. Which is the faster runner?

$$768\overline{)6000.00}\quad 7.81$$

$\therefore$ A is the faster runner.

39. How many miles an hour does a person walk who takes 2 steps a second and 1900 steps in a mile?

$$\begin{array}{r} 60 \\ \times 2 \\ \hline 120 \\ \times 60 \\ \hline 7200 \end{array} \text{ steps.}$$

$$19\overline{)72}\quad 3\tfrac{15}{19} \text{ mi. } \textit{Ans.}$$

40. If an ounce troy of gold be worth \$20, what is the value of a pound avoirdupois?

$$\begin{array}{r} \$20 \\ \times 12 \\ \hline \$240 \end{array} \text{ per lb. troy.}$$

$$\frac{\overset{175}{\cancel{7000}}}{\underset{3}{\cancel{\cancel{5760}}}} \times \frac{\overset{5}{\$\cancel{240}}}{1} = \frac{875}{3} = \$291.67. \quad \textit{Ans.}$$

(with 5760 reduced to 144, then to 3)

41. Two stars cross the meridian at 6 hrs. 4 min. 42.3 sec. and 7 hrs. 2 min. 57.21 sec., respectively. What is the interval between the observations?

hrs.	min.	sec.
7	2	57.21
6	4	42.3
	58	14.91 *Ans.*

42. How long will it take to fill $\frac{13}{25}$ of a cistern, when the whole requires 6 hrs. 10 min.?

6 hrs. 10 min.

10 min. $= \frac{10}{60}$ hr. $= \frac{1}{6}$ hr.

$6\frac{1}{6}$ hrs.

$6\frac{1}{6} \times \frac{13}{25} = \frac{37}{6} \times \frac{13}{25}$

$= \frac{481}{150} = 3\frac{31}{150}$ hrs.

$\frac{31}{150}$ of 60 min. $= 12\frac{2}{5}$ min.

$\frac{2}{5}$ of 60 sec. $= 24$ sec.

3 hrs. 12 min. 24 sec. *Ans.*

43. The circumference of a circle is 6 yds. 1 ft. 5.1 in., and is divided into 360 degrees. What is the length of 55 degrees?

$\frac{55}{360} = \frac{11}{72}$

$$\begin{array}{l} 6 \text{ yds. } 1 \text{ ft. } 5.1 \text{ in.} \\ \times 3 \\ \hline 19 \text{ ft.} \\ \times 12 \\ \hline 228 \\ \times 5.1 \\ \hline 233.1 \text{ in.} \end{array}$$

$$72\overline{)233.1000}\quad 3.2375$$

$$\begin{array}{r} 3.2375 \text{ in.} \\ \times 11 \\ \hline 12\overline{)35.6125} \text{ in.} \\ 2 \text{ ft. } 11.6125 \text{ in. } \textit{Ans.} \end{array}$$

44. Multiply 2 t. 16 cwt. $63\frac{2}{5}$ lbs. by $1\frac{4}{9}$.

	t.	cwt.	lbs.
	2	16	$63\frac{2}{5}$
			$\times 4$
9)	11	6	$53\frac{3}{5}$
	1	5	$17\frac{1}{15}$
	2	16	$63\frac{2}{5}$
	4	1	$80\frac{7}{15}$. *Ans.*

45. Into how many shares has £120 been divided when each share is £3 8 *s.* $6\frac{6}{7}$ *d.*?

$$6\tfrac{6}{7}\,d. = \frac{6\frac{6}{7}}{12}\,s. = \tfrac{4}{7}\,s.$$

$$8\tfrac{4}{7}\,s. = £\frac{8\frac{4}{7}}{20} = £\tfrac{3}{7}.$$

$$\frac{120}{3\frac{3}{7}} = \frac{7}{\cancel{24}} \times \frac{\overset{5}{\cancel{120}}}{1} = 35. \text{ Ans.}$$

46. If $\frac{13}{15}$ of one line be equal to $\frac{8}{9}$ of another line, which is the greater? and what fraction of it is the less?

$$\frac{13}{15},\ \frac{8}{9} = \frac{39,\ 40}{45}.$$

$\therefore$ the former is the greater. *Ans.*

$$\frac{\frac{13}{15}}{\frac{8}{9}} = \frac{\overset{3}{\cancel{9}}}{8} \times \frac{13}{\underset{5}{\cancel{15}}} = \frac{39}{40}. \text{ Ans.}$$

47. Multiply 5 mi. 206 rds. 2 ft. 2 in. by 786.

$$\begin{array}{r} 2 \text{ in.} \\ \times\ 786 \\ \hline 12)\ 1572 \text{ in.} \\ \hline 131 \text{ ft.} \end{array} \qquad \begin{array}{r} 2 \text{ ft.} \\ \times\ 786 \\ \hline 1572 \\ 131 \\ \hline 3\,|\,1703 \\ 5\frac{1}{2}\,|\ \ 567\ \ldots\ 2 \text{ ft.} \\ \times\ 2 \\ \hline 11)\ 1134 \\ \hline 103\ \ldots\ \frac{1}{2} \text{ yd.} \end{array} \qquad \begin{array}{r} 206 \\ \times\ 786 \\ \hline 161916 \\ 103 \\ \hline 320)\ 162019 \\ \hline 506\ \ldots\ 99 \text{ rds.} \end{array}$$

$$\begin{array}{r} 5 \\ \times\ 786 \\ \hline 3930 \\ 506 \\ \hline 4436 \text{ mi.} \end{array}$$

4436 mi. 99 rds. $\frac{1}{2}$ yd. 2 ft.
= 4436 mi. 99 rds. 1 yd. 6 in. *Ans.*

48. The returns of a gold mine are 241 t. of ore yielding 2 oz. 1 dwt. 15 grs. of fine gold a ton, and 193 t. yielding 1 oz. 12 dwt. 9 grs. a ton. Find the value of the whole yield, at $19.45 an ounce.

lbs.	oz.	dwt.	grs.
	2	1	15
			× 241
41	9	11	15
	1	12	9
			× 193
26	0	8	9
41	9	11	15
67	10		

$$\begin{array}{r} 67 \\ \times\ 12 \\ \hline 804 \\ 10 \\ \hline 814 \text{ oz.} \end{array} \qquad \begin{array}{r} \$19.45 \\ \times\ 814 \\ \hline \$15{,}832.30 \end{array} \text{ Ans.}$$

49. Divide 93 long tons 56 lbs. by 23 lbs. 5 oz.

$$\begin{array}{r}
93 \text{ l. t. } 56 \text{ lbs.} \\
\times 20 \\
\hline
1860 \text{ l. cwt.} \\
\times 112 \\
\hline
208320 \\
56 \\
\hline
208376 \text{ lbs.} \\
\times 16 \\
\hline
3334016 \text{ oz.}
\end{array}
\qquad
\begin{array}{r}
23 \text{ lbs. } 5 \text{ oz.} \\
\times 16 \\
\hline
368 \\
5 \\
\hline
373 \text{ oz.}
\end{array}$$

$$\begin{array}{r}
8938\tfrac{142}{373} \ \textit{Ans.} \\
\hline
373)3334016
\end{array}$$

50. Telegraph poles on railroads are generally erected at intervals of 88 yds. Show that if a passenger count the number of poles which the train passes in three minutes, that number will express the number of miles an hour the train is going.

$$1760 \text{ yds.} = 1 \text{ mi.} \qquad 60 \text{ min.} = 1 \text{ hr.}$$
$$88 \text{ yds.} = \tfrac{1}{20} \text{ mi.} \qquad 3 \text{ min.} = \tfrac{1}{20} \text{ hr.}$$

51. If Greenwich time be 5 hrs. 8 min. 16 sec. later than Washington time, and Chicago be 87° 35′ W., what is the difference between Washington and Chicago time?

$$\begin{array}{rl}
87^\circ\ 35' & = 4 \times (87 \text{ min. } 35 \text{ sec.}) \\
& = 5 \text{ hrs. } 50 \text{ min. } 20 \text{ sec.} \\
& \ 5 \text{ hrs. } \ \ 8 \text{ min. } 16 \text{ sec.} \\
\hline
& \qquad 42 \text{ min. } \ 4 \text{ sec. } \textit{Ans.}
\end{array}$$

EXERCISE LVIII.

1. A train from New York to Philadelphia, 90 miles, makes the whole distance in 2 hrs. 5 min. What is its rate?

$$2 \text{ hrs. } 5 \text{ min.} = 2\tfrac{1}{12} \text{ hrs.}$$
$$90 \text{ mi.} \div 2\tfrac{1}{12} = 43\tfrac{1}{5} \text{ mi. } \textit{Ans.}$$

2. Winlock, in 1869, found that electricity went through 7200 miles of wire in $\frac{2}{3}$ of a second. What was its rate per second?

$$7200 \text{ mi.} \div \tfrac{2}{3} = 10{,}800 \text{ mi. } \textit{Ans.}$$

3. If the time required for a signal to pass through the cable from Brest to Duxbury, 3799 miles, be 0.816 of a second, what is the rate per second?

$$3799 \text{ mi.} \div 0.816 = 4655.637 \text{ mi. } \textit{Ans.}$$

4. If the report of a gun $1\frac{1}{4}$ miles distant is heard in $5\frac{5}{8}$ seconds after the flash is seen, what is the velocity of sound, in feet, per second?

$$1\tfrac{1}{4} \text{ mi.} \div 5\tfrac{5}{8} = \tfrac{2}{9} \text{ mi.} = 1173\tfrac{1}{3} \text{ ft. } \textit{Ans.}$$

5. If a man walk $3\frac{1}{5}$ miles in 46 minutes, what is his rate per hour?

$$3\tfrac{1}{5} \text{ mi.} \div \tfrac{46}{60} = 4\tfrac{4}{23} \text{ mi. } \textit{Ans.}$$

6. If a horse go $47\frac{1}{2}$ miles in 10 hrs. 40 min., what is his average rate per hour?

$$10 \text{ hrs. } 40 \text{ min.} = 10\tfrac{2}{3} \text{ hrs.}$$

$$47\tfrac{1}{2} \text{ mi.} \div 10\tfrac{2}{3} = 4\tfrac{29}{64} \text{ mi. } \textit{Ans.}$$

7. If a stone on a glacier move $95\frac{1}{2}$ feet in 188 days, what is its rate, in inches, per day?

$$95\tfrac{1}{2} \text{ ft.} \div 188 = \tfrac{191}{376} \text{ ft.} = 6\tfrac{9}{94} \text{ in. } \textit{Ans.}$$

8. If a horse trot $\frac{5}{6}$ of a mile in $2\frac{1}{3}$ minutes, in what time can he trot a mile?

$$2\tfrac{1}{3} \text{ min.} \div \tfrac{5}{6} = 2\tfrac{4}{5} \text{ min. } \textit{Ans.}$$

9. If a train run 18 miles in 39 minutes, how long does it take to run one mile?

$$39 \text{ min.} \div 18 = 2\tfrac{1}{6} \text{ min. } \textit{Ans.}$$

10. If sound travel 1125 feet a second, how long will it take to travel one mile?

$$1 \text{ mi.} = 5280 \text{ ft.}$$

$$5280 \div 1125 = 4.7 \text{ sec. } \textit{Ans.}$$

11. If a train require 3 hours to travel $104\frac{1}{4}$ miles, find its average time for travelling a mile.

$$104\tfrac{1}{4} \text{ mi.} \div 3 = 34\tfrac{3}{4} \text{ mi.}$$

$$60 \text{ min.} \div 34\tfrac{3}{4} = 1\tfrac{101}{139} \text{ min.} = 1 \text{ min. } 43\tfrac{83}{139} \text{ sec. } \textit{Ans.}$$

12. If a mower cut $7\frac{1}{2}$ acres of grass in $3\frac{1}{2}$ days, what part of a day will it take him to cut one acre? If a day consist of 10 working-hours, what part of an acre does he cut in an hour?

$$3\tfrac{1}{2} \text{ dys.} \div 7\tfrac{1}{2} = \tfrac{7}{15} \text{ dy. } \textit{Ans.}$$
$$3\tfrac{1}{2} \text{ dys. of 10 hrs.} = 35 \text{ hrs.,}$$
$$\text{and } 7\tfrac{1}{2} \text{ A.} \div 35 = \tfrac{3}{14} \text{ A. } \textit{Ans.}$$

13. If a mower cut $3\frac{1}{2}$ square rods in $\frac{1}{8}$ of an hour, how many acres can he cut in a day of 10 hours?

$$3\tfrac{1}{2} \text{ sq. rds.} \div \tfrac{1}{8} = 28 \text{ sq. rds.}$$
$$10 \times 28 \text{ sq. rds.} = 280 \text{ sq. rds.} = 1\tfrac{3}{4} \text{ A. } \textit{Ans.}$$

14. If a fountain yield $117\frac{1}{2}$ gallons in $\frac{3}{4}$ of an hour, at what rate per hour is it flowing?

$$117\tfrac{1}{2} \text{ gals.} \div \tfrac{3}{4} = 156\tfrac{2}{3} \text{ gals. } \textit{Ans.}$$

15. If a merchant's profits be \$3147 in $7\frac{1}{2}$ months, what are his profits for a year?

$$7\tfrac{1}{2} \text{ mo.} = \frac{7\frac{1}{2}}{12} \text{ yr.} = \tfrac{5}{8} \text{ yr.}$$
$$\$3147 \div \tfrac{5}{8} = \$5035.20. \textit{ Ans.}$$

16. If a wheel turn 17° 30′ in 35 minutes, in how many hours does it make a complete revolution?

$$17^\circ\, 30' \div 35 = 30' = \tfrac{1}{2}^\circ \text{ in one min.}$$
$$360 \div \tfrac{1}{2} = 720 \text{ min.} = 12 \text{ hrs. } \textit{Ans.}$$

17. If a man's expenditures be \$4358 in $13\frac{1}{3}$ months, what is his yearly rate of expenditure?

$$13\tfrac{1}{3} \text{ m.} = \frac{13\frac{1}{3}}{12} \text{ yr.} = \tfrac{10}{9} \text{ yr.}$$
$$\$4358 \div \tfrac{10}{9} = \$3922.20. \textit{ Ans.}$$

18. If a cistern lose by leakage 7 gals. 1 pt. in 49 hrs. 40 min., what is its hourly rate of loss?

$$49 \text{ hrs. } 40 \text{ min.} = 49\tfrac{2}{3} \text{ hrs.}$$
$$7 \text{ gals. } 1 \text{ pt.} = 57 \text{ pts.}$$
$$57 \text{ pts.} \div 49\tfrac{2}{3} = 1\tfrac{22}{149} \text{ pts.}$$

19. If the circumference of the earth at the equator be 24,900 miles, at what rate per hour is a person there carried round, one whole rotation being made in 23 hrs. 56 min.?

$$23 \text{ hrs. } 56 \text{ min.} = 23\tfrac{14}{15} \text{ hrs.}$$
$$24{,}900 \text{ mi.} \div 23\tfrac{14}{15} = 1040\tfrac{140}{359} \text{ mi. } \textit{Ans.}$$

20. If a man travel $3\frac{3}{5}$ miles in $7\frac{1}{2}$ minutes, how many miles will he travel in 50 minutes? and how long will he take to travel 50 miles?

$$7\tfrac{1}{2} \text{ min.} \div 3\tfrac{3}{5} = 2\tfrac{1}{12} \text{ min.}$$
$$50 \text{ mi.} \div 2\tfrac{1}{12} = 24 \text{ mi. } \textit{Ans.}$$
$$3\tfrac{3}{5} \text{ mi.} \div 7\tfrac{1}{2} = \tfrac{12}{25} \text{ mi.}$$
$$50 \div \tfrac{12}{25} = 104\tfrac{1}{6} \text{ min.}$$
$$= 1 \text{ hr. } 44 \text{ min. } 10 \text{ sec. } \textit{Ans.}$$

21. If A can mow a certain meadow in 4 days, and B in 3 days, how long will it take both?

If A can mow it in 4 days, in one day he can mow $\frac{1}{4}$ of it.
If B can mow it in 3 days, in one day he can mow $\frac{1}{3}$ of it.
Both together can mow $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$ of it in one day.
$\therefore$ both together can mow the whole in $\frac{12}{7}$ days, or $1\frac{5}{7}$ days. *Ans.*

22. If A can lay a certain wall in $4\frac{1}{2}$ days, and B in $5\frac{1}{2}$ days, how long will it take both?

If A can do it in $4\frac{1}{2}$ days, in one day he can do $\dfrac{1}{4\frac{1}{2}} = \frac{2}{9}$ of it.

If B can do it in $5\frac{1}{2}$ days, in one day he can do $\dfrac{1}{5\frac{1}{2}} = \frac{2}{11}$ of it.

Both together can do $\frac{2}{9} + \frac{2}{11} = \frac{40}{99}$ of it in one day.
$\therefore$ both together can do the whole in $\frac{99}{40}$ days, or $2\frac{19}{40}$ days. *Ans.*

23. If a pipe will fill a vessel in $4\frac{1}{2}$ hours, and another in $3\frac{1}{2}$ hours, how long will it take both to fill the vessel?

If one pipe will fill it in $4\frac{1}{2}$ hrs., in one hr. it will fill $\dfrac{1}{4\frac{1}{2}} = \frac{2}{9}$ of it.

If another pipe will fill it in $3\frac{1}{2}$ hrs., in one hr. it will fill $\dfrac{1}{3\frac{1}{2}} = \frac{2}{7}$ of it.

Both pipes together will fill $\frac{2}{9} + \frac{2}{7} = \frac{32}{63}$ of it in one hour.
$\therefore$ both pipes together will fill it in $\frac{63}{32} = 1\frac{31}{32}$ hrs. $= 1$ hr. 58 min. $7\frac{1}{2}$ sec. *Ans.*

24. If A can go from Boston to Albany in $9\frac{1}{4}$ hours, and B from Albany to Boston in $11\frac{1}{3}$ hours, and they start at the same time, in how many hours will they meet?

If A can go in $9\frac{1}{4}$ hrs., in 1 hour he can go $\dfrac{1}{9\frac{1}{4}} = \frac{4}{37}$ of the distance.

If B can go in $11\frac{1}{3}$ hrs., in 1 hour he can go $\dfrac{1}{11\frac{1}{3}} = \frac{3}{34}$ of the distance.

Both together can go $\frac{4}{37} + \frac{3}{34} = \frac{247}{1258}$ of the distance in 1 hour.
$\therefore$ they will meet in $\frac{1258}{247}$ hrs. $= 5\frac{23}{247}$ hrs.

25. A requires 4 days, B 3 days, and C $4\frac{1}{2}$ days, to do a certain piece of work. How long will it take all three working together?

If A can do it in 4 days, in one day he can do $\frac{1}{4}$ of it.
If B can do it in 3 days, in one day he can do $\frac{1}{3}$ of it.

If C can do it in $4\frac{1}{2}$ days, in one day he can do $\dfrac{1}{4\frac{1}{2}} = \frac{2}{9}$ of it.

All together can do $\frac{1}{4} + \frac{1}{3} + \frac{2}{9} = \frac{29}{36}$ of it in one day.
$\therefore$ it will take them, all working together, $\frac{36}{29}$ days $= 1\frac{7}{29}$. *Ans.*

26. A can mow $\frac{5}{9}$ of a field in 3 days; B can mow $\frac{2}{3}$ of it in 4 days. How long will it take both to mow the field?

3 days $\div \frac{5}{9} = 5\frac{2}{5}$ days, and 4 days $\div \frac{2}{3} = 6$ days.

If A can mow it in $5\frac{2}{5}$ days, in one day he can mow $\dfrac{1}{5\frac{2}{5}} = \frac{5}{27}$ of it.

If B can mow it in 6 days, in one day he can mow $\frac{1}{6}$ of it.
Both together can mow $\frac{5}{27} + \frac{1}{6} = \frac{19}{54}$ of it in one day.
$\therefore$ both together can mow it in $\frac{54}{19}$ days $= 2\frac{16}{19}$ days. *Ans.*

27. One pipe can fill a cistern half full in $\frac{3}{4}$ of an hour, and another can fill it three-quarters full in $\frac{1}{2}$ an hour. How long will it take both pipes to fill the cistern?

$\frac{3}{4}$ hr. $\div \frac{1}{2} = 1\frac{1}{2}$ hrs., and $\frac{1}{2}$ hr. $\div \frac{3}{4} = \frac{2}{3}$ hr.

If one pipe fills it in $1\frac{1}{2}$ hrs., in one hour it will fill $\dfrac{1}{1\frac{1}{2}} = \frac{2}{3}$ of it.

If another pipe fills it in $\frac{2}{3}$ hr., in one hour it will fill $\dfrac{1}{\frac{2}{3}} = \frac{3}{2}$ of it.

Both together will fill $\frac{2}{3} + \frac{3}{2} = \frac{13}{6}$ of it in one hour.
$\therefore$ both together will fill it in $6 \div 13 = \frac{6}{13}$ hr. *Ans.*

28. A cistern which holds 100 gallons can be filled from a pipe in 25 minutes, and emptied by a waste-pipe in 45 minutes. If both are opened together, how long will it take to fill the cistern, and how much water will be wasted?

The water-pipe fills $\frac{1}{25}$ every minute.
The waste-pipe empties $\frac{1}{45}$ every minute.
When both are open, $\frac{1}{25} - \frac{1}{45} = \frac{4}{225}$ is gained every minute.
$\therefore$ the whole will be filled in $\frac{225}{4} = 56\frac{1}{4}$ min. *Ans.*
If $\frac{1}{45}$ of the cistern is wasted every minute, in $56\frac{1}{4}$ minutes $56\frac{1}{4} \times \frac{1}{45}$ would be wasted.
Now, as the cistern holds 100 gals., the number of gallons wasted would be $56\frac{1}{4} \times \frac{1}{45} \times 100$ gals.

$$56\tfrac{1}{4} \times \tfrac{1}{45} \times 100 = \frac{\overset{5}{\cancel{225}}}{\cancel{4}} \times \frac{1}{\cancel{45}} \times \frac{\overset{25}{\cancel{100}}}{1} = 125 \text{ gals. } \textit{Ans.}$$

29. A pipe can fill a cistern one-third full in $\frac{1}{4}$ of an hour; a waste-pipe can empty $\frac{1}{4}$ of the cistern in 20 minutes. If both pipes are opened, in what time will the cistern be filled?

$\frac{1}{4}$ hr. $\times 3 = \frac{3}{4}$ hr. $= 45$ min., and 20 min. $\times 4 = 80$ min.
The water-pipe fills $\frac{1}{45}$ every minute.
The waste-pipe empties $\frac{1}{80}$ every minute.
When both are open, $\frac{1}{45} - \frac{1}{80} = \frac{7}{720}$ is gained every minute.
$\therefore$ the whole will be filled in $\frac{720}{7} = 102\frac{6}{7}$ min.
$= 1$ hr. 42 min. $51\frac{3}{7}$ sec. *Ans.*

30. If one pipe runs into a cistern at the rate of 2 gallons in 3 minutes, and another at the rate of 5 gallons in 4 minutes, while the water is running out of a third pipe at the rate of 4 gallons in 5 minutes, how long will it take to gain 71 gallons in the cistern?

2 gals. $\div 3 = \frac{2}{3}$ gals., 5 gals. $\div 4 = \frac{5}{4}$ gals., and 4 gals. $\div 5 = \frac{4}{5}$ gals.
If one pipe pours in $\frac{2}{3}$ gals. per minute, another pours in $\frac{5}{4}$ gals. per minute, and another empties $\frac{4}{5}$ gals. per minute, the cistern gains $\frac{2}{3} + \frac{5}{4} - \frac{4}{5} = \frac{67}{60}$ gals. per minute.
$\therefore$ it will take as many minutes to gain 71 gals. as $71 \div \frac{67}{60} = 63\frac{39}{67}$ min. $= 1$ hr. 3 min. $34\frac{62}{67}$ sec.

31. A and B can do a piece of work in $2\frac{1}{2}$ days; A and C in $3\frac{1}{3}$ days; B and C in $4\frac{1}{4}$ days. Required the time in which all three, working together, can do the work, and in which each can do it alone.

If A and B can do it in $2\frac{1}{2}$ days, they can do $\dfrac{1}{2\frac{1}{2}} = \frac{2}{5}$ of it in one day.

If A and C can do it in $3\frac{1}{3}$ days, they can do $\dfrac{1}{3\frac{1}{3}} = \frac{3}{10}$ of it in one day.

If B and C can do it in $4\frac{1}{4}$ days, they can do $\dfrac{1}{4\frac{1}{4}} = \frac{4}{17}$ of it in one day.

All can do $\frac{2}{5} + \frac{3}{10} + \frac{4}{17} = \frac{159}{170}$ in 2 days, or $\frac{159}{340}$ of it in one day.

$\therefore$ all can do it in $\frac{340}{159} = 2\frac{22}{159}$ days. *Ans.*

If A, B, and C can do $\frac{159}{340}$, and B and C $\frac{4}{17}$ of it in one day, A can do $\frac{159}{340} - \frac{4}{17} = \frac{79}{340}$ of it in one day.

$\therefore$ A can do the whole in $\frac{340}{79} = 4\frac{24}{79}$ days. *Ans.*

If A, B, and C can do $\frac{159}{340}$, and A and C $\frac{3}{10}$ of it in one day, B can do $\frac{159}{340} - \frac{3}{10} = \frac{57}{340}$ of it in one day.

$\therefore$ B can do the whole in $\frac{340}{57} = 5\frac{55}{57}$ days. *Ans.*

If A, B, and C can do $\frac{159}{340}$, and A and B $\frac{2}{5}$ of it in one day, C can do $\frac{159}{340} - \frac{2}{5} = \frac{23}{340}$ of it in one day.

$\therefore$ C can do the whole in $\frac{340}{23} = 14\frac{18}{23}$ days. *Ans.*

32. Sampson & Reed sold $\frac{5}{8}$ of a lot of wheat to one party, $\frac{3}{4}$ of the remainder to another, and had 93 bushels left. How much had they at first?

After selling $\frac{5}{8}$ of the wheat they had $\frac{3}{8}$ left.

After selling $\frac{3}{4}$ of $\frac{3}{8}$ of the wheat they had $\frac{1}{4}$ of $\frac{3}{8} = \frac{3}{32}$ left.

Then 93 bush. $= \frac{3}{32}$ of the lot.

$\therefore$ the whole lot $=$ 93 bush. $\div \frac{3}{32} =$ 992 bush. *Ans.*

33. In a certain school $\frac{9}{16}$ of the scholars are girls, $\frac{4}{7}$ of the boys are over 16 years old, and 6 boys are under 16. How many girls, and how many scholars in all?

After the girls, or $\frac{9}{16}$ of the school, are taken out, there remain $\frac{7}{16}$ of the school, or the boys.

After $\frac{4}{7}$ of $\frac{7}{16}$ are taken out, $\frac{3}{7}$ of $\frac{7}{16} = \frac{3}{16}$ are left.

$\therefore$ the whole number of scholars is $6 \div \frac{3}{16} = 32$ scholars. *Ans.*

$\frac{9}{16}$ of 32 scholars $= 18$, number of girls. *Ans.*

34. In a certain school $\frac{13}{24}$ are boys; $\frac{9}{22}$ of the girls are under **16**, and 13 girls are over 16. How many boys and how many girls in the school?

After the boys, or $\frac{13}{24}$ of the school, are taken out, there remains $\frac{11}{24}$ of the school, or the girls.

After $\frac{9}{22}$ of $\frac{11}{24}$ are taken out, $\frac{13}{22}$ of $\frac{11}{24} = \frac{13}{48}$ are left.

$\therefore$ the whole number of scholars $= 13 \div \frac{13}{48} = 48$ scholars.

$\frac{11}{24}$ of $48 = 22$ girls. *Ans.* $\frac{13}{24}$ of $48 = 26$ boys. *Ans.*

35. If from a certain number $\frac{3}{4}$ of it be subtracted, then $\frac{1}{5}$ of the remainder, then $\frac{1}{7}$ of that remainder, and 6 still remain, what is the number?

After $\frac{3}{4}$ of it is subtracted, $\frac{1}{4}$ is left.
After $\frac{1}{5}$ of $\frac{1}{4}$ is subtracted, $\frac{4}{5}$ of $\frac{1}{4} = \frac{1}{5}$ is left.
After $\frac{1}{7}$ of $\frac{1}{5}$ is subtracted, $\frac{6}{7}$ of $\frac{1}{5} = \frac{6}{35}$ is left.
$\therefore$ the number $= 6 \div \frac{6}{35} = 35$. *Ans.*

36. 20 is $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{2}{3}$ of what number?

$$\frac{\cancel{5}}{\underset{2}{\cancel{\underset{\cancel{4}}{\cancel{8}}}}} \text{ of } \frac{\cancel{2}}{\cancel{5}} \text{ of } \frac{\cancel{2}}{3} = \frac{1}{6}. \qquad \therefore \text{the number} = 20 \div \tfrac{1}{6} = 120. \text{ *Ans.*}$$

37. 6 is $\frac{6}{7}$ of $\frac{4}{5}$ of $\frac{1}{4}$ of what number?

$$\frac{6}{7} \text{ of } \frac{\cancel{4}}{5} \text{ of } \frac{1}{\cancel{4}} = \frac{6}{35}. \qquad \therefore \text{the number} = 6 \div \tfrac{6}{35} = 35. \text{ *Ans.*}$$

38. Express $\frac{1}{29}$ of 1 lb. troy + $\frac{1}{29}$ of 1 lb. avoirdupois as troy and as avoirdupois weights.

$$\frac{1}{29} \text{ lb. av.} = \frac{1}{29} \text{ of } \frac{\overset{175}{\cancel{7000}}}{\underset{144}{\cancel{5760}}} = \frac{175}{4176} \text{ lbs. troy.}$$

$$\frac{1}{29} + \frac{175}{4176} = \frac{144 + 175}{4176} = \frac{319}{4176} = \frac{11}{144} \text{ lbs. troy.}$$

$\frac{11}{144}$ lbs. $= \frac{11}{144}$ of 12 oz. $= \frac{11}{12}$ oz. $= \frac{11}{12}$ of 20 dwt. $= 18\frac{1}{3}$ dwt.
$= 18$ dwt. 8 grs. *Ans.*

$$\frac{11}{144} \text{ lbs. troy} = \frac{11}{\cancel{144}} \text{ of } \frac{\overset{\cancel{144}}{\cancel{5760}}}{\underset{175}{\cancel{7000}}} = \frac{11}{175} \text{ lbs. av.}$$

$\frac{11}{175}$ lbs. $= \frac{11}{175}$ of 16 oz. $= 1\frac{1}{175}$ oz. av. *Ans.*

39. The cargo of a ship, worth \$45,000, belongs to three partners. A owns $\frac{7}{9}$ of $\frac{3}{5}$ of it, B's share is equal to $3\frac{3}{14}$ of $\frac{2}{9}$ of A's share, and C owns the remainder. What ought each to receive from the sale?

$\frac{7}{9}$ of $\frac{3}{5} = \frac{7}{15}$ of ship. $\frac{7}{15}$ of \$45,000 = \$21,000, A's share.
$3\frac{3}{14}$ of $\frac{2}{9}$ of $\frac{7}{15} = \frac{1}{3}$ of ship. $\frac{1}{3}$ of \$45,000 = \$15,000, B's share.
\$45,000 − (\$21,000 + \$15,000) = \$9000, C's share.

40. Find the largest number which is contained an integral number of times in each of the following: $2\frac{5}{9}$, $6\frac{7}{18}$, $11\frac{1}{2}$, $19\frac{1}{6}$.

$2\frac{5}{9}, 6\frac{7}{18}, 11\frac{1}{2}, 19\frac{1}{6} = \frac{23}{9}, \frac{115}{18}, \frac{23}{2}, \frac{115}{6}$.
G. C. M. of 23, 115, 23, 115 = 23.
L. C. M. of 9, 18, 2, 6 = 18.
∴ G. C. M. of fractions $= \frac{23}{18} = 1\frac{5}{18}$. *Ans.*

41. A person bequeathed $\frac{5}{12}$ of his property to A, $\frac{1}{4}$ of it to B, $\frac{1}{8}$ to C, $\frac{1}{6}$ to D, and the remainder, \$550, to E. What was the value of the whole property?

$\frac{5}{12} + \frac{1}{4} + \frac{1}{8} + \frac{1}{6} = \frac{23}{24}$.
After $\frac{23}{24}$ is subtracted, $\frac{1}{24}$ remains.
∴ whole money = \$550 ÷ $\frac{1}{24}$ = \$13,200. *Ans.*

42. Arrange in descending order of magnitude, $\frac{13}{25}$, $\frac{15}{26}$, $\frac{16}{27}$.

$$\frac{13}{25}, \frac{15}{26}, \frac{16}{27} = \frac{9126,\ 10125,\ 10400}{17550}.$$
∴ the order of magnitude is $\frac{16}{27}$, $\frac{15}{26}$, $\frac{13}{25}$. *Ans.*

43. A bankrupt's debts are \$2520, and the value of his property is \$1890. How much can he pay on a dollar?

$$\frac{\$1890}{2520} = \frac{\$3}{4} = \$0.75. \textit{ Ans.}$$

44. A bankrupt's debts are \$4264, and he pays $62\frac{1}{2}$ cents on a dollar; what are his assets?

$$0.62\tfrac{1}{2} = \frac{5}{8}. \qquad \frac{5}{\cancel{8}} \text{ of } \frac{\overset{533}{\cancel{\$4264}}}{1} = \$2665. \textit{ Ans.}$$

45. If 15 yards of silk cost \$18.75, how much will $20\frac{1}{3}$ yards cost?

If 15 yds. cost $\$18\frac{3}{4}$, 1 yd. costs $\frac{\$18\frac{3}{4}}{15}$,

and $20\frac{1}{3}$ yds. cost $20\frac{1}{3} \times \frac{\$18\frac{3}{4}}{15}$.

$$20\tfrac{1}{3} \times \frac{\$18\frac{3}{4}}{15} = \frac{61}{3} \times \frac{1}{\cancel{15}} \times \frac{\$\overset{5}{\cancel{75}}}{4} = \frac{\$305}{12} = \$25.42. \textit{ Ans.}$$

46. If $3\frac{2}{5}$ pounds of tea cost \$3.80, how much can I buy for \$21.87?

If $3\frac{2}{5}$ lbs. cost \$3.80, 1 lb. costs $\frac{\$3.80}{3\frac{2}{5}}$, and as much can be

bought for \$21.87 as $2187 \div \frac{380}{3\frac{2}{5}}$.

$$2187 \div \frac{380}{3\frac{2}{5}} = \frac{17}{5} \times \frac{1}{380} \times \frac{2187}{1} = \frac{37179}{1900} = 19\tfrac{1079}{1900} \text{ lbs. } \textit{Ans.}$$

47. If $\frac{3}{14}$ of a ton of coal cost \$1.12, what is the price of $5\frac{1}{2}$ cwt.?

$5\frac{1}{2}$ cwt. $= \frac{5\frac{1}{2}}{20}$ t. $= \frac{11}{40}$ t.

If $\frac{3}{14}$ t. cost \$1.12, 1 t. costs $\frac{\$1.12}{\frac{3}{14}}$, and $\frac{11}{40}$ t. costs $\frac{11}{40} \times \frac{\$1.12}{\frac{3}{14}}$.

$$\frac{11}{\underset{5}{\cancel{40}}} \times \frac{\overset{14}{\cancel{\$1.12}}}{1} \times \frac{14}{3} = \frac{\$21.56}{15} = \$1.44. \textit{ Ans.}$$

48. If $\frac{2}{11}$ of a piece of work be done in 25 days, how much will be done in $11\frac{2}{3}$ days?

If $\frac{2}{11}$ can be done in 25 days, the whole can be done in $\frac{25}{\frac{2}{11}}$ days,

and as much can be done in $11\frac{2}{3}$ days as $11\frac{2}{3} \div \frac{25}{\frac{2}{11}}$.

$$11\tfrac{2}{3} \div \frac{25}{\frac{2}{11}} = \frac{2}{11} \times \frac{1}{\underset{5}{\cancel{25}}} \times \frac{\overset{7}{\cancel{35}}}{3} = \frac{14}{165}. \textit{ Ans.}$$

49. A man walks 18 mi. 106 rds. $3\frac{2}{3}$ yards in $5\frac{1}{2}$ hours. How long does he take to walk a mile and a half?

18 mi. 106 rds. $3\frac{2}{3}$ yds.:

$3\frac{2}{3}$ yds. $= \dfrac{3\frac{2}{3}}{5\frac{1}{2}}$ rd. $= \frac{2}{3}$ rd.

$106\frac{2}{3}$ rds. $= \dfrac{106\frac{2}{3}}{320} = \frac{1}{3}$ mi.

18 mi. 106 rds. $3\frac{2}{3}$ yds. $= 18\frac{1}{3}$ mi.

If he walks $18\frac{1}{3}$ mi. in $5\frac{1}{2}$ hrs., he will walk $\dfrac{18\frac{1}{3}}{5\frac{1}{2}}$ mi. in 1 hr.,

and it will take him as long to walk $1\frac{1}{2}$ miles as $1\frac{1}{2} \div \dfrac{18\frac{1}{3}}{5\frac{1}{2}}$.

$$1\frac{1}{2} \div \frac{18\frac{1}{3}}{5\frac{1}{2}} = \frac{\cancel{11}}{2} \times \frac{3}{\underset{5}{\cancel{55}}} \times \frac{3}{2} = \frac{9}{20} \text{ hr.} = 27 \text{ min. } \textit{Ans.}$$

50. When an ounce of gold is worth \$19.45, what is the value of 0.04 of a pound?

$\$19.45 \times 12 \times 0.04 = \$9.336$. *Ans.*

51. If 9 horses can plow 46 acres in a certain time, how many acres can 12 horses plow in the same time?

Since 9 horses can plow 46 acres in a certain time,
1 horse can plow $\frac{1}{9}$ of 46 acres in the same time,
and 12 horses can plow $12 \times \frac{1}{9}$ of $46 = 61\frac{1}{3}$ acres. *Ans.*

52. If 12 men can reap a field in 4 days, in what time will 32 men reap it?

Since 12 men can reap a field in 4 days,
1 man can reap it in 12×4 days,
and 32 men can reap it in $\dfrac{12 \times 4}{32}$ days $= 1\frac{1}{2}$ days. *Ans.*

53. If 72 men dig a trench in 63 days, in how many days will 42 men dig another trench three times as great?

Since 72 men can dig a trench in 63 days,
1 man can dig it in 72×63 days.
1 man can dig one 3 times as large in $3 \times 72 \times 63$ days,
and 42 men can dig it in $\dfrac{3 \times 72 \times 63}{42}$ days $= 324$ days. *Ans.*

54. If a man travels 540 miles in 24 days, walking 6 hours a day, how many miles can he travel in 3 days, walking 8 hours a day?

Since he can go 540 mi. in 24×6 hrs. $= 144$ hrs.,
in one hour he can go $\frac{1}{144}$ of 540 mi.,
and in 3×8 hrs. $= 24$ hrs. he can go $24 \times \frac{1}{144}$ of 540 mi.
$= 90$ mi. *Ans.*

55. If 15 men can perform a piece of work in 22 days, how many men will finish another piece of work four times as large in $\frac{1}{5}$ of the time?

Since 15 men can do the work in 22 days,
it will take 4×15 men to do 4 times the work in 22 days,
and to do 4 times the work in $\frac{1}{5}$ of 22 days it will take
$5 \times 4 \times 15$ men $= 300$ men. *Ans.*

56. A garrison of 2100 has provisions for 9 months, but receives reinforcements of 600 men. How long will the provisions last?

Since the provisions will last 2100 men 9 months,
they will last 1 man 2100×9 months,
and they will last $2100 + 600 = 2700$ men

$$\frac{2100 \times 9}{2700} \text{ mos.} = 7 \text{ mos. } \textit{Ans.}$$

57. If a cubic foot of ice weigh $57\frac{3}{8}$ pounds, how many cubic feet of ice will weigh a ton?

Since 1 cubic foot of ice weighs $57\frac{3}{8}$ lbs.; to weigh a ton

it will take $\frac{2000}{57\frac{3}{8}}$ cu. ft. $= 34\frac{394}{459}$ cu. ft. *Ans.*

58. How many bushels of wheat will serve 72 people 8 days when 4 bushels serve 6 people 24 days?

72 people will eat twelve times as much as 6 people in the same time.

And the same number of people will eat $\frac{1}{3}$ as much in 8 days as in 24 days.

Hence, 72 people in 8 days will eat $12 \times \frac{1}{3}$ times as much as 6 people in 24 days.

$12 \times \frac{1}{3} \times 4$ bushels $= 16$ bushels. *Ans.*

59. If 2 horses eat 8 bushels of oats in 16 days, how many horses will eat 3000 bushels in 24 days?

In 16 days 8 bu. can be eaten by 2 horses.
In 1 day 8 bu. can be eaten by 16×2 horses.
In 1 day 1 bu. can be eaten by $\frac{16 \times 2}{8}$ horses.
In 24 days 1 bu. can be eaten by $\frac{16 \times 2}{24 \times 8}$ horses.
In 24 days 3000 bu. can be eaten by $\frac{3000 \times 16 \times 2}{24 \times 8}$ horses,
$= 500$ horses. *Ans.*

60. If a man travel 150 miles in 5 days, when the days are 12 hours long, in how many days of 10 hours each will he travel 500 miles?

He can go 150 miles in 5 days of 12 hours $= 60$ hours.
He can go 1 mile in $\frac{60}{150}$ hour.
He can go 500 miles in $\frac{500 \times 60}{150}$ hours.
He can go 500 miles in $\frac{500 \times 60}{150 \times 10}$ days of 10 hours,
$= 20$ days. *Ans.*

61. If a regiment of 939 soldiers consume 351 bushels of wheat in 21 days, how many soldiers will consume 1404 bushels in 7 days?

1404 bu. will last the same number of men four times as long as 351 bu.
And the same amount will last three times the number of men for 7 days as for 21 days.
$3 \times 4 \times 939$ soldiers $= 11{,}268$ soldiers. *Ans.*

62. If 5 men can reap a field of $12\frac{1}{2}$ acres in $3\frac{1}{2}$ days, working 16 hours a day, in what time can 7 men reap a field of 15 acres, working 12 hours a day?

5 men can reap $12\frac{1}{2}$ acres in $3\frac{1}{2}$ days of 16 hours $= 56$ hours.
1 man can reap $12\frac{1}{2}$ acres in 5×56 hours.
1 man can reap 1 acre in $\frac{5 \times 56}{12\frac{1}{2}}$ hours.

1 man can reap 15 acres in $\frac{15 \times 5 \times 56}{12\frac{1}{2}}$ hours.

7 men can reap 15 acres in $\frac{15 \times 5 \times 56}{7 \times 12\frac{1}{2}}$ hours.

7 men can reap 15 acres in $\frac{15 \times 5 \times 56}{12 \times 7 \times 12\frac{1}{2}}$ days of 12 hours,

$= 4$ days. *Ans.*

63. If 7 men mow 22 acres in 8 days, working 11 hours a day, in how many days, working 10 hours a day, will 11 men mow 360 acres?

7 men can mow 22 acres in 8 days of 11 hours $= 88$ hours,

1 man can mow 22 acres in 7×88 hours.

1 man can mow 1 acre in $\frac{7 \times 88}{22}$ hours.

12 men can mow 1 acre in $\frac{7 \times 88}{12 \times 22}$ hours.

12 men can mow 360 acres in $\frac{360 \times 7 \times 88}{12 \times 22}$ hours.

12 men can mow 360 acres in $\frac{360 \times 7 \times 88}{10 \times 12 \times 22}$ days of 10 hours,

$= 84$ days. *Ans.*

64. If 44 cannon, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day?

44 cannon firing 30 rounds for 3 hours consume
300 bbls. in 5 days.

44 cannon firing 30 rounds for 1 hour consume
300 bbls. in 3×5 days.

44 cannon firing 1 round for 1 hour consume
300 bbls. in $30 \times 3 \times 5$ days.

1 cannon firing 1 round for 1 hour consumes
300 bbls. in $44 \times 30 \times 3 \times 5$ days.

1 cannon firing 1 round for 1 hour consumes
1 bbl. in $\frac{44 \times 30 \times 3 \times 5}{300}$ days.

66 cannon firing 1 round for 1 hour consume

$$1 \text{ bbl. in } \frac{44 \times 30 \times 3 \times 5}{300 \times 66} \text{ days.}$$

66 cannon firing 40 rounds for 1 hour consume

$$1 \text{ bbl. in } \frac{44 \times 30 \times 3 \times 5}{40 \times 300 \times 66} \text{ days.}$$

66 cannon firing 40 rounds for 5 hours consume

$$1 \text{ bbl. in } \frac{44 \times 30 \times 3 \times 5}{5 \times 40 \times 300 \times 66} \text{ days.}$$

66 cannon firing 40 rounds for 5 hours consume

$$400 \text{ bbls. in } \frac{400 \times 44 \times 30 \times 3 \times 5}{5 \times 40 \times 300 \times 66} \text{ days} = 2 \text{ days. } \textit{Ans.}$$

65. How many times will a wheel $2\frac{4}{7}$ feet in circumference turn round in travelling over $12\frac{6}{7}$ yards?

The wheel revolves once in going $2\frac{4}{7}$ feet.

$\therefore$ it will turn around as many times in going $12\frac{6}{7}$ yds. $= 38\frac{4}{7}$ ft. as $38\frac{4}{7} \div 2\frac{4}{7} = 15$. *Ans.*

66. How much ground will be travelled over by a wheel $1\frac{3}{7}$ yards in circumference, when it has made $4\frac{1}{2}$ turns?

If the wheel turns once in going $1\frac{3}{7}$ yds., in making $4\frac{1}{2}$ turns it will go $4\frac{1}{2}$ times $1\frac{3}{7}$ yds. $= 6\frac{3}{7}$ yds. *Ans.*

67. Find the circumference of a wheel which makes 9 turns in travelling over $7\frac{1}{5}$ yards.

If it makes 9 turns in going $7\frac{1}{5}$ yds., it will go $7\frac{1}{5}$ yds. $\div 9 = \frac{4}{5}$ yd. $= 2\frac{2}{5}$ ft. in making one turn.

$\therefore$ the circumference of the wheel is $2\frac{2}{5}$ feet. *Ans.*

68. If the circumference of a wheel be $\frac{22}{7}$ of 1 yd. $1\frac{1}{8}$ ft., how many times will it turn in travelling $3\frac{3}{7}$ miles?

1 yd. $1\frac{1}{8}$ ft. $= 1\frac{1\frac{1}{8}}{3}$ yds. $= 1\frac{3}{8}$ yds. $\quad 3\frac{3}{7}$ mi. $= 3\frac{3}{7} \times 1760$ yds.

If the wheel makes 1 turn in going $\frac{22}{7}$ of $1\frac{3}{8}$ yds., it will make as many turns in going $3\frac{3}{7} \times 1760$ yds. as $\dfrac{3\frac{3}{7} \times 1760}{\frac{22}{7} \times \frac{11}{8}} = 1396\frac{4}{11}$. *Ans.*

69. If the wheel of a locomotive be $3\frac{1}{7}$ times 5.52 feet in circumference, how many times does it turn in a minute, when the locomotive is running at the rate of 13.34 miles an hour?

$5.52 = 5\frac{13}{25}$, and $13.34 = 13\frac{17}{50}$.

If it is going at the rate of $13\frac{17}{50}$ miles per hour, it is going at the rate of $\frac{13\frac{17}{50}}{60}$ miles per minute, or at the rate of $\frac{13\frac{17}{50} \times 5280}{60}$ feet per minute.

If it turns once in going $3\frac{1}{7} \times 5\frac{13}{25}$ feet, it will turn as many times in going $\frac{13\frac{17}{50} \times 5280}{60}$ feet as $\frac{13\frac{17}{50} \times 5280}{60} \div (3\frac{1}{7} \times 5\frac{13}{25}) = 67\frac{2}{3}$.

Ans.

70. A can run $\frac{4}{55}$ of a mile in $\frac{3}{5}$ of a minute, B can run $\frac{5}{56}$ of a mile in $\frac{2}{3}$ of a minute, and C $\frac{2}{19}$ of a mile in $\frac{5}{7}$ of a minute. Which is the fastest runner? and if he can run a certain distance in 3 min. 10 sec., how much longer will each of the others take to run the same distance?

If A can run $\frac{4}{55}$ mi. in $\frac{3}{5}$ min., to run a mile it will take him $\frac{3}{5}$ min. $\div \frac{4}{55} = 8\frac{1}{4}$ min.

If B can run $\frac{5}{56}$ mi. in $\frac{2}{3}$ min., to run a mile it will take him $\frac{2}{3}$ min. $\div \frac{5}{56} = 7\frac{7}{15}$ min.

If C can run $\frac{2}{19}$ mi. in $\frac{5}{7}$ min., to run a mile it will take him $\frac{5}{7}$ min. $\div \frac{2}{19}$ min. $= 6\frac{11}{14}$ min.

$\therefore$ C is the fastest runner.

3 min. 10 sec. $= 3\frac{1}{6}$ min.

If C can run a certain distance in $3\frac{1}{6}$ min., and a mile in $6\frac{11}{14}$ min., the distance is that part of a mile which $3\frac{1}{6}$ is of $6\frac{11}{14} = \frac{7}{15}$.

If A can run a mile in $8\frac{1}{4}$ min., he can run $\frac{7}{15}$ mi. in $\frac{7}{15}$ of $8\frac{1}{4}$ min. $= 3\frac{17}{20}$ min. $=$ 3 min. 51 sec.

But C can run it in 3 min 10 sec. $\therefore$ it takes A 41 sec. longer than C.

If B can run a mile in $7\frac{7}{15}$ min., he can run $\frac{7}{15}$ mi. in $\frac{7}{15}$ of $7\frac{7}{15}$ min. $= 3\frac{109}{225} =$ 3 min. $29\frac{1}{15}$ sec.

But C can run it in 3 min. 10 sec. $\therefore$ it takes B $19\frac{1}{15}$ sec. longer than C.

Find the amount of the following bills:

71.

Mr. Richard Rowe, *Boston, Nov. 23, 1880.*

To John Doe, Dr.

To 125 *lbs. sugar*	@ 10 *cts.*	$12	50
" 1 *bag coffee,* 115 *lbs.*	@ 32 *cts.*	36	80
" 25 *gals. molasses*	@ 62 *cts.*	15	50
" 8 *lbs. Japan tea*	@ 92 *cts.*	7	36
" 28 *lbs. crackers*	@ 8 *cts.*	2	24
" 2 *bbls. flour*	@ $7.50	15	00
		$89	40

Received Payment,

John Doe.

72.

Mr. James Hardy, *Boston, Feb. 29, 1888.*

To C. H. Mills, Dr.

To 275 *bbls. flour*	@ $6.75	$1856	25
" 324 *bbls. flour*	@ $6.25	2025	00
" 300 *bu. potatoes*	@ 48 *cts.*	144	00
" 1578 *lbs. butter*	@ 32 *cts.*	504	96
" 2000 *bbls. apples*	@ $1.25	2500	00
" 1 *car-load oats,* 20,000 *lbs.,* 625 *bu.,*	@ 42 *cts.*	262	50
" 1 *car-load corn,* 28,575 *lbs.,* 510.27 *bu.,*	@ 55 *cts.*	280	65
		$7573	36

Received Payment,

C. H. Mills.

73.

James Harlow, *Boston, Jan. 1, 1888.*

To John Pike, Dr.

To 12 *bales Texas cotton,* 5760 *lbs.*	@ $9\frac{1}{4}$ *cts.* . .	$532	80
" 7 *bales upland cotton,* 3514 *lbs.*	@ $10\frac{1}{4}$ *cts.* . .	360	19
" 3 *bales low middling,* 1476 *lbs.*	@ $9\frac{3}{4}$ *cts.* . .	143	91
" 8 *bales good ordinary,* 9220 *lbs.*	@ 9 *cts.* . .	793	80
		$1830	70

Received Payment,

John Pike.

Exercise LIX.

1. What length of board 15 in. wide will contain 11 sq. ft. 36 sq. in.?

$$11 \text{ sq. ft. } 36 \text{ sq. in.} = 11\tfrac{1}{4} \text{ sq. ft.}$$
$$15 \text{ in.} = 1\tfrac{1}{4} \text{ ft.}$$
$$11\tfrac{1}{4} \div 1\tfrac{1}{4} = 9 \text{ ft. } \textit{Ans.}$$

2. What length of road 44 ft. wide will contain an acre?

$$1 \text{ A.} = 160 \text{ sq. rds.}$$
$$44 \text{ ft.} = 2\tfrac{3}{4} \text{ rds.}$$
$$160 \div 2\tfrac{2}{3} = 60 \text{ rds. } \textit{Ans.}$$

3. Find the area of a rectangular field 13.12 chains long, 10.35 chains broad.

$$\begin{array}{r} 13.12 \text{ ch.} \\ \times\, 10{,}35 \text{ ch.} \\ \hline 10)\underline{135.792} \text{ sq. ch.} \\ 13 \text{ A. } 5.792 \text{ sq. ch. } \textit{Ans.} \end{array}$$

4. A path 216 ft. long measured 72 sq. yds. Find its breadth.

$$72 \text{ sq. yds.} = 648 \text{ sq. ft.}$$
$$648 \div 216 = 3 \text{ ft. } \textit{Ans.}$$

5. A rectangular field of 21.66 acres is 250.8 yds. broad. Find its length.

$$1 \text{ A.} = 4840 \text{ sq. yds.}$$
$$\frac{4840 \times 21.66}{250.8} = 418 \text{ yds. } \textit{Ans.}$$

6. What is the area of a table if length and breadth be 4 ft. $3\frac{3}{7}$ in. and 2 ft. $9\frac{3}{5}$ in., respectively?

$$4 \text{ ft. } 3\tfrac{3}{7} \text{ in.} = 4\tfrac{2}{7} \text{ ft.}$$
$$2 \text{ ft. } 9\tfrac{3}{5} \text{ in.} = 2\tfrac{4}{5} \text{ ft.}$$
$$4\tfrac{2}{7} \times 2\tfrac{4}{5} = 12 \text{ sq. ft. } \textit{Ans.}$$

7. From each corner of a square, the side of which is 2 ft. 5 in., a square measuring 5 in. on a side is cut out. Find the area of the remainder of the figure.

$$2 \text{ ft. } 5 \text{ in.} = 2\tfrac{5}{12} \text{ ft.}$$
$$2\tfrac{5}{12} \times 2\tfrac{5}{12} = 5\tfrac{121}{144} \text{ sq. ft.}$$
$$= 5 \text{ sq. ft. } 121 \text{ sq. in.}$$
$$5 \times 5 = 25 \text{ sq. in.}$$
$$25 \text{ sq. in.} \times 4 = 100 \text{ sq. in.}$$
$$5 \text{ sq. ft. } 121 \text{ sq. in.} - 100 \text{ sq. in.}$$
$$= 5 \text{ sq. ft. } 21 \text{ sq. in. } \textit{Ans.}$$

8. The length and breadth of a map are $4\frac{1}{2}$ ft. and $3\frac{1}{3}$ ft., respectively. If the map represent 77,760 sq. mi. of country, how many miles are there to a square inch?

$$4\tfrac{1}{2} \times 3\tfrac{1}{3} = 15 \text{ sq. ft.}$$
$$= 2160 \text{ sq. in.}$$
$$77{,}760 \div 2160 = 36 \text{ sq. mi. } \textit{Ans.}$$

9. In rolling a grass plot 24 yds. long and containing 400 sq. yds., how many times must a roller 3 ft. 4 in. wide be drawn

over it lengthwise so that the whole may be rolled?

$400 \div 24 = 16\frac{2}{3}$ yds.

3 ft. 4 in. $= 3\frac{1}{3}$ ft. $= 1\frac{1}{9}$ sq. yds.

$16\frac{2}{3} \div 1\frac{1}{9} = 15$. *Ans.*

10. How many sods, each 2 ft. $3\frac{1}{2}$ in. long and $8\frac{1}{4}$ in. broad, would be required to turf an acre of ground?

2 ft. $3\frac{1}{2}$ in. $= 27\frac{1}{2}$ in.

1 A. $= 6,272,640$ sq. in.

$$\frac{6272640}{27\frac{1}{2} \times 8\frac{1}{4}} = 27,648. \textit{ Ans.}$$

11. Find the area of a picture-frame $2\frac{1}{4}$ in. broad and having an outside measurement of 4 ft. $6\frac{1}{2}$ in. in length and 2 ft. 8 in. in width.

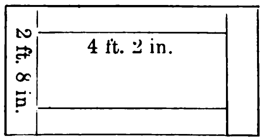
$2\frac{1}{4}$ in. $\times 2 = 4\frac{1}{2}$ in.

4 ft. $6\frac{1}{2}$ in. $- 4\frac{1}{2}$ in. $= 4$ ft. 2 in.

4 ft. 2 in. + 2 ft. 8 in. = 6 ft. 10 in.

6 ft. 10 in. $\times 2 = 13$ ft. 8 in. $= 13\frac{2}{3}$ ft.

$2\frac{1}{4}$ in. $= \frac{3}{16}$ ft.



$$13\frac{2}{3} \times \frac{3}{16} = \frac{41}{\cancel{3}} \times \frac{\cancel{3}}{16} = \frac{41}{16} = 2\frac{9}{16} \text{ sq. ft.}$$

$2\frac{9}{16}$ sq. ft. = 2 sq. ft. 81 sq. in. *Ans.*

12. Find the expense of glazing four windows, each containing 12 panes, the panes being each a foot long and 10 in. wide, and the price of the glass 38 cents per square foot.

10 in. $= \frac{5}{6}$ ft. $\qquad 1 \times \frac{5}{6} = \frac{5}{6}$ sq. ft.

$$\overset{2}{\cancel{12}} \times \frac{5}{\cancel{6}} = 10 \text{ sq. ft.} \qquad 10 \text{ sq. ft.} \times 4 = 40 \text{ sq. ft.}$$

$40 \times \$0.38 = \$15.20$. *Ans.*

13. A garden 76 yds. long and 56 yds. broad, enclosed by a wall, has a border 4 ft. wide within the wall, and within this a path 5 ft. wide, the middle being grass. Find the areas of the border, path, and grass, respectively.

4 ft. $= 1\frac{1}{3}$ yds. $\quad 2 \times 1\frac{1}{3}$ yds. $= 2\frac{2}{3}$ yds.

76 yds. $- 2\frac{2}{3}$ yds. $= 73\frac{1}{3}$ yds.

$73\frac{1}{3}$ yds. + 56 yds. $= 129\frac{1}{3}$ yds.

$129\frac{1}{3}$ yds. $\times 2 = 258\frac{2}{3}$ yds. = perimeter.

$258\frac{2}{3} \times 1\frac{1}{3} = \frac{776}{3} \times \frac{4}{3} = \frac{3104}{9} = 344\frac{8}{9}$ sq. yds. (1) *Ans.*

[illegible] = [illegible] yds.

[illegible] = [illegible] yds.

[illegible] = [illegible] yds.

[illegible] = [illegible] yds.

[illegible] = [illegible] × [illegible]

= [illegible]

= 411⅓ sq. yds.

(2) *Ans.*

[illegible] sq. yds. [illegible] *Ans.*

17. [illegible] of a cistern [illegible] the area of the [illegible]?

[illegible] = [illegible] ft. Radius.

[illegible] = [illegible] sq. ft.

3.1416

× [illegible] sq. ft.

[illegible] sq. ft. *Ans.*

18. [illegible] of the clock [illegible] London, are each [illegible] What is [illegible] in square feet?

[illegible] = [illegible] ft.

[illegible] = [illegible] sq. ft.

3.1416

× [illegible] sq. ft.

[illegible] sq. ft. *Ans.*

19. [illegible] square inches [illegible] of a ball 3 in. in [illegible]?

3 × 3 = 9 sq. in.

3.1416

× 9 sq. in.

28.2744 sq. in. *Ans.*

20. How many square inches of surface in a spherical blackboard 12 in. in diameter?

$$12 \times 12 = 144 \text{ sq. in.}$$

$$\begin{array}{r} 3.1416 \\ \times\ 144 \text{ sq. in.} \\ \hline 452.39\cancel{0}\cancel{4} \text{ sq. in.} \end{array}$$

21. What is the interior surface of a hemispherical vase 20 in. in diameter?

$$20 \times 20 = 400 \text{ sq. in.}$$

$$400 \text{ sq. in.} \div 2 = 200 \text{ sq. in.}$$

$$\begin{array}{r} 3.1416 \\ \times\ 200 \text{ sq. in.} \\ \hline 628.32 \text{ sq. in.} \end{array}$$

$$= 4 \text{ sq. ft. } 52.32 \text{ sq. in.}$$

22. How many yards of carpeting $\frac{3}{4}$ of a yard wide will be required for a floor 26 ft. long, $15\frac{3}{4}$ ft. wide, if the strips run lengthwise? How many if the strips run across the room? How much will be turned under in each case?

$$15\tfrac{3}{4} \text{ ft.} = 5\tfrac{1}{4} \text{ yds.}$$

$$5\tfrac{1}{4} \div \tfrac{3}{4} = 7 \text{ strips.}$$

$$26 \text{ ft.} = 8\tfrac{2}{3} \text{ yds.}$$

$$7 \times 8\tfrac{2}{3} \text{ yds.} = 60\tfrac{2}{3} \text{ yds. } \textit{Ans.}$$

None to turn under.

$$8\tfrac{2}{3} \div \tfrac{3}{4} = 11\tfrac{5}{9} = 12 \text{ strips.}$$

$$12 \times 5\tfrac{1}{4} \text{ yds.} = 63 \text{ yds. } \textit{Ans.}$$

$$1 - \tfrac{5}{9} = \tfrac{4}{9}.$$

$$\frac{\cancel{4}}{\underset{3}{\cancel{9}}} \times \frac{\cancel{3}}{\cancel{4}} = \frac{1}{3} \text{ yd. to turn under.}$$

23. How many yards $\frac{7}{8}$ of a yard wide will be required for a room $8\frac{1}{2}$ yds. long and 17 ft. wide, if the strips run lengthwise, and there is a waste of $\frac{1}{16}$ of a yard in each strip, in matching patterns?

$$17 \text{ ft.} = 5\tfrac{2}{3} \text{ yds.}$$

$$5\tfrac{2}{3} \div \tfrac{7}{8} = 6\tfrac{10}{21} = 7 \text{ strips.}$$

$$7 \times \tfrac{1}{16} \text{ yd.} = \tfrac{7}{16} \text{ yd.}$$

$$7 \times 8\tfrac{1}{2} \text{ yds.} = 59\tfrac{1}{2} \text{ yds.}$$

$$59\tfrac{1}{2} \text{ yds.} + \tfrac{7}{16} \text{ yds.}$$

$$= 59\tfrac{15}{16} \text{ yds. } \textit{Ans.}$$

24. How many square yards of oil-cloth will be required for a hall floor $5\frac{1}{4}$ yds. long and 10 ft. wide?

$$10 \text{ ft.} = 3\tfrac{1}{3} \text{ yds.}$$

$$5\tfrac{1}{4} \times 3\tfrac{1}{3} = 17\tfrac{1}{2} \text{ sq. yds. } \textit{Ans.}$$

25. What will be the cost of carpet $\frac{5}{4}$ of a yard wide for a room $28\frac{1}{2}$ ft. by $18\frac{3}{4}$ ft., if the strips run lengthwise, and the cost per yard is 92 cents?

$$18\tfrac{3}{4} \text{ ft.} = 6\tfrac{1}{4} \text{ yds.}$$

$$6\tfrac{1}{4} \div \tfrac{5}{4} = 5 \text{ strips.}$$

$$28\tfrac{1}{2} \text{ ft.} = 9\tfrac{1}{2} \text{ yds.}$$

$$9\tfrac{1}{2} \times 5 = 47\tfrac{1}{2} \text{ yds.}$$

$$47\tfrac{1}{2} \times \$0.92 = \$43.70. \ \textit{Ans.}$$

26. Find the cost of carpet 30 in. wide, at $1.25 per yd. for a room 18 ft. by 14 ft., if the strips run lengthwise; if the strips run across the room.

$30 \text{ in.} = \frac{5}{6} \text{ yd.}$
$18 \text{ ft.} = 6 \text{ yds.}$
$14 \text{ ft.} = 4\frac{2}{3} \text{ yds.}$
$4\frac{2}{3} \div \frac{5}{6} = 5\frac{3}{5} = 6 \text{ strips.}$
$6 \times 6 \text{ yds.} = 36 \text{ yds.}$
$36 \times \$1\frac{1}{4} = \$45.$ *Ans.*
$6 \div \frac{5}{6} = 7\frac{1}{5} = 8 \text{ strips.}$
$8 \times 4\frac{2}{3} \times \$1\frac{1}{4} = \$46\frac{2}{3}$
$= \$46.67.$ *Ans.*

27. Find the cost of carpeting 27 inches wide, at $1.12½ per yard, for a room 29 ft. 9 in. by 23 ft. 6 in., if the strips run across the room.

$27 \text{ in.} = \frac{3}{4} \text{ yd.}$
$29 \text{ ft. } 9 \text{ in.} = 29\frac{3}{4} \text{ ft.} = 9\frac{11}{12} \text{ yds.}$
$23 \text{ ft. } 6 \text{ in.} = 23\frac{1}{2} \text{ ft.} = 7\frac{5}{6} \text{ yds.}$
$9\frac{11}{12} \div \frac{3}{4} = 13\frac{2}{9}.$
$14 \times 7\frac{5}{6} = 109\frac{2}{3} \text{ yds.}$
$= 14 \text{ strips.}$
$109\frac{2}{3} \times \$1.125 = \$123.38.$ *Ans.*

28. Find the cost of carpeting ¾ of a yard wide, at $2.75 per yard, for a room 34 ft. 8 in. by 13 ft. 3 in., if the strips run lengthwise, and if there be a waste of ¼ of a yard on each strip in matching the pattern.

$34 \text{ ft. } 8 \text{ in.} = 34\frac{2}{3} \text{ ft.} = 11\frac{5}{9} \text{ yds.}$
$13 \text{ ft. } 3 \text{ in.} = 13\frac{1}{4} \text{ ft.} = 4\frac{5}{12} \text{ yds.}$
$4\frac{5}{12} \div \frac{3}{4} = 5\frac{8}{9} = 6 \text{ strips.}$
$6 \times 11\frac{5}{9} = 69\frac{1}{3} \text{ yds.}$
$6 \times \frac{1}{4} \text{ yd.} = 1\frac{1}{2} \text{ yds. waste.}$
$69\frac{1}{3} + 1\frac{1}{2} = 70\frac{5}{6} \text{ yds.}$
$70\frac{5}{6} \times \$2.75 = \$194.79.$ *Ans.*

29. Which way must the strips of carpet ¾ of a yard wide run in order to carpet most economically a room 20 ft. 6 in. long and 19 ft. 6 in. wide, if there be no waste for matching the pattern?

$20 \text{ ft. } 6 \text{ in.} = 20\frac{1}{2} \text{ ft.} = 6\frac{5}{6} \text{ yds.}$
$19 \text{ ft. } 6 \text{ in.} = 19\frac{1}{2} \text{ ft.} = 6\frac{1}{2} \text{ yds.}$
$6\frac{1}{2} \div \frac{3}{4} = 8\frac{2}{3} = 9 \text{ strips.}$
$9 \times 6\frac{5}{6} = 61\frac{1}{2} \text{ yds. lengthwise.}$
$6\frac{5}{6} \div \frac{3}{4} = 9\frac{1}{9} = 10 \text{ strips.}$
$10 \times 6\frac{1}{2} = 65 \text{ yds. across.}$
$\therefore$ the strips must run lengthwise.

30. Find the number of yards of plastering in the walls of a room 21¾ ft. long, 16½ ft. wide, and 11 ft. high, if 12 sq. yds. be allowed for doors, windows, and base-boards.

$21\frac{3}{4} + 16\frac{1}{2} = 38\frac{1}{4} \text{ ft.}$
$2 \times 38\frac{1}{4} \text{ ft.} = 76\frac{1}{2} \text{ ft.}$
$76\frac{1}{2} \times 11 = 841\frac{1}{2} \text{ sq. ft.}$
$= 93\frac{1}{2} \text{ sq. yds.}$
$93\frac{1}{2} \text{ sq. yds.} - 12 \text{ sq. yds.}$
$= 81\frac{1}{2} \text{ sq. yds.}$ *Ans.*

31. How many square yards of plastering in the walls and ceiling of a room 30 ft. 8 in. long, 26 ft. 5 in. wide, 10 ft. 6 in. high, if 24 sq. yds. be allowed for doors, windows, and base-boards?

$30 \text{ ft. } 8 \text{ in.} = 30\frac{2}{3} \text{ ft.}$
$26 \text{ ft. } 5 \text{ in.} = 26\frac{5}{12} \text{ ft.}$
$30\frac{2}{3} + 26\frac{5}{12} = 57\frac{1}{12} \text{ ft.}$
$2 \times 57\frac{1}{12} = 114\frac{1}{6} \text{ ft.}$
$114\frac{1}{6} \times 10\frac{1}{2} = 133\frac{7}{36} \text{ sq. yds.}$
$30\frac{2}{3} \times 26\frac{5}{12} = 90\frac{1}{81} \text{ sq. yds.}$
$133\frac{7}{36} + 90\frac{1}{81} = 223\frac{67}{324}.$
$223\frac{67}{324} - 24 = 199\frac{67}{324} \text{ sq. yds.}$ *Ans.*

32. What will be the cost of plastering the walls and ceiling of a room 27 ft. 4 in. long, 20 ft. wide, and 12 ft. 6 in. high, at 27 cents per square yard, if 20 sq. yds. be deducted for doors, windows, and base-board?

27 ft. 4 in. $= 27\frac{1}{3}$ ft. $= 9\frac{1}{9}$ yds.
20 ft. $= 6\frac{2}{3}$ yds.
12 ft. 6 in. $= 12\frac{1}{2}$ ft. $= 4\frac{1}{6}$ yds.
$9\frac{1}{9} + 6\frac{2}{3} = 15\frac{7}{9}$ yds.
$2 \times 15\frac{7}{9} = 31\frac{5}{9}$ yds.
$31\frac{5}{9} \times 4\frac{1}{6} = 131\frac{13}{27}$ sq. yds.
$9\frac{1}{9} \times 6\frac{2}{3} = 60\frac{20}{27}$ sq. yds.
$131\frac{13}{27} \times 60\frac{20}{27} = 192\frac{2}{9}$ sq. yds.
$192\frac{2}{9} - 20 = 172\frac{2}{9}$ sq. yds.
$172\frac{2}{9} \times \$0.27 = \$46.50$. *Ans.*

33. Find the cost of whitening the ceiling and walls of a room 14 ft. 4 in. wide, 15 ft. 6 in. long, 10 ft. 6 in. high, at 5 cents per square yard, allowing 9 sq. yds. for doors and windows.

14 ft. 4 in. $= 14\frac{1}{3}$ ft. $= 4\frac{7}{9}$ yds.
15 ft. 6 in. $= 15\frac{1}{2}$ ft. $= 5\frac{1}{6}$ yds.
10 ft. 6 in. $= 10\frac{1}{2}$ ft. $= 3\frac{1}{2}$ yds.
$4\frac{7}{9} + 5\frac{1}{6} = 9\frac{17}{18}$ yds.
$2 \times 9\frac{17}{18} = 19\frac{8}{9}$ yds.
$19\frac{8}{9} \times 3\frac{1}{2} = 69\frac{11}{18}$ sq. yds.
$4\frac{7}{9} \times 5\frac{1}{6} = 24\frac{37}{54}$ sq. yds.
$69\frac{11}{18} + 24\frac{37}{54} = 94\frac{8}{27}$ sq. yds.
$94\frac{8}{27} - 9 = 85\frac{8}{27}$ sq. yds.
$85\frac{8}{27} \times \$0.05 = \$4.26$. *Ans.*

34. Find the cost of plastering a room 21 ft. long, 15 ft. wide, 12 ft. high, at 40 cents per square yard, allowing for a door 7 ft. high, 3 ft. wide; 3 windows, each 5 ft. high, 3 ft. wide; and a dado 2 ft. 9 in. high around the room.

$21 + 15 = 36$ ft.
2×36 ft. $= 72$ ft.
$72 \times 12 = 1179$ sq. ft.
$3 \times 7 = 21$ sq. ft. door.
$5 \times 3 \times 3 = 45$ sq. ft. windows.
$2\frac{3}{4} \times 72 = 198$ sq. ft. dado.

$21 + 45 + 198 = 264$ sq. ft.
$1179 - 264 = 915$ sq. ft.
915 sq. ft. $= 101\frac{2}{3}$ sq. yds.
$101\frac{2}{3} \times \$0.40 = \$40.67$. *Ans.*

35. Find the cost of papering a room 20 ft. 6 in. long, 17 ft. 4 in. wide, 9 ft. high, with paper 18 in. wide, 8 yards in a roll, at 75 cents a roll; allowing for 2 doors, each 7 ft. high, 3 ft. wide, and for 3 windows, each 5 ft. 6 in. high and 3 ft. 3 in. wide.

20 ft. 6 in. $= 20\frac{1}{2}$ ft. $= 6\frac{5}{6}$ yds.
17 ft. 4 in. $= 17\frac{1}{3}$ ft. $= 5\frac{7}{9}$ yds.
9 ft. $= 3$ yds.
$6\frac{5}{6} + 5\frac{7}{9} = 12\frac{11}{18}$ yds. $2 \times 12\frac{11}{18} = 25\frac{2}{9}$.
$25\frac{2}{9} \times 3 = 75\frac{2}{3}$ sq. yds.
$2 \times 7 \times 3 = 42$ sq. ft. $= 4\frac{2}{3}$ sq. yds. doors.

$3 \times 5\frac{1}{2} \times 3\frac{1}{4} = 53\frac{5}{8}$ sq. ft. $= 5\frac{23}{24}$ sq. yds. windows.
$4\frac{2}{3} + 5\frac{23}{24} = 10\frac{5}{8}$ sq. yds.
$75\frac{2}{3} - 10\frac{5}{8} = 65\frac{1}{24}$ sq. yds.
18 in. $= \frac{1}{2}$ yd. $8 \times \frac{1}{2} = 4$ sq. yds.
$65\frac{1}{24} \div 4 = 16\frac{25}{96} = 17$ rolls.
$17 \times \$0.75 = \$12.75$. *Ans.*

36. Find the cost of papering a room 32 ft. long, 22 ft. wide, 13 ft. high, with paper 18 in. wide, 8 yards in a roll, at \$1.25 a roll, if 50 sq. yds. be allowed for doors, windows, and base-board.

$32 + 22 = 54$ ft. $2 \times 54 = 108$ ft.
$108 \times 13 = 1404$ sq. ft. $= 156$ sq. yds.
$156 - 50 = 106$ sq. yds.
18 in. $= \frac{1}{2}$ yd. $8 \times \frac{1}{2} = 4$ sq. yds.
$106 \div 4 = 26\frac{1}{2} = 27$ rolls.
$27 \times \$1.25 = \$33.75$. *Ans.*

37. Find the cost of papering a room 26 ft. long, 21 ft. wide, 12 ft. high, with paper 20 in. wide, 8 yards in a roll, at \$1.50 a roll, and a border at 25 cents per running foot; allowing for a fire-place 5 ft. 3 in. by 4 ft., a door 7 ft. by $4\frac{1}{2}$ ft., and 3 windows, each 6 ft. by $3\frac{1}{2}$ ft.

26 ft. $= 8\frac{2}{3}$ yds. 21 ft. $= 7$ yds. 12 ft. $= 4$ yds.
$8\frac{2}{3} + 7 = 15\frac{2}{3}$ yds. $2 \times 15\frac{2}{3} = 31\frac{1}{3}$ yds.
$31\frac{1}{3} \times 4 = 125\frac{1}{3}$ sq. yds.
5 ft. 3 in. $= 5\frac{1}{4}$ ft.

$5\frac{1}{4} \times 4 = 21$ sq. ft. $= 2\frac{1}{3}$ sq. yds. fire-place.
$7 \times 4\frac{1}{2} = 31\frac{1}{2}$ sq. ft. $= 3\frac{1}{2}$ sq. yds. door.
$3 \times 6 \times 3\frac{1}{2} = 63$ sq. ft. $= 7$ sq. yds. windows.

$12\frac{5}{6}$ sq. yds. to be deducted.

$125\frac{1}{3} - 12\frac{5}{6} = 112\frac{1}{2}$ sq. yds.
20 in. $= \frac{5}{9}$ yd. $8 \times \frac{5}{9} = 4\frac{4}{9}$ sq. yds.
$112\frac{1}{2} \div 4\frac{4}{9} = 25\frac{5}{16} = 26$ rolls.
$26 \times \$1.50 = \$39.00$.
Perimeter $= 31\frac{1}{3}$ yds. $= 94$ ft.
$94 \times \$0.25 = \$23.50$.
$\$39.00 + \$23.50 = \$62.50$. *Ans.*

How many feet board measure in:

38. A board 18 ft. long, 9 in. wide, $\frac{7}{8}$ in. thick?

A board 16 ft. long, 11 in. wide, 1 in. thick?

$$9 \text{ in.} = \tfrac{3}{4} \text{ ft.}$$

$$18 \times \tfrac{3}{4} = 13\tfrac{1}{2} \text{ ft. } \textit{Ans.}$$

$$11 \text{ in.} = \tfrac{11}{12} \text{ ft.}$$

$$16 \times \tfrac{11}{12} = 14\tfrac{2}{3} \text{ ft. } \textit{Ans.}$$

39. Twenty boards averaging 14 ft. long, 10 in. wide, $\frac{1}{2}$ in. thick?

$$10 \text{ in.} = \tfrac{5}{6} \text{ ft.}$$

$$\frac{\overset{7}{\cancel{14}}}{1} \times \frac{5}{\underset{3}{\cancel{6}}} \times \frac{20}{1} = \frac{700}{3} = 233\tfrac{1}{3} \text{ ft. } \textit{Ans.}$$

40. Three joists 13 ft. long, 8 in. wide, 3 in. thick?

$$8 \text{ in.} = \tfrac{2}{3} \text{ ft.}$$

$$\text{Hence 1 joist} = 3 \text{ boards 13 by } \tfrac{2}{3}.$$

$$\frac{13}{1} \times \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{1} \times \frac{3}{1} = 78 \text{ ft. } \textit{Ans.}$$

41. A stick of timber 8 in. by 9 in. and 27 ft. long?

$$8 \text{ in.} = \tfrac{2}{3} \text{ ft.}$$

$$\text{Hence 1 stick} = 9 \text{ boards 27 ft. by } \tfrac{2}{3} \text{ ft.}$$

$$\frac{\overset{9}{\cancel{27}}}{1} \times \frac{2}{\cancel{3}} \times \frac{9}{1} = 162 \text{ ft. } \textit{Ans.}$$

42. Two beams, each 6 in. by 9 in. and 23 ft. long?

$$6 \text{ in.} = \tfrac{1}{2} \text{ ft.}$$

$$\text{Hence 1 beam} = 9 \text{ boards 23 ft. by } \tfrac{1}{2} \text{ ft.}$$

$$\frac{23}{1} \times \frac{1}{\cancel{2}} \times \frac{9}{1} \times \frac{\cancel{2}}{1} = 207 \text{ ft. } \textit{Ans.}$$

43. Three joists, each 3 in. by 4 in. and 11 ft. long?

$$3 \text{ in.} = \tfrac{1}{4} \text{ ft.}$$

Hence 1 joist = 4 boards 11 ft. by $\frac{1}{4}$ ft.

$$\frac{11}{1} \times \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{1} \times \frac{3}{1} = 33 \text{ ft. } \textit{Ans.}$$

44. Five joists, each 6 in. by 4 in. and 14 ft. long?

$$6 \text{ in.} = \tfrac{1}{2} \text{ ft.}$$

Hence 1 joist = 4 boards 14 ft. by $\frac{1}{2}$ ft.

$$\frac{14}{1} \times \frac{1}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{1} \times \frac{5}{1} = 140 \text{ ft. } \textit{Ans.}$$

45. A stick of timber 10 in. square and 36 ft. long?

$$10 \text{ in.} = \tfrac{5}{6} \text{ ft.}$$

Hence 1 stick = 10 boards 36 ft. by $\frac{5}{6}$ ft.

$$\frac{\overset{6}{\cancel{36}}}{1} \times \frac{5}{\cancel{6}} \times \frac{10}{1} = 300 \text{ ft. } \textit{Ans.}$$

46. Ten planks, each 13 ft. long, 15 in. wide, 2 in. thick?

$$15 \text{ in.} = \tfrac{5}{4} \text{ ft.}$$

Hence 1 plank = 2 boards 13 ft. by $\frac{5}{4}$ ft.

$$\frac{13}{1} \times \frac{5}{4} \times \frac{2}{1} \times \frac{10}{1} = 325 \text{ ft. } \textit{Ans.}$$

Find the cost of:

47. Nine joists, each 15 ft. long, $3\frac{1}{2}$ in. by 5 in., at \$12 per M.

5 in. = $\frac{5}{12}$ ft. \$12 per M. = $\frac{\$12}{1000}$ per sq. ft.

Hence 1 joist = $3\frac{1}{2}$ boards 15 ft. by $\frac{5}{12}$ ft.

$$15 \times \frac{7}{2} \times \frac{5}{12} \times 9 \times \frac{\$12}{1000} = \frac{\$169}{80} = \$2.36. \ \textit{Ans.}$$

48. Thirty planks, each 12 ft. long, 11 in. wide, 3 in. thick, at \$15 per M.

$$11 \text{ in.} = \tfrac{11}{12} \text{ ft.} \quad \$15 \text{ per M.} = \frac{\$15}{1000} \text{ per sq. ft.}$$

Hence 1 plank = 3 boards 12 ft. by $\frac{11}{12}$ ft.

$$12 \times \frac{11}{12} \times 3 \times 30 \times \frac{\$15}{1000} = \frac{\$297}{20} = \$14.85. \textit{ Ans.}$$

49. Four sticks of timber, each 8 in. by 9 in. and 23 ft. long, at \$18 per M.

$$9 \text{ in.} = \tfrac{3}{4} \text{ ft.} \quad \$18 \text{ per M.} = \frac{\$18}{1000} \text{ per sq. ft.}$$

Hence 1 stick = 8 boards 23 ft. by $\frac{3}{4}$ ft.

$$\frac{23}{1} \times \frac{3}{4} \times \frac{8}{1} \times \frac{4}{1} \times \frac{\$18}{1000} = \frac{\$1242}{125} = \$9.94. \textit{ Ans.}$$

50. A board 24 ft. long, 23 in. wide at one end and 17 in. at the other, and $1\frac{1}{2}$ in. thick, at \$30 per M.

$$\frac{23+17}{2} = 20 \text{ in. average width.}$$

$$20 \text{ in.} = 1\tfrac{2}{3} \text{ ft}$$

$$\$30 \text{ per M.} = \frac{\$30}{1000} \text{ per sq. ft.}$$

Hence the board = $1\frac{1}{2}$ boards 24 ft. by $1\frac{2}{3}$ ft.

$$24 \times \frac{5}{3} \times \frac{3}{2} \times \frac{\$30}{1000} = \frac{\$9}{5} = \$1.80. \textit{ Ans.}$$

51. A stick of timber 29 ft. long, 10 in. by 12 in. at \$13.50 per M.

$$12 \text{ in.} = 1 \text{ ft.} \quad \$13.50 \text{ per M.} = \frac{\$13\frac{1}{2}}{1000} = \frac{\$27}{2000} \text{ per ft.}$$

Hence 1 stick = 10 boards 29 ft. by 1 ft.

$$29 \times \cancel{10} \times 1 \times \frac{\$27}{200\cancel{0}} = \frac{\$783}{200} = \$3.92. \textit{ Ans.}$$

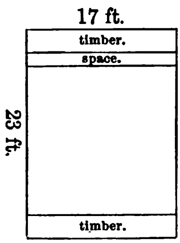
52. The flooring for two floors, each 23 ft. by 17 ft., each floor double, and of boards $\frac{7}{8}$ in. thick; the lower floor at \$18, and the upper at \$24, per M.

Boards $\frac{7}{8}$ in. thick are reckoned as 1 in.
Hence each floor, being double, will require $2 \times 23 \times 17 = 782$ sq. ft.
Average cost per floor $= \$21$ per M.

$\therefore$ Average cost both floors $= \$42$ per M.

$\therefore$ Whole cost $= 782 \times \frac{\$42}{1000} = \$32.84$. *Ans.*

53. The flooring timbers for a room 23 ft. by 17 ft. at \$18 per M, if they are 2 in. by 10 in., 17 ft. long, and are placed on edge, two close to the walls, and the others with spaces of $\frac{39}{40}$ of a foot between them.



The room being 17 ft. wide and the timbers 17 ft. long, the timbers must run across the room. After a timber is placed against the wall at one end, the remaining distance to be occupied with timbers and spaces $= 23$ ft. $- 2$ in. $= 22\frac{5}{6}$ ft. The distance occupied by a timber and a space $= \frac{1}{6} + \frac{39}{40} = \frac{137}{120}$ ft.

$\therefore$ $22\frac{5}{6} \div \frac{137}{120} =$ number of timbers required for remaining space.

$$22\tfrac{5}{6} \div \frac{137}{120} = \frac{\cancel{137}}{\cancel{6}} \times \frac{\overset{20}{\cancel{120}}}{\cancel{137}} = 20.$$

1 timber is supposed to have been placed.

$\therefore$ $20 + 1 = 21$, the whole number of timbers required.

$$\cancel{10} \times \frac{1}{\cancel{6}} \times 17 \times \frac{\$\overset{3}{\cancel{18}}}{100\cancel{0}} = \frac{\$51}{100} = \$0.51, \text{ cost of one timber.}$$

$\therefore$ $21 \times \$0.51 = \$10.71$, whole cost.

54. A log 14 ft. long, 17 in. in diameter.

$$17^2 - 2 \times 17 = 289 - 34 = 255.$$

$$\tfrac{21}{40} \text{ of } \tfrac{14}{10} \text{ of } 255 = 187 \text{ ft. } \textit{Ans.}$$

55. A log 11 ft. long, 13 in. in diameter.

$$13^2 - 2 \times 13 = 169 - 26 = 143.$$

$$\tfrac{21}{40} \text{ of } \tfrac{11}{10} \text{ of } 143 = 83 \text{ ft. } \textit{Ans.}$$

56. A log 16 ft. long, 20 in. in diameter.

$$20^2 - 2 \times 20 = 400 - 40 = 360.$$
$$\tfrac{21}{40} \text{ of } \tfrac{16}{10} \text{ of } 360 = 302 \text{ ft. } \textit{Ans.}$$

57. A log 12 ft. long, 15 in. in diameter.

$$15^2 - 2 \times 15 = 225 - 30 = 195.$$
$$\tfrac{21}{40} \text{ of } \tfrac{12}{10} \text{ of } 195 = 123 \text{ ft. } \textit{Ans.}$$

Find the value, at $9 per M. of:

58. A log 17 ft. long, averaging 11 in. in diameter.

$$11^2 - 2 \times 11 = 121 - 22 = 99.$$
$$\tfrac{21}{40} \text{ of } \tfrac{17}{10} \text{ of } 99 = 88.3575 \text{ ft.}$$
$$88.3575 \times \$0.009 = \$0.80. \textit{ Ans.}$$

59. A log 18 ft. long, averaging 13 in. in diameter.

$$13^2 - 2 \times 13 = 169 - 26 = 143.$$
$$\tfrac{21}{40} \text{ of } \tfrac{18}{10} \text{ of } 143 = 135.135 \text{ ft.}$$
$$135.135 \times \$0.009 = \$1.22. \textit{ Ans.}$$

60. A log 13 ft. long, 16 in. in diameter.

$$16^2 - 2 \times 16 = 256 - 32 = 224.$$
$$\tfrac{21}{40} \text{ of } \tfrac{13}{10} \text{ of } 224 = 153 \text{ ft.}$$
$$153 \times \$0.009 = \$1.38. \textit{ Ans.}$$

61. A log 14 ft. long, 12 in. in diameter.

$$12^2 - 2 \times 12 = 144 - 24 = 120.$$
$$\tfrac{21}{40} \text{ of } \tfrac{14}{10} \text{ of } 120 = 88 \text{ ft.}$$
$$88 \times \$0.009 = \$0.79. \textit{ Ans.}$$

62. How many clapboards will be required to cover the front of a house 60 ft. long and 20 ft. high, if they are laid 4 in. to the weather, and if 120 sq. ft. be deducted for doors and windows?

$$60 \times 20 = 1200.$$
$$1200 - 120 = 1080.$$
$$4 \times \tfrac{1}{3} = 1\tfrac{1}{3} \text{ sq. ft.}$$
$$1080 \div 1\tfrac{1}{3} = \tfrac{3}{4} \text{ of } 1080.$$
$$= 810. \textit{ Ans.}$$

63. If one thousand shingles cover 120 sq. ft. of roof, what is the average width of a shingle?

$$\tfrac{120}{1000} = \tfrac{3}{25} \text{ sq. ft.} = 17\tfrac{7}{25} \text{ sq. in.}$$
$$\tfrac{1}{3} \text{ of } 16 \text{ in.} = 5\tfrac{1}{3} \text{ in.}$$
$$17\tfrac{7}{25} \div 5\tfrac{1}{3} = 3\tfrac{6}{25} \text{ in. } \textit{Ans.}$$

64. Allowing one thousand shingles for 120 sq. ft., how many thousand will be required to cover the pitched roof of a house 60 ft. long, if the width of each side of the roof be $24\frac{1}{2}$ ft.?

$$24\tfrac{1}{2} \times 60 = 1470 \text{ sq. ft.}$$
$$1470 \div 120 = 24\tfrac{1}{2}.$$
$$24\tfrac{1}{2} \text{ thousand. } \textit{Ans.}$$

Exercise LX.

1. Find the volume of a rectangular solid whose length, breadth, and thickness are 7 ft., 2 ft. 6 in., and 11 in. respectively.

$$7 \times 2\tfrac{1}{2} \times \tfrac{11}{12} = 16\tfrac{1}{24} \text{ cu. ft.}$$
$$= 16 \text{ cu. ft. } 72 \text{ cu. in. } \textit{Ans.}$$

2. How many cubic feet of air in a hall 54 ft. long, 33 ft. wide, 21 ft. 4 in. high?

$$54 \times 33 \times 21\tfrac{1}{3} = 38{,}016 \text{ cu. ft. } \textit{Ans.}$$

3. Find the volume of a cube whose edge is $2\frac{1}{2}$ yds.

$$2\tfrac{1}{2} \times 2\tfrac{1}{2} \times 2\tfrac{1}{2} = 15\tfrac{5}{8} \text{ cu. yds.}$$
$$= 15 \text{ cu. yds. } 16 \text{ cu. ft. } 1512 \text{ cu. in. } \textit{Ans.}$$

4. A cellar is dug 21 ft. long, 17 ft. 3 in. wide, 9 ft. deep. How many cubic yards of earth are taken out?

$$\frac{21 \times 17\frac{1}{4} \times 9}{27} = 120\tfrac{3}{4} \text{ cu. yds. } \textit{Ans.}$$

5. How many cubic feet of water does a cistern hold whose length, breadth, and height are 5 ft. 4 in., 3 ft. 6 in., 2 ft. 10 in., respectively?

$$5\tfrac{1}{3} \times 3\tfrac{1}{2} \times 2\tfrac{5}{6} = 52\tfrac{8}{9} \text{ cu. ft. } \textit{Ans.}$$

6. If the dimensions of a brick be 8 in. by $3\frac{1}{2}$ in. by $2\frac{1}{4}$ in., find its volume.

$$8 \times 3\tfrac{1}{2} \times 2\tfrac{1}{4} = 63 \text{ cu. in. } \textit{Ans.}$$

7. In a bar of iron 21 ft. long, 3 in. wide, 2 in. thick, how many cubic inches are there?

$$21 \text{ ft.} = 252 \text{ in.}$$
$$252 \times 3 \times 2 = 1512 \text{ cu. in. } \textit{Ans.}$$

8. What is the value of a bar of gold 8 in. long and $\frac{3}{4}$ of an inch square, at \$190 a cubic inch?

$$8 \times \tfrac{3}{4} \times \tfrac{3}{4} \times \$190. = \$855. \textit{ Ans.}$$

9. A reservoir whose length and breadth are 15 yds. and 12 yds., respectively, holds 330 cu. yds. of water. What is its depth?

$$\frac{330}{15 \times 12} = 1\tfrac{5}{6} \text{ yds. } \textit{Ans.}$$

10. What length must be cut off a beam 9 in. by 15 in. to contain $2\frac{1}{2}$ cu. ft.?

$$\frac{2\frac{1}{2}}{\frac{3}{4} \text{ of } \frac{5}{4}} = 2\tfrac{2}{3} \text{ ft.}$$
$$= 2 \text{ ft. } 8 \text{ in. } \textit{Ans.}$$

11. How high should a room be made, if its length be 31 ft. 3 in. and breadth 24 ft., in order that it may contain 10,000 cu. ft. of air?

$$\frac{10000}{31\frac{1}{4} \times 24} = 13\tfrac{1}{3} \text{ ft. } \textit{Ans.}$$

12. A piece of wood 5 ft. long, 1 ft. broad, and 9 in. thick, is cut up into matches $2\frac{1}{2}$ in. long and 0.1 of an inch square. How many will there be if no allowance be made for waste in cutting?

5 ft. = 60 in.; 1 ft. = 12 in.

$$\frac{60 \times 12 \times 9}{2\frac{1}{2} \times \frac{1}{10} \times \frac{1}{10}} = 259{,}200. \textit{ Ans.}$$

13. How long a wall 6 ft. high, $12\frac{3}{4}$ in. thick, could be built with the bricks forming a pile 17 ft. 6 in. long, 5 ft. wide, 4 ft. 3 in. high?

$$12\tfrac{3}{4} \text{ in.} = 1\tfrac{1}{16} \text{ ft.}$$

$$\frac{17\frac{1}{2} \times 5 \times 4\frac{1}{4}}{6 \times 1\frac{1}{16}} = 58\tfrac{1}{3} \text{ ft. } \textit{Ans.}$$

14. Find the surface of a cube whose edge is 3 ft. $5\frac{2}{3}$ in.

$$5\tfrac{2}{3} \text{ in.} = \tfrac{17}{36} \text{ ft.}$$
$$3\tfrac{17}{36} \times 3\tfrac{17}{36} \times 6 = 72\tfrac{73}{216} \text{ sq. ft.}$$
$$= 72 \text{ sq. ft. } 48\tfrac{2}{3} \text{ sq. in. } \textit{Ans.}$$

15. Find the surface of a rectangular block of stone 4 ft. long, $2\frac{1}{2}$ ft. broad, $1\frac{1}{4}$ ft. thick.

$$4 \times 2\tfrac{1}{2} \times 2 = 20.$$
$$2\tfrac{1}{2} \times 1\tfrac{1}{4} \times 2 = 6\tfrac{1}{4}.$$
$$4 \times 1\tfrac{1}{4} \times 2 = 10.$$
$$20 + 6\tfrac{1}{4} + 10 = 36\tfrac{1}{4} \text{ sq. ft.}$$
$$= 36 \text{ sq. ft. } 36 \text{ sq. in. } \textit{Ans.}$$

16. A lake whose area is 45 A. is covered with ice 3 in. thick. Find the weight of the ice in tons, if a cubic foot weigh 920 oz. avoirdupois.

1 A. = 43,560 sq. ft.

$$\begin{array}{r} 43560 \\ \times\ 45 \\ \hline 1960200 \text{ sq. ft.} \end{array}$$

3 in. = $\frac{1}{4}$ ft.

$$\begin{array}{r} 4\overline{)1960200} \\ 490050 \text{ cu. ft.} \\ \times\ 920 \\ \hline 450846000 \end{array}$$

32,000 oz. = 1 t.

$$\begin{array}{r} 14088\tfrac{15}{16} \text{ t. } \textit{Ans.} \\ 32\overline{)450846} \end{array}$$

17. How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 16 in. thick, each brick being 8 in. long, 4 in. wide, $2\frac{1}{4}$ in. thick?

$$\frac{75\times 6\times 1\frac{1}{3}}{\frac{2}{3}\times\frac{1}{3}\times\frac{3}{16}}=14{,}400.\ \textit{Ans.}$$

18. Find the cost of making a road 110 yds. in length and 18 ft. wide, the soil being first removed to the depth of 1 ft. at a cost of 25 cents a cubic yard; rubble being then laid 8 in. deep, at 25 cents a cubic yard, and gravel placed on top 9 in. thick, at $62\frac{1}{2}$ cents a cubic yard.

$$18\text{ ft.}=6\text{ yds.}$$

$$\frac{110}{1}\times\frac{6}{1}\times\frac{1}{3}\times\frac{\$25}{100}=\$55.$$

$$8\text{ in.}=\tfrac{2}{9}\text{ yd.}$$

$$\frac{110}{1}\times\frac{6}{1}\times\frac{2}{9}\times\frac{\$25}{100}=\$36\tfrac{2}{3}.$$

$$9\text{ in.}=\tfrac{1}{4}\text{ yd.}$$

$$\frac{110}{1}\times\frac{6}{1}\times\frac{1}{4}\times\frac{\$625}{1000}=\$103\tfrac{1}{8}.$$

Whole cost = $194.79. *Ans.*

19. A room whose length is 27 ft., breadth 24 ft., height 10 ft., is to have its ceiling raised so as to increase the space by 84 cu. yds. What will then be its height?

$$27\text{ ft.}=9\text{ yds.}$$
$$24\text{ ft.}=8\text{ yds.}$$

$$\frac{84}{9\times 8}=1\tfrac{1}{6}\text{ yds.}=3\tfrac{1}{2}\text{ ft.}$$

$$10+3\tfrac{1}{2}=13\tfrac{1}{2}\text{ ft.}\ \textit{Ans.}$$

20. A block of wood 5 ft. 4.8 in. long, 1 ft. 9 in. wide and thick, weighs 7.56 cwt. Determine the weight, in pounds, of a cubic foot.

$$5\text{ ft. }4.8\text{ in.}=5\tfrac{2}{5}\text{ ft.}$$
$$7.56\text{ cwt.}=756\text{ lbs.}$$

$$\frac{756}{5\frac{2}{5}\times 1\frac{3}{4}\times 1\frac{3}{4}}=45\tfrac{5}{7}\text{ lbs.}\ \textit{Ans.}$$

21. How many cords in a pile of wood 40 ft. long, 4 ft. wide, 5 ft. 4 in. high?

$$\frac{40\times 4\times 5\frac{1}{3}}{8\times 4\times 4}=6\tfrac{2}{3}.\ \textit{Ans.}$$

22. A pile of wood containing $67\frac{1}{2}$ cords is 270 ft. long and 4 ft. wide. How high is it?

$$\frac{67\frac{1}{2}\times 128}{270\times 4}=8\text{ ft.}\ \textit{Ans.}$$

23. What will be the cost of a pile of wood 25 ft. long, 4 ft. wide, 4 ft. 8 in. high, at $3.75 a cord?

$$\frac{25\times 4\times 4\frac{2}{3}\times \$3\frac{3}{4}}{128}=\$13.67.\ \textit{Ans.}$$

24. What must be the length of a load of wood $3\frac{1}{2}$ ft. high and 5 ft. wide, to contain a cord?

$$\frac{128}{5 \times 3\frac{1}{2}} = \frac{128 \times 2}{5 \times 7} = 7\tfrac{11}{35} \text{ ft. } \textit{Ans.}$$

25. How high must manure be in a cart 6 ft. by 4 ft., in order to be $\frac{1}{2}$ a cord.

$$\frac{\frac{1}{2} \times 128}{6 \times 4} = 2\tfrac{2}{3} \text{ ft. } \textit{Ans.}$$

26. Find the number of bushels in a bin that is 8 ft. long, 4 ft. wide, 3 ft. deep.

$8 \times 4 \times 3 = 96.$

$96 - \frac{1}{5}$ of $96 = 96 - 19\frac{1}{5} = 76\frac{4}{5}$ bu. *Ans.*

27. Find the number of bushels in a bin 9 ft. long, 6 ft. 6 in. wide, 3 ft. 4 in. deep.

$9 \times 6\frac{1}{2} \times 3\frac{1}{3} = 195.$

$195 - \frac{1}{5}$ of $195 = 195 - 39$

$= 156$ bu. *Ans.*

28. Find the depth of a bin to hold 360 bu., if its length be 12 ft. and its width 6 ft.

$360 - \frac{1}{4}$ of $360 = 450$ cu. ft.

$$\frac{450}{12 \times 6} = 6\tfrac{1}{4} \text{ ft. } \textit{Ans.}$$

29. Find the length of a bin that is 6 ft. wide and 5 ft. deep, if it hold 400 bu.

$400 + \frac{1}{4}$ of $400 = 500$ cu. ft.

$$\frac{500}{6 \times 5} = \frac{500}{30} = \frac{50}{3} = 16\tfrac{2}{3} \text{ ft. } \textit{Ans.}$$

30. Find the number of bushels that will fill a bin 8.5 ft. long, 4.5 ft. wide, 3.5 ft. deep.

$8.5 \times 4.5 \times 3.5 = 133.875$ cu. ft.

$133.875 - \frac{1}{5}$ of 133.875

$= 107.1$ bu. *Ans.*

31. A bin 20 ft. long, 12 ft. wide, and 6 ft. deep, is full of wheat. What is its value, at \$1.25 a bushel?

$20 \times 12 \times 6 = 1440$ cu. ft.

$1440 - \frac{1}{5}$ of $1440 = 1152$ bu.

$1152 \times \$1\frac{1}{4} = \$1440.$ *Ans.*

32. If a ton of coal occupy 40 cu. ft., how many tons will a bin hold that is 21 ft. long, 10 ft. wide, 5 ft. deep?

$$\frac{21 \times 10 \times 5}{40} = 26\tfrac{1}{4} \text{ t. } \textit{Ans.}$$

33. If a ton of Lehigh coal occupy 35 cu. ft., how many tons will a bin hold that is 8 ft. long, 5 ft. 9 in. wide, 4 ft. 6 in. deep?

$$\frac{8 \times 5\frac{3}{4} \times 4\frac{1}{2}}{35} = 5\tfrac{32}{35} \text{ t. } \textit{Ans.}$$

34. Find the number of gallons that a cistern will hold that is 13 ft. long, 6 ft. wide, 7 ft. 4 in. deep.

$$13 \times 6 \times 7\tfrac{1}{3} = 572 \text{ cu. ft.}$$
$$= 988{,}416 \text{ cu. in.}$$

$$231\overline{)988416}\quad 4278\tfrac{6}{7} \text{ gals. } Ans.$$

35. Find the number of gallons that a tank will hold that is 4 ft. long, 2 ft. 8 in. wide, 1 ft. 8 in. deep.

$$4 \times 2\tfrac{2}{3} \times 1\tfrac{2}{3} = 17\tfrac{7}{9} \text{ cu. ft.}$$

$$1728 \times 17\tfrac{7}{9}$$

$$231\overline{)30720}(132\tfrac{68}{77} \text{ gals. } Ans.$$

36. Find the number of gallons in a cubic foot.

$$231\overline{)1728}\quad 7\tfrac{37}{77} \text{ gals. } Ans.$$

37. Find the capacity of a cistern, in cubic feet, that will hold 200 barrels of water.

$$\frac{200 \times 31\tfrac{1}{2}}{7\tfrac{1}{2}} = 840 \text{ cu. ft. } Ans.$$

38. Find the number of gallons that a round cistern will hold that is 6 ft. in diameter and 7 ft. deep.

$$3^2 = 9.$$
$$3.1416 \times 9 \times 7 \times 7\tfrac{1}{2}$$
$$= 1484.41 \text{ gals. } Ans.$$

39. Find the number of gallons that a vessel will hold that is 12 in. in diameter and 10 in. deep.

$$6 \text{ in.} = \tfrac{1}{2} \text{ ft.} \quad (\tfrac{1}{2})^2 = \tfrac{1}{4}.$$
$$10 \text{ in.} = \tfrac{5}{6} \text{ ft.}$$
$$\tfrac{1}{4} \times \tfrac{5}{6} \times 3.1416 \times 7\tfrac{1}{2} = 4.91 \text{ gals. } Ans.$$

40. How many quarts will a round vessel hold $5\tfrac{1}{8}$ in. in diameter and 6 in. deep?

$$(5\tfrac{1}{8})^2 = 26\tfrac{17}{64}.$$

$$0.7854 \times 26\tfrac{17}{64} = 20.9658$$
$$20.9658 \times 6 = 125.7948 \text{ cu. in.}$$

$$30 \text{ qts.} = 1 \text{ cu. ft.}$$
$$\therefore\ 1 \text{ qt.} = \tfrac{1}{30} \text{ of } 1728 \text{ cu. in.}$$
$$= 57.6 \text{ cu. in.}$$

$$576\overline{)1257.95}\quad 2.18 \text{ qts. } Ans.$$

41. Find the number of cubic inches in a sphere 11 in. in diameter.

$$11 \times 11 \times 11 \times 0.5236$$
$$= 696.9 \text{ cu. in. } Ans.$$

42. How many quarts will a sphere hold that is 12 in. in diameter?

$$1^3 = 1.$$
$$0.5236 \times 1 = 0.5236 \text{ cu. ft.}$$
$$30 \text{ qts.} = 1 \text{ cu. ft.}$$
$$0.5236 \times 30 = 15.708 \text{ qts. } \textit{Ans.}$$

43. What part of a bushel will a hemispherical bowl hold that is 13 in. in diameter?

$$13^3 = 13 \times 13 \times 13 = 2197.$$

$$\begin{array}{r} 0.5236 \\ \times\ 2197 \\ \hline 2)\underline{1150.3492} \text{ cu. in.} \\ 575.1746 \text{ cu. in.} \end{array}$$

$$1 \text{ bu.} = 2150.42 \text{ cu. in.}$$

$$\begin{array}{r} 0.267. \ \textit{Ans.} \\ 215042\overline{)57517.460} \end{array}$$

44. If a cubical box 2 ft. on an edge contain a solid sphere 2 ft. in diameter, how many gallons of water can be poured into the box?

$$2 \times 2 \times 2 = 8.$$
$$8 \times 0.5236 = 4.1888.$$

$$\begin{array}{r} 8.0000 \\ \underline{4.1888} \\ 3.8112 \text{ cu. ft.} \\ \underline{\times\ 1728} \\ 6585.5536 \text{ cu. in.} \end{array}$$

$$\begin{array}{r} 28.51 \text{ gals. } \textit{Ans.} \\ 231\overline{)6585.55} \end{array}$$

45. If 64 qts. of water be poured into a vessel that will hold 2 bu. of wheat, what part of the vessel will be filled?

$$2 \text{ bu.} = 64 \text{ dry qts.}$$
$$64 \text{ dry qts.} = 64 \times 67\tfrac{1}{5} \text{ cu. in.}$$
$$64 \text{ liquid qts.} = 64 \times 57\tfrac{3}{4} \text{ cu. in.}$$
$$\frac{64 \times 57\frac{3}{4}}{64 \times 67\frac{1}{5}} = \tfrac{55}{64} = 0.859. \ \textit{Ans.}$$

46. Find the number of cubic inches in 1 oz. (av.) of water.

$$\begin{array}{r} 1000\underline{)1728.000} \\ 1.728 \text{ cu. in. } \textit{Ans.} \end{array}$$

47. Find the weight in ounces (av.) of 1 cu. in. of water.

$$\tfrac{1000}{1728} = \tfrac{125}{216} \text{ oz. } \textit{Ans.}$$

48. Find the weight in ounces (av.) of 1 pt. of water.

$$\frac{1000}{7\frac{1}{2} \times 8} = 16\tfrac{2}{3} \text{ oz. } \textit{Ans.}$$

49. Find the number of pints in 1 lb. of water.

$$1 \text{ pt. of water weighs } 16\tfrac{2}{3} \text{ oz.}$$
$$1 \text{ lb.} = 16 \text{ oz.}$$
$$\frac{16}{16\frac{2}{3}} = \tfrac{24}{25} \text{ pts. } \textit{Ans.}$$

50. Find the weight, in grains, of 1 cu. in. of water.

1 cu. in. of water weighs $\frac{1000}{1728}$ oz. or $\frac{1000}{1728 \times 16}$ lbs.

$$\text{or } \frac{1000 \times 7000}{1728 \times 16} \text{ grs.} = 253\tfrac{79}{432} \text{ grs. } \textit{Ans.}$$

51. Find the specific gravity of a bar of iron 5 in. long, and 2 in. square, if it weigh 5 lbs.

$5 \times 2 \times 2 = 20$ cu. in.; if 20 cu. in. weigh 5 lbs., 4 cu. in. weigh 1 lb., and 1 cu. in. weighs 4 oz. 1 cu. in. of water weighs $\frac{125}{216}$ oz.

$$\frac{4}{\frac{125}{216}} = 6\tfrac{114}{125} = 6.912. \textit{ Ans.}$$

52. Find the specific gravity of a bar of iron 18 in. long, $2\frac{1}{3}$ in. wide, $1\frac{3}{4}$ in. thick, if it weigh 18 lbs. 9 oz.

$18 \times 2\frac{1}{3} \times 1\frac{3}{4} = 73\frac{1}{2}$; if $73\frac{1}{2}$ cu. in. weigh 18 lbs. 9 oz. or 297 oz., 1 cu. in. weighs $\frac{297}{73\frac{1}{2}}$.

$$\frac{297}{73\frac{1}{2}} \div \frac{125}{216} = 6\tfrac{6023}{6125} = 6.983. \textit{ Ans.}$$

53. Find the number of cubic inches to the pound of iron, if its specific gravity be 7.48.

$$1 \text{ oz. of water} = \frac{216}{125} \text{ cu. in.}$$

$$1 \text{ lb. of water} = \frac{216 \times 16}{125} \text{ cu. in.}$$

$$1 \text{ lb. of iron (specific gravity 7.48)} = \frac{216 \times 16}{7.48 \times 125} = 3\tfrac{651}{935} \text{ cu. in. } \textit{Ans.}$$

54. Find the number of cubic inches in 2 lbs. $6\frac{1}{2}$ oz. of gold, if its specific gravity be 19.36.

$$1 \text{ oz. (av.) of water} = \frac{216}{125} \text{ cu. in.}$$

$$1 \text{ lb. (av.) of water} = \frac{216 \times 16}{125} \text{ cu. in.}$$

$$1 \text{ lb. (troy) of water} = \frac{216 \times 16 \times 5760}{125 \times 7000} \text{ cu. in.}$$

$$2\tfrac{13}{24} \text{ lbs. (troy) of water} = \frac{216 \times 16 \times 5760 \times 2\frac{13}{24}}{125 \times 7000}.$$

$2\frac{13}{24}$ lbs. of gold (specific gravity 19.36 or $19\frac{9}{25}$)

$$= \frac{216 \times 16 \times 5760 \times 2\frac{13}{24}}{125 \times 7000 \times 19\frac{9}{25}}$$

$$= \frac{316224}{105875} \text{ cu. in.} = 2.987 \text{ cu. in. } \textit{Ans.}$$

55. How many pounds does a boy lift in raising a cubic foot of stone under water, if its specific gravity be $2\frac{1}{2}$?

Specific gravity of stone $= 2\frac{1}{2}$.
Specific gravity of water $= 1.$

Difference $= 1\frac{1}{2}$.
1 cu. ft. of water weighs 62.5 lbs.

$1\frac{1}{2} \times 62.5$ lbs. $= 93.75$ lbs. *Ans.*

56. A square-built scow 12 ft. long, $6\frac{1}{2}$ ft. wide, sinks 5 in. in water. What does it weigh, and how many pounds will be required to sink it 7 in. deeper?

$12 \times 6\frac{1}{2} \times \frac{5}{12} = 32\frac{1}{2}$ cu. ft.

$$\begin{array}{r} 62.5 \\ \times\, 32\frac{1}{2} \\ \hline 2031.2\frac{1}{4} \text{ lbs.} \end{array}$$

$= 1$ t. $31\frac{1}{4}$ lbs. (1) *Ans.*

$6\frac{1}{2} \times 12 \times 1 = 78$

$$\begin{array}{r} 78 \\ \times\, 62\frac{1}{2} \\ \hline 4875 \text{ lbs.} \end{array}$$

$$\begin{array}{rrr} = 2 \text{ t.} & 8 \text{ cwt.} & 75 \text{ lbs.} \\ 1 & & 31\frac{1}{4} \\ \hline 1 \text{ t.} & 8 \text{ cwt.} & 43\frac{3}{4} \text{ lbs.} \end{array}$$

(2) *Ans.*

57. A square-built scow 11 ft. long, $5\frac{1}{4}$ ft. wide, weighs 320 lbs. and is loaded with 750 lbs. of stone. How deep will it sink in the water?

$11 \times 5\frac{1}{4} \times \frac{1}{12} = 4\frac{13}{16}$ cu. ft.
$4\frac{13}{16} \times 62.5$ lbs. $= 300.78125$ lbs.
$320 + 750 = 1070$ lbs.

$$\begin{array}{r} 3.557 \text{ in. } \textit{Ans.} \\ 30078125\overline{)107000000.000} \end{array}$$

58. How many tons of ice, specific gravity 0.93, can be packed in a building 50 ft. long, 40 ft. wide, 20 ft. high?

$$\frac{50 \times 40 \times 20 \times 62\frac{1}{2} \times 0.93}{2000} = 1162\tfrac{1}{2} \text{ t. } \textit{Ans.}$$

59. If an iceberg weigh 0.9 of an equal bulk of sea-water, how many cubic yards in an iceberg 40 rds. long, 6 yds. wide, and rising 160 ft. out of the sea?

$$40 \text{ rds.} = 660 \text{ ft.}$$
$$6 \text{ yds.} = 18 \text{ ft.}$$
$$660 \times 18 \times 160 = 1900800 \text{ cu. ft.} = 70{,}400 \text{ cu. yds.}$$

Now, if the iceberg weighs 0.9 of the weight of an equal bulk of sea-water, only $\frac{1}{10}$ of the iceberg is above the water.

$$10 \times 70{,}400 = 704{,}000 \text{ cu. yds. } \textit{Ans.}$$

60. If a cubic foot of brick wall weigh 90 lbs. and contain 22 bricks, with the mortar, what is the weight and specific gravity of a brick and its share of mortar?

$$90 \div 22 = 4\tfrac{1}{11} \text{ lbs. (1) } \textit{Ans.}$$
$$62\tfrac{1}{2} \div 22 = 2\tfrac{37}{44} \text{ lbs.}$$
$$4\tfrac{1}{11} \div 2\tfrac{37}{44} = \tfrac{36}{25} = 1.444. \text{ (2) } \textit{Ans.}$$

61. What is the weight of a brick wall 40 ft. long, 20 ft. high, and 1 ft. thick, if the specific gravity of a brick with its mortar be 1.46; and how many thousand bricks will be required for the wall, allowing 22 for a cubic foot?

$$40 \times 20 \times = 1\ 800 \text{ cu. ft.}$$
$$800 \times 1.46 \times 62.5 \text{ lbs.} = 73000 \text{ lbs.}$$
$$= 36\tfrac{1}{2} \text{ t. (1) } \textit{Ans.}$$
$$800 \times 22 = 17{,}600. \text{ (2) } \textit{Ans.}$$

Exercise LXI.

1. Reduce 24 gals. to liters.

$$24 \text{ gals.} = 96 \text{ qts.}$$
$$96 \times 0.946^{l} = 90.816^{l}. \textit{ Ans.}$$

2. Reduce 10 lbs. troy to kilograms.

$$10 \text{ lbs.} = 120 \text{ oz.}$$
$$120 \times 31.104^{g} = 3732.480^{g}$$
$$= 3.73\cancel{248}^{kg}. \textit{ Ans.}$$

3. Reduce 50.5 cu. yds. to cu. meters.

$50.5 \times 0.765^{cbm} = 38.63^{cbm}$. *Ans.*

4. Reduce $69\frac{17}{100}$ mi. to kilometers.

$69.17 \times 1.609^{km} = 111.2945^{km}$.

5. Reduce 12 A. 12 rds. to hektars.

12 sq. rds. = 0.075 A.

$12.075 \times 0.405^{ha} = 4.89^{ha}$. *Ans.*

6. Reduce 10 cords to sters.

$10 \times 3.624^{st} = 36.24^{st}$. *Ans.*

7. Reduce 4 cwt. 24 lbs. to kilograms.

4 cwt. 24 lbs. = 424 lbs.

$424 \times 0.454^{kg} = 192.496^{kg}$. *Ans.*

8. Reduce 25 bu. 2 pks. to hektoliters.

25 bu. 2 pks. = 816 qts.

816×1.101 qts. $= 898.416^{l}$

$= 8.98\cancel{4}\cancel{1}\cancel{6}^{hl}$. *Ans.*

9. Reduce to the common system 15^{km}.

$$\begin{array}{r} 0.621 \\ \times 15 \\ \hline 9.315 \text{ mi.} \\ \times 320 \\ \hline 100.8 \text{ rds.} \\ \times 5\frac{1}{2} \\ \hline 4 \text{ yds. nearly.} \end{array}$$

9 mi. 100 rds. 4 yds. nearly. *Ans.*

10. Reduce to the common system 3^{ha}.

$$\begin{array}{r} 2.471 \text{ A.} \\ \times 3 \\ \hline 7.413 \text{ A.} \\ \times 160 \\ \hline 66.08 \text{ sq. rds.} \\ \times 30\frac{1}{4} \\ \hline 2 \text{ sq. yds. nearly.} \end{array}$$

7 A. 66 sq. rds. 2 sq. yds. nearly. *Ans.*

11. Reduce to the common system 12.125^{cbm}.

$$\begin{array}{r} 1.308 \text{ cu. yds.} \\ \times 12\frac{1}{8} \\ \hline 15.8295 \text{ cu. yds.} \\ \times 27 \\ \hline 23.3965 = 23.4 \text{ cu. ft.} \end{array}$$

15 cu. yds. 23.4 cu. ft. *Ans.*

12. Reduce to the common system 101.25^{l}.

$$\begin{array}{r} 1.0567 \text{ liquid qts.} \\ \times 101\frac{1}{4} \\ \hline 106.9908 \end{array}$$

= 107 liquid qts. nearly. *Ans.*

$$\begin{array}{r} 0.908 \text{ dry qts.} \\ \times 101\frac{1}{4} \\ \hline 91.935 \end{array}$$

= 92 dry qts. nearly. *Ans.*

13. Reduce to the common system 20.25^{hl}.

$$20.25^{hl} = 2025^{l}$$

$$\begin{array}{r} 1.0567 \text{ liquid qts.} \\ \times 2025 \\ \hline 4)\underline{2139.8175} \text{ qts.} \\ 535 \text{ gals. nearly. } \textit{Ans.} \end{array}$$

$$\begin{array}{r} 0.908 \text{ dry qts.} \\ \times 2025 \\ \hline 32)\underline{1838.7} \\ 57 \text{ bu. nearly. } \textit{Ans.} \end{array}$$

14. Reduce to the common system (troy weight) 5^{kg}.

$$5^{kg} = 5000^{g}.$$

$$\begin{array}{r|l} & 15.432 \text{ grs.} \\ & \times 5000 \\ \hline 24 & 77160 \text{ grs.} \\ 20 & 3215 \text{ dwt.} \\ 12 & 160 \text{ oz. } 15 \text{ dwt.} \\ & 13 \text{ lbs. } 4 \text{ oz.} \end{array}$$

13 lbs. 4 oz. 15 dwt. *Ans.*

15. Reduce to the common system 24^{st}.

$$\begin{array}{r} 0.276^{c} \\ \times 24 \\ \hline 6.624 \\ \times 128 \\ \hline 79.872 \end{array}$$

$= 80$ cu. ft. nearly.

6 c. 80 cu. ft. nearly. *Ans.*

16. Reduce to the common system 62.5^{qm}.

$$\begin{array}{r} 1.196 \text{ sq. yds.} \\ \times 62\tfrac{1}{2} \\ \hline 74.75 \text{ sq. yds. } \textit{Ans.} \end{array}$$

17. Reduce to the common system (avoirdupois weight) 1001^{kg}.

$$\begin{array}{r} 2.205 \text{ lbs.} \\ \times 1001 \\ \hline 2207.205 \text{ lbs.} \end{array}$$

$$\begin{array}{r|l} 100 & 2207 \text{ lbs.} \\ 20 & 22 \text{ cwt. } 7 \text{ lbs.} \\ & 1 \text{ t. } 2 \text{ cwt.} \end{array}$$

1 t. 2 cwt. 7 lbs. nearly. *Ans.*

18. Find in acres, etc., the area of a field if its length be 100^{m} and breadth 75^{m}.

$$100 \times 75 = 7500^{qm}.$$

$$\begin{array}{r|l} & 1.196 \text{ sq. yds.} \\ & \times 7500 \\ \hline 30\tfrac{1}{4} & 8970 \text{ sq. yds.} \\ 160 & 296 \text{ sq. rds. } 16 \text{ sq. yds.} \\ & 1 \text{ A. } 136 \text{ sq. rds.} \end{array}$$

1 A. 136 sq. rds. 16 sq. yds. *Ans.*

19. Determine the number of cubic meters in a box 2 yds. long, 3 ft. wide, $2\frac{1}{2}$ ft. deep.

$$2 \text{ yds.} = 6 \text{ ft.}$$

$$6 \times 3 \times 2\tfrac{1}{2} = 45 \text{ cu. ft.}$$

$$= 1\tfrac{2}{3} \text{ cu. yds.}$$

$$1\tfrac{2}{3} \times 0.765^{cbm} = 1.275^{cbm}. \textit{ Ans.}$$

20. Determine the number of cubic yards in a box 2^{m} long, 75^{cm} wide, 50^{cm} deep.

$2 \times 0.75 \times 0.50 = 0.75^{cbm}$.
0.75×1.308 cu. yds.
$= 0.981$ cu. yds. *Ans.*

21. If a man walk 75^{m} a minute, what is his rate in miles per hour?

$75 \times 60 = 4500^{m} = 4.5^{km}$.
4.5×0.621 mi. $= 2.795$ mi. *Ans.*

22. If cast-iron weigh 7.113^{g} per cubic centimeter, how many pounds does a cubic foot weigh?

As the weight of the iron is 7.113^{g} per cubic centimeter, the specific gravity is 7.113.

7.113×62.5 lbs. $= 444.5625$ lbs. *Ans.*

23. How many steps 2 ft. 6 in. long will a man take in walking a kilometer?

$$\begin{array}{r} 0.621 \text{ mi.} \\ \times 5280 \\ \hline 32788.8 \text{ ft.} \end{array}$$

$$25\overline{)32788.8} = 1311.5$$

1311.5 steps = 1312 steps. *Ans.*

24. Find the value of a carboy (17 qts.) of sulphuric acid, of 1.841 specific gravity, at $2\frac{1}{4}$ cents a pound.

$$\begin{array}{r} 0.946^{l} \\ \times 17 \\ \hline 16.082^{l} \end{array}$$

$$\begin{array}{r} 16.082^{kg} \\ \times 2.205 \text{ lbs.} \\ \hline 35.461 \text{ lbs.} \\ \times 1.841 \\ \hline 65.284 \text{ lbs.} \end{array}$$

$$\begin{array}{r} 65.284 \\ \times \$0.02\frac{1}{4} \\ \hline \$1.47. \end{array}$$ *Ans.*

25. Find the value of a carboy ($17\frac{1}{2}^{l}$) of nitric acid, of 1.451 specific gravity, at 15 cents a pound.

$$\begin{array}{r} 2.205 \text{ lbs.} \\ \times 17\frac{1}{2} \\ \hline 38.588 \text{ lbs.} \\ \times 1.451 \\ \hline 55.971 \text{ lbs.} \\ \times \$0.15 \\ \hline \$8.40. \end{array}$$ *Ans.*

26. Find the weight in pounds and in kilograms of $31\frac{1}{6}$ gals. of the best alcohol, specific gravity 0.792.

$31\frac{1}{6}$ gals. $= 124\frac{2}{3}$ qts.

$$\begin{array}{r} 0.946^{l} \\ \times 124\frac{2}{3} \\ \hline 117.935^{l} \end{array}$$ of water.

$$\begin{array}{r} 117.935^{kg} \\ \times\, 0.792 \\ \hline 93.405^{kg}. \end{array}$$ (1) *Ans.*

$$\begin{array}{r} 93.405 \\ 2.205 \text{ lbs.} \\ \hline 205.958 \text{ lbs.} \end{array}$$ (2) *Ans.*

27. If the specific gravity of sea-water be 1.026, and that of olive-oil be 0.915, what will be the weight of a hektoliter of each in pounds and in kilograms?

$$1^{hl} = 100^{l} = 100^{kg}.$$

$$\begin{array}{r} 1.026 \\ \times\, 100^{kg} \\ \hline 102.6^{kg}. \end{array}$$ (1) *Ans.*

$$\begin{array}{r} \times\, 2.205 \text{ lbs} \\ \hline 226.233 \text{ lbs.} \end{array}$$ (2) *Ans.*

$$\begin{array}{r} 0.915 \\ \times\, 100^{kg} \\ \hline 91.5^{kg}. \end{array}$$ (3) *Ans.*

$$\begin{array}{r} \times\, 2.205 \text{ lbs.} \\ \hline 201.76 \text{ lbs.} \end{array}$$ (4) *Ans.*

28. Find the weight in pounds and in kilograms of the air, specific gravity 0.00129206, in a room 7^{m} by 5^{m}, and 3.5^{m} high.

$$7 \times 5 \times 3\tfrac{1}{2} = 122\tfrac{1}{2}^{cbm}.$$

$$122\tfrac{1}{2}^{cbm} \text{ of water} = 122{,}500^{kg}.$$

$$\begin{array}{r} 0.00129206 \\ \times\, 122500^{kg} \\ \hline 158.277\cancel{35}^{kg}. \end{array}$$ (1) *Ans.*

$$\begin{array}{r} 158.277 \\ \times\, 2.205 \text{ lbs.} \\ \hline 349 \text{ lbs.} \end{array}$$ (2) *Ans.*

29. Find the weight in pounds and in kilograms of the air, specific gravity 0.00129206, in a room 23 ft. long, 16 ft. wide, and 10 ft. high.

$$23 \times 16 \times 10 = 3680 \text{ cu. ft.}$$

$$\begin{array}{r} 3680 \\ \times\, 62\tfrac{1}{2} \text{ lbs.} \\ \hline 230000 \text{ lbs.} \\ \times\, 0.00129206 \\ \hline 297.1738 \text{ lbs.} \end{array}$$ (1) *Ans.*

$$\begin{array}{r} 297.1738 \\ \times\, 0.454^{kg} \\ \hline 134.9169^{kg}. \end{array}$$ (2) *Ans.*

30. If a balloon weigh 2^{kg}, and contain $10{,}000^{l}$ of hydrogen gas, specific gravity 0.00008929, what is its lifting force in kilograms and in pounds when the air has a specific gravity of 0.00129206?

$$\begin{array}{r} 0.00129206 \\ 0.00008929 \\ \hline 0.00120277 \\ \times\, 10000^{kg} \\ \hline 12.0277^{kg} \\ 2. \\ \hline 10.0277^{kg}. \end{array}$$ (1) *Ans.*

$$\begin{array}{r} 10.0277 \\ \times 2.205 \text{ lbs.} \\ \hline 22.111 \text{ lbs. (2) } \textit{Ans.} \end{array}$$

31. If a pile of wood be 1.2^{m} wide, 7^{m} long, and 2^{m} high, how much is it worth, at $4.50 a cord?

$1.2 \times 7 \times 2 = 16.8^{st}$.
$16.8 \times 0.276^{c} = 4.6368^{c}$.
$4.6368 \times \$4\frac{1}{2} = \$20.87$. *Ans.*

32. How many miles will be travelled in 1 hr. 28 min. 21 sec., at the rate of 50^{km} an hour?

1 hr. 28 min. 21 sec. = 1.4725 hr.
$50^{km} = 50 \times 0.621$ mi. = 31.05 mi.
1.4725×31.05 mi. = 45.721 mi. *Ans.*

33. Find the time of travelling 31 mi. 180 yds. at 1 min. 25 sec. per kilometer.

$1^{km} = 0.621$ mi.
1 min. 25 sec. = $1\frac{5}{12}$ min.
31 mi. 180 yds. = 31.1023 mi.
$31102.3 \div 621 = 50.08$
$50.08 \times 1\frac{5}{12}$ min. = 70.95 min.
= 1 hr. 11 min., nearly. *Ans.*

34. What is the weight of 12 cu. yds. 16 cu. ft. 720 cu. in. of earth of which a cubic meter weighs 1 t. 17 cwt.?

12 cu. yds. 16 cu. ft. 720 cu. in.
= 12.608 cu. yds.
$1^{cbm} = 1.308$ cu. yds.
$12608 \div 1308 = 9.639$.
1 t. 17 cwt. = 37 cwt.
9.639×37 cwt. = 356.643 cwt.
= 17 t. 16 cwt. 64 lbs. *Ans.*

35. Find the weight in grams of a liter of mercury, of which a cubic inch weighs 0.4925 of a pound avoirdupois.

$1^{l} = 61.03$ cu. in.
0.4925 lbs. $\times$ 61.03 = 30.057 lbs.
$30.057 \times 453.59^{g} = 13633.55^{g}$.

36. How many yards of cloth, at $3.12½ a meter, should be given in exchange for 15^{m} at $2.75 a yard?

1 yd. = 0.914^{m}.
$0.914 \times \$3\frac{1}{8} = \$2.856$ per yd.
15×1.0936 yds. = 16.404 yds.
$16.404 \times \$2\frac{3}{4} = \$45.111$.
$45111 \div 2856 = 15.79$ yds. *Ans.*

37. If a wine merchant buy 3^{hl} of wine for 1600 francs, at what rate, United States money, does he pay a gallon, reckoning 25 francs equal to $4.85?

$3^{hl} = 300^{l}$.
1600 francs $\div$ 300 = $5\frac{1}{3}$ francs.
$\$4.850 \div 25 = \$0.194$.
$\$0.194 \times 5\frac{1}{3} = \$1.035$ per l.
$1^{l} = 1.0567$ qts.
= 0.264 gals.
$1035 \div 264 = \$3.92$. *Ans.*

38. A mill-wheel is turned by a stream of water running at the rate of a yard per second in a channel 5 ft. wide and 9 in. deep. Determine the weight of water in metric tons, supplied in 12 hrs., if a cubic foot of water weigh 1000 oz.

$3 \times 5 \times \frac{3}{4} = 11\frac{1}{4}$ cu. ft.
12 hrs. = 43,200 sec.

$43200 \times 11\frac{1}{4}$ cu. ft.
= 486000 cu. ft.

486000×1000 oz.
= 486000000 oz.
= 30375000 lbs.

0.00045359 m. t. $\times$ 30375000
= 13777.796 m. t.
Ans.

Exercise LXII.

1. Which is the greater ratio, 5 : 8 or 6 : 9?

$5 : 8 = \frac{5}{8} = \frac{15}{24}$.
$6 : 9 = \frac{6}{9} = \frac{2}{3} = \frac{16}{24}$.
$\therefore$ 6 : 9 is greater.

2. Which is the greater ratio, 7 : 10 or 9 : 12?

$7 : 10 = \frac{7}{10} = \frac{14}{20}$.
$9 : 12 = \frac{9}{12} = \frac{3}{4} = \frac{15}{20}$.
$\therefore$ 9 : 12 is greater.

3. Which is the greater ratio, 8 : 9 or 10 : 12?

$8 : 9 = \frac{8}{9} = \frac{16}{18}$.
$10 : 12 = \frac{10}{12} = \frac{5}{6} = \frac{15}{18}$.
$\therefore$ 8 : 9 is greater.

4. Which is the greater ratio, 6 : 12 or 8 : 14?

$6 : 12 = \frac{6}{12} = \frac{1}{2} = \frac{7}{14}$.
$8 : 14 = \frac{8}{14}$.
$\therefore$ 8 : 14 is greater.

5. Which is the greater ratio, 10 cwt. : 15 cwt. or \$7 : \$9?

$$10 \text{ cwt.} : 15 \text{ cwt.} = \frac{10 \text{ cwt.}}{15 \text{ cwt.}} = \frac{2}{3} = \frac{6}{9}.$$

$$\$7 : \$9 = \frac{\$7}{\$9} = \frac{7}{9}. \qquad \therefore \$7 : \$9 \text{ is greater.}$$

6. Which is the greater ratio, 5 dys. : 7 dys. or 8 ft. : 11 ft.?

$$5 \text{ dys.} : 7 \text{ dys.} = \frac{5 \text{ dys.}}{7 \text{ dys.}} = \frac{5}{7} = \frac{55}{77}.$$

$$8 \text{ ft.} : 11 \text{ ft.} = \frac{8 \text{ ft.}}{11 \text{ ft.}} = \frac{8}{11} = \frac{56}{77}. \qquad \therefore 8 \text{ ft.} : 11 \text{ ft. is greater.}$$

7. Which is the greater ratio, 9 yds. : 6 yds. or 5 : 3?

$$9 \text{ yds.} : 6 \text{ yds.} = \frac{9 \text{ yds.}}{6 \text{ yds.}} = \tfrac{9}{6}.$$

$$5 : 3 = \tfrac{5}{3} = \tfrac{10}{6}. \qquad \therefore\ 5 : 3 \text{ is greater.}$$

8. Which is the greater ratio, $\frac{2}{3}$ lb. : $\frac{1}{2}$ lb. or $\frac{5}{8}$ yd. : $\frac{3}{8}$ yd.?

$$\tfrac{2}{3} \text{ lb.} : \tfrac{1}{2} \text{ lb.} = \frac{\frac{2}{3} \text{ lb.}}{\frac{1}{2} \text{ lb.}} = \tfrac{4}{3}.$$

$$\tfrac{5}{8} \text{ yd.} : \tfrac{3}{8} \text{ yd.} = \frac{\frac{5}{8} \text{ yd.}}{\frac{3}{8} \text{ yd.}} = \tfrac{5}{3}. \qquad \therefore\ \tfrac{5}{8} \text{ yd.} : \tfrac{3}{8} \text{ yd. is greater.}$$

Exercise LXIII.

1. If 24 men can finish some work in 14 days, how long will it take 21 men to do it?

$$21 : 24 : : 14 \text{ dys.} : \text{what?}$$

$$\frac{\overset{8}{\cancel{24}} \times \overset{2}{\cancel{14}}}{\underset{3}{\cancel{21}}} \text{ dys.} = 16 \text{ dys. } \textit{Ans.}$$

2. A well is dug in 13 days of 9 hours each. How many days of 10 hours each would it have taken?

$$10 : 9 : : 13 \text{ dys.} : \text{what?}$$

$$\frac{9 \times 13}{10} \text{ dys.} = \frac{117}{10} = 11\tfrac{7}{10} \text{ dys. } \textit{Ans.}$$

3. A man who steps 2 ft. 5 in. takes 2480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?

2 ft. 5 in. = 29 in. 2 ft. 7 in. = 31 in.

$$31 : 29 : : 2480 : \text{what?}$$

$$\frac{29 \times \overset{80}{\cancel{2480}}}{\cancel{31}} = 2320. \qquad 2320 \text{ steps. } \textit{Ans.}$$

4. If $\frac{5}{13}$ ton cost \$6, what will $7\frac{5}{9}$ cwt. cost, at the same rate?

$$7\tfrac{5}{9}\text{ cwt.} = \frac{7\frac{5}{9}}{20}\text{ t.} = \tfrac{17}{45}\text{ t.}$$

$$\tfrac{5}{13} : \tfrac{17}{45} :: \$6 : \text{what?}$$

$$\frac{\frac{17}{45}\times 6}{\frac{5}{13}} = \frac{17\times 13\times \$\overset{2}{\cancel{6}}}{\underset{15}{\cancel{45}}\times 5} = \frac{\$442}{75} = \$5.89.\ \textit{Ans.}$$

5. If 42 yds. of carpet 2 ft. 3 in. wide are required for a room, how many yards 2 ft. 4 in. wide will be required?

2 ft. 3 in. = 27 in. 2 ft. 4 in. = 28 in.

$$28 : 27 :: 42\text{ yds.} : \text{what?}$$

$$\frac{27\times\overset{3}{\cancel{42}}}{\underset{2}{\cancel{28}}} = \frac{81}{2}\text{ yds.} = 40\tfrac{1}{2}\text{ yds. } \textit{Ans.}$$

6. A court was paved with 950 stones, each $1\frac{5}{6}$ sq. ft., and is repaved with 836 stones of a uniform size. Find the size of each.

$$836 : 950 :: 1\tfrac{5}{6}\text{ sq. ft.} : \text{what?}$$

$$\frac{950\times 1\frac{5}{6}}{836} = \frac{\overset{25}{\cancel{950}}\times \cancel{11}}{\underset{\underset{2}{\cancel{22}}}{\cancel{836}}\times 6} = 2\tfrac{1}{12}\text{ sq. ft. } \textit{Ans.}$$

7. If a train, at the rate of $\frac{5}{13}$ of a mile per minute, take $3\frac{1}{4}$ hrs. to reach a station, how long will it take at the rate of $\frac{7}{15}$ of a mile a minute?

$$\tfrac{7}{15} : \tfrac{5}{13} :: 3\tfrac{1}{4}\text{ hrs.} : \text{what?}$$

$$\frac{\frac{5}{13}\times 3\frac{1}{4}}{\frac{7}{15}} = \frac{15}{7}\times\frac{5}{\cancel{13}}\times\frac{\cancel{13}}{4} = \frac{75}{28} = 2\tfrac{19}{28}\text{ hrs. } \textit{Ans.}$$

8. If a post 4 ft. 8 in. high cast a shadow 7 ft. 3 in. long, how long a shadow will a post 11 ft. high cast?

4 ft. 8 in. = $4\frac{2}{3}$ ft. 7 ft. 3 in. = $7\frac{1}{4}$ ft.

$$4\tfrac{2}{3} : 11 :: 7\tfrac{1}{4}\text{ ft.} : \text{what?}$$

$$\frac{11\times 7\frac{1}{4}}{4\frac{2}{3}} = \frac{3\times 11\times 29}{14\times 4} = 17\tfrac{5}{56}\text{ ft.} = 17\text{ ft. } 1\tfrac{1}{14}\text{ in. } \textit{Ans.}$$

9. When a shadow 8 ft. 5 in. long is cast by a post 5 ft. 7 in. high, how high is a steeple that casts a shadow of 211 ft. at the same time?

$$8\tfrac{5}{12} : 211 :: 5\tfrac{7}{12} \text{ ft.} : \text{what?}$$

$$\frac{211 \times 5\frac{7}{12}}{8\frac{5}{12}} = \frac{\cancel{12} \times 211 \times 67}{101 \times \cancel{12}} = 139\tfrac{98}{101} \text{ ft.} = 139 \text{ ft. } 11\tfrac{65}{101} \text{ in.}$$

10. If 4 men can mow a certain field in 10 hrs., how many men will it take to mow it in 5 hrs.?

$$5 : 10 :: 4 \text{ men} : \text{what?} \qquad \frac{\overset{2}{\cancel{10}} \times 4}{\cancel{5}} \text{ men} = 8 \text{ men. } \textit{Ans.}$$

11. A tap discharging 4 gals. a minute empties a cistern in 3 hrs. How long will it take a tap discharging 7 gals. a minute to empty it?

$$7 : 4 :: 3 \text{ hrs.} : \text{what?} \qquad \frac{4 \times 3}{7} \text{ hrs.} = 1\tfrac{5}{7} \text{ hrs. } \textit{Ans.}$$

12. A pipe discharging 3 gals. 1 pt. a minute fills a tub in 4 min. 20 sec. How long will it take a pipe discharging 83 qts. a minute to fill it?

3 gals. 1 pt. = 25 pts. 83 qts. = 166 pts.
4 min. 20 sec. = 260 sec.

$$166 : 25 :: 260 \text{ sec.} : \text{what?} \qquad \frac{25 \times \overset{130}{\cancel{260}}}{\underset{83}{\cancel{166}}} = 39\tfrac{13}{83} \text{ sec. } \textit{Ans.}$$

13. If both pipes of Ex. 12 discharge at the same time into the tub, how long will it take to fill it?

$4\frac{1}{3}$ min. = 260 sec.

$$95\tfrac{1}{2} : 12\tfrac{1}{2} :: 260 \text{ sec.} : \text{what?}$$

$$\frac{12\frac{1}{2} \times 260}{95\frac{1}{2}} = \frac{\cancel{2}}{191} \times \frac{25}{\cancel{2}} \times \frac{260}{1} = \tfrac{6500}{191} \text{ sec.} = 34\tfrac{6}{191} \text{ sec. } \textit{Ans.}$$

14. How long will it take to fill a cistern of 165 gals. by a pipe that fills one of 120 gals. in 7 min. 16 sec.?

16 sec. = $\frac{4}{15}$ min.

$$120 : 165 :: 7\tfrac{4}{15} \text{ min.} : \text{what?}$$

$$\frac{165 \times 7\frac{4}{15}}{120} = \frac{\overset{11}{\cancel{165}} \times 109}{120 \times \cancel{15}} = 9\tfrac{119}{120} \text{ min.} = 9 \text{ min. } 59\tfrac{1}{2} \text{ sec. } \textit{Ans.}$$

15. A ship has sailed 1800 mi. in a fortnight. How long, at the same rate, will it take for a voyage of 5000 mi.?

1800 : 5000 : : 2 wks. : what?

$$\frac{\overset{25}{\cancel{5000}} \times 2}{\underset{9}{\cancel{1800}}} = \frac{50}{9} = 5\tfrac{5}{9} \text{ wks.}$$

$5\frac{5}{9}$ wks. = 5 wks. 4 dys. nearly. *Ans.*

16. The wheels of a carriage are 6 ft. 9 in. and 9 ft. 6 in. in circumference. How many times will the larger turn while the smaller turns 3762 times?

6 ft. 9 in. = $6\frac{3}{4}$ ft. 9 ft. 6 in. = $9\frac{1}{2}$ ft.

$9\frac{1}{2} : 6\frac{3}{4}$: : 3762 : what?

$$\frac{6\frac{3}{4} \times 3762}{9\frac{1}{2}} = \frac{\cancel{2} \times 27 \times \overset{99}{\cancel{\overset{198}{\cancel{3762}}}}}{\cancel{19} \times \underset{2}{\cancel{4}}} = 2673. \ \textit{Ans.}$$

17. If $\frac{3}{25}$ of a ship be worth \$2167, what is the value of $\frac{7}{17}$ of it?

$\frac{3}{25} : \frac{7}{17}$: : \$2167 : what?

$$\frac{\frac{7}{17} \times \$2167}{\frac{3}{25}} = \frac{25 \times 7 \times 2167}{3 \times 17} = \frac{\$379225}{51} = \$7435.78. \ \textit{Ans.}$$

18. What will be the weight of 18 cu. ft. 432 cu. in. of stone of which 10 cu. ft. 864 cu. in. weigh 14 cwt. 7 lbs.?

$10\frac{1}{2} : 18\frac{1}{4}$: : 1407 lbs. : what?

$$\frac{18\frac{1}{4} \times 1407}{10\frac{1}{2}} = \frac{\cancel{2} \times 73 \times \overset{67}{\cancel{1407}}}{\cancel{21} \times \underset{2}{\cancel{4}}} = \frac{4891}{2} \text{ lbs.} = 2445\tfrac{1}{2} \text{ lbs.}$$

$$= 1 \text{ t. } 4 \text{ cwt. } 45\tfrac{1}{2} \text{ lbs. } \textit{Ans.}$$

19. If 280 lbs. of flour make 360 lbs. of bread, how many four pound loaves can be made from 1 cwt. of flour?

280 : 100 : : 360 lbs. : what?

$$\frac{100 \times \overset{9}{\cancel{360}}}{\underset{7}{\cancel{280}}} = \frac{900}{7} = 128\tfrac{4}{7} \text{ lbs.} \qquad 128\tfrac{4}{7} \div 4 = 32\tfrac{1}{7}. \ \textit{Ans.}$$

20. If a column of mercury 27.93 in. high weigh 0.76 of a pound, what will be the weight of a column of the same diameter 29.4 in. high?

$$27.93 : 29.4 :: 0.76 \text{ lb.} : \text{what?}$$

$$\frac{\overset{0.2}{\cancel{29.4}} \times \overset{4}{\cancel{0.76}}}{\underset{0.19}{\cancel{27.93}}} \text{ lb.} = 0.8 \text{ lb. } \textit{Ans.}$$

21. How many francs will pay a bill of £100, when £42 10*s.* 8*d.* is equivalent to 1090.98 francs?

$$£42\ 10s.\ 8d. = £42\tfrac{8}{15}.$$

$$42\tfrac{8}{15} : 100 :: 1090.98 \text{ francs} : \text{what?}$$

$$\frac{1090.98 \times 100}{42\frac{8}{15}} \text{ francs} = \frac{15}{\cancel{638}} \times \frac{\overset{171}{\cancel{109098}}}{\cancel{100}} \times \frac{\cancel{100}}{1} = 2565 \text{ francs.}$$

22. What will be the weight of a cube whose edge is 2 ft. 2 in., when a cube of the same material whose edge is 1 ft. 4 in. weighs 537.6 lbs.?

$$2 \text{ ft. } 2 \text{ in.} = 2\tfrac{1}{6} \text{ ft.} \qquad 1 \text{ ft. } 4 \text{ in.} = 1\tfrac{1}{3} \text{ ft.}$$

$$(1\tfrac{1}{3})^3 : (2\tfrac{1}{6})^3 :: 537.6 \text{ lbs.} : \text{what?}$$

$$\tfrac{64}{27} : \tfrac{2197}{216} :: 537.6 \text{ lbs.} : \text{what?}$$

$$\frac{\cancel{27}}{\underset{2}{\cancel{64}}} \times \frac{2197}{\underset{8}{\cancel{216}}} \times \frac{\overset{21}{\cancel{\overset{672}{\cancel{5376}}}}}{10} = \frac{46137}{20} \text{ lbs.} = 2306.85 \text{ lbs. } \textit{Ans.}$$

23. If a square field measuring 50 yds. $10\frac{2}{7}$ in. on each side be worth $\$2710\frac{10}{17}$, what is the value of a square field 62 yds. 1 ft. each way?

$$50 \text{ yds. } 10\tfrac{2}{7} \text{ in.} = 50\tfrac{2}{7} \text{ yds.} \qquad 52 \text{ yds. } 1 \text{ ft.} = 62\tfrac{1}{3} \text{ yds.}$$

$$(50\tfrac{2}{7})^2 : (62\tfrac{1}{3})^2 :: \$2710\tfrac{10}{17} : \text{what?}$$

$$\frac{123904}{49} : \frac{34969}{9} :: \frac{\$46080}{17} : \text{what?}$$

$$\frac{49}{\underset{121}{\cancel{123904}}} \times \frac{\overset{17}{\cancel{\overset{2057}{\cancel{34969}}}}}{\cancel{9}} \times \frac{\overset{5}{\cancel{\overset{5120}{\cancel{46080}}}}}{\cancel{17}} = \$4165. \ \textit{Ans.}$$

24. A gains 4 yds. on B in running 30 yds. How much will he gain while B is running $97\frac{1}{2}$ yds.?

$$30 : 97\tfrac{1}{2} :: 4 \text{ yds.} : \text{what?}$$

$$\frac{\overset{13}{\cancel{195}} \times \overset{2}{\cancel{4}}}{\cancel{2} \times \underset{\cancel{15}}{\cancel{30}}} \text{ yds.} = 13 \text{ yds. } \textit{Ans.}$$

25. If 10 cu. in. of gold weigh as much as 193 cu. in. of water, what is the size of a nugget weighing as much as a cubic foot of water?

$$193 : 1728 :: 10 \text{ cu. in.} : \text{what?}$$

$$\frac{1728 \times 10}{193} = \frac{17280}{193} = 89\tfrac{103}{193} \text{ cu. in. } \textit{Ans.}$$

26. If a garrison of 1500 men have provisions for 13 months, how long will the provisions last if it be increased by 700 men?

$$1500 + 700 = 2200.$$

$$2200 : 1500 :: 13 \text{ mo.} : \text{what?}$$

$$\frac{\overset{15}{\cancel{1500}} \times 13}{\underset{22}{\cancel{2200}}} \text{ mo.} = \tfrac{195}{22} \text{ mo.} = 8\tfrac{19}{22} \text{ mo. } \textit{Ans.}$$

27. If a tree 38 ft. high be represented by a drawing $1\frac{1}{2}$ in. high, what, on the same scale, will represent the height of a house 45 ft. high?

$$38 : 45 :: 1\tfrac{1}{2} \text{ in.} : \text{what?}$$

$$\frac{45 \times 3}{38 \times 2} \text{ in.} = \tfrac{135}{76} \text{ in.} = 1\tfrac{59}{76} \text{ in. } \textit{Ans.}$$

28. If a country 630 mi. long be represented on a raised map by a length of $5\frac{1}{2}$ ft., by what height ought a mountain of 15,750 ft. be represented on the map?

630 mi. = 3,326,400 ft. $\qquad 5\frac{1}{2}$ ft. = 66 in.

$$3326400 : 15750 :: 66 \text{ in.} : \text{what?}$$

$$\frac{\overset{5}{\cancel{15750}} \times \cancel{66}}{\underset{\underset{16}{\cancel{50400}}}{\cancel{3326400}}} \text{ in.} = \tfrac{5}{16} \text{ in. } \textit{Ans.}$$

29. A train travels $\frac{1}{4}$ of a mile in 18 sec. How many miles an hour does it travel?

$$1 \text{ hr.} = 3600 \text{ sec.} \qquad 18 : 3600 :: \tfrac{1}{4} \text{ mi.} : \text{what?}$$

$$\frac{\overset{200}{\cancel{3600}} \times \frac{1}{4}}{\cancel{18}} = 50 \text{ mi. } \textit{Ans.}$$

30. If $4\frac{1}{2}$ tons of coal fill a bin 9 ft. long, 5 ft. broad, 5 ft. high, how many cubic feet will be required for the coal of a steamer carrying 3 weeks' consumption at 20 tons a day?

$$9 \times 5 \times 5 = 225 \text{ cu. ft.} \qquad 3 \text{ wks.} = 21 \text{ dys.}$$

$$21 \times 20 \text{ t.} = 420 \text{ t.} \qquad 4\tfrac{1}{2} : 420 :: 225 \text{ cu. ft.} : \text{what?}$$

$$\frac{420 \times \overset{25}{\cancel{225}} \times 2}{\cancel{9}} \text{ cu ft.} = 21{,}000 \text{ cu. ft. } \textit{Ans.}$$

31. If 2 lbs. of rosin be melted with 5 oz. of mutton tallow, to make a grafting wax, how many ounces of tallow will 20 oz. of the wax contain?

$$2 \text{ lbs.} + 5 \text{ oz.} = 2 \text{ lbs. } 5 \text{ oz.} = 37 \text{ oz.}$$

$$37 : 20 :: 5 \text{ oz.} : \text{what?}$$

$$\frac{20 \times 5}{37} = \frac{100}{37} \text{ oz.} = 2\tfrac{26}{37} \text{ oz. } \textit{Ans.}$$

Exercise LXIV.

1. How many days 8 hours long will 60 men take to finish some work which 24 men can do in 15 days, working 10 hours a day?

$$\left.\begin{array}{r|l} 8 & 10 \\ 60 & 24 \end{array}\right. :: 15 \text{ dys.} : \text{what?}$$

$$\frac{\overset{5}{\cancel{10}} \times \overset{3}{\cancel{24}} \times \cancel{15}}{\cancel{8} \times \underset{\underset{2}{\cancel{4}}}{\cancel{60}}} \text{ dys.} = \tfrac{15}{2} \text{ dys.} = 7\tfrac{1}{2} \text{ dys. } \textit{Ans.}$$

2. What will be the expense of covering a room with drugget 4 ft. wide, at 91⅔ cts. a yard, when carpet 2 ft. 3 in. wide for the room costs $70.50, at $1.37½ a yard?

$$\$0.91\tfrac{2}{3} = \$\tfrac{11}{12}. \qquad \$1.37\tfrac{1}{2} = \$1\tfrac{3}{8}.$$

$$\left.\begin{array}{r|l} 4 & 2\frac{1}{4} \\ 1\frac{3}{8} & \frac{11}{12} \end{array}\right. :: \$70\tfrac{1}{2} : \text{what?}$$

$$\frac{\overset{3}{\cancel{9}}}{4} \times \frac{11}{\underset{4}{\cancel{12}}} \times \frac{141}{\cancel{2}} \times \frac{1}{\cancel{4}} \times \frac{\overset{\cancel{2}}{\cancel{8}}}{11} = \frac{\$423}{16} = \$26.44. \text{ *Ans.*}$$

3. If 4418 tons of iron ore produce $36,190 worth of metal, when iron is at $37.50 a ton, what will be the value of the iron from 2275 tons of ore, at $47 a ton?

$$\left.\begin{array}{r|l} 37\frac{1}{2} & 47 \\ 4418 & 2275 \end{array}\right. :: \$36{,}190 : \text{what?}$$

$$\frac{2 \times \cancel{47} \times \overset{91}{\cancel{2275}} \times \$\overset{335}{\cancel{36190}}}{\underset{3}{\cancel{75}} \times \underset{94}{\cancel{4418}}} = \frac{\$70070}{3} = \$23{,}356.67. \text{ *Ans.*}$$

4. If a bar of iron 3⅓ ft. long, 3 in. wide, 2¾ in. thick weigh 93 lbs., what will be the weight of a bar 3⅔ ft. long, 4 in. wide, and 2½ in. thick?

$$\left.\begin{array}{r|l} 3\frac{1}{3} & 3\frac{2}{3} \\ 3 & 4 \\ 2\frac{3}{4} & 2\frac{1}{2} \end{array}\right. :: 93 \text{ lbs.} : \text{what?}$$

$$\frac{\cancel{11}}{\cancel{3}} \times \frac{\overset{\cancel{2}}{\cancel{4}}}{\cancel{3}} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{31}{\cancel{93}}}{1} \times \frac{\cancel{3}}{\underset{\cancel{2}}{\cancel{10}}} \times \frac{4}{\cancel{11}} = 124 \text{ lbs. *Ans.*}$$

5. If 40 bu. of wheat can be grown on the same area as 48 bu. of barley, and 28 acres produce 840 bu. of wheat, how much barley will be obtained from 38 acres?

$$\left.\begin{array}{r|l} 40 & 48 \\ 28 & 38 \end{array}\right. :: 840 \text{ bu.} : \text{what?} \qquad \frac{\overset{6}{\cancel{48}} \times 38 \times \overset{\overset{6}{\cancel{30}}}{\cancel{840}}}{\underset{\cancel{5}}{\cancel{40}} \times \cancel{28}} = 1368 \text{ bu.}$$

6. If 18 men can dig a trench 150 ft. long, 6 ft. broad, and 4 ft. 6 in. deep in 12 days, how long will 16 men take for a trench 210 ft. long, 5 ft. broad, and 4 ft. deep?

$$\begin{array}{r|l} 16 & 18 \\ 150 & 210 \\ 6 & 5 \\ 4\frac{1}{2} & 4 \end{array} :: 12 \text{ dys.} : \text{what?}$$

$$\frac{\overset{4}{\cancel{18}} \times \overset{7}{\cancel{210}} \times \cancel{5} \times \cancel{4} \times \overset{2}{\cancel{12}}}{\underset{4}{\cancel{16}} \times \underset{30}{\cancel{150}} \times \cancel{6} \times \cancel{4\frac{1}{2}}} \text{ dys.} = 14 \text{ dys. } \textit{Ans.}$$

7. In the reprint of a book consisting of 810 pages, 50 lines are contained in a page, instead of 40, and 72 letters in a line, instead of 60. Of how many pages will the new edition consist?

$$\begin{array}{r|l} 50 & 40 \\ 72 & 60 \end{array} :: 810 : \text{what?} \qquad \frac{\overset{10}{\cancel{40}} \times \overset{6}{\cancel{60}} \times \overset{\overset{9}{\cancel{45}}}{\cancel{810}}}{\underset{5}{\cancel{50}} \times \underset{4}{\cancel{72}}} = 540. \textit{ Ans.}$$

8. If 3280 42-lb. shot cost \$3000, how many 32-lb. shot can be bought for \$4200?

$$\begin{array}{r|l} 3000 & 4200 \\ 32 & 42 \end{array} :: 3280 : \text{what?} \qquad \frac{\overset{7}{\cancel{4200}} \times \overset{21}{\cancel{42}} \times \overset{\overset{41}{\cancel{205}}}{\cancel{3280}}}{\underset{5}{\cancel{3000}} \times \underset{2}{\cancel{32}}} = 6027.$$

9. What must be the rate of wages, that 12 men may earn in 10 days the same amount that 9 men earn in 14 days, at \$1.50 a day?

$$\begin{array}{r|l} 12 & 9 \\ 10 & 14 \end{array} :: \$1.50 : \text{what?}$$

$$\frac{9 \times \overset{7}{\cancel{14}} \times \$\overset{\overset{0.05}{0.\cancel{15}}}{\cancel{1.50}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{12}} \times \underset{\cancel{2}}{\cancel{10}}} = \frac{\$3.15}{2} = \$1.575. \textit{ Ans.}$$

10. A reservoir 15 yds. long and 4 ft. deep holds 32,500 gals. Determine the quantity of water it will hold when it has been increased in length by 18 ft. and in depth 1 ft.

$$\left.\begin{array}{r|l} 15 & 21 \\ 4 & 5 \end{array}\right. :: 32{,}500 \text{ gals.} : \text{what?}$$

$$\frac{\overset{7}{\cancel{21}} \times \cancel{5} \times \overset{8125}{\cancel{32500}}}{\underset{\cancel{3}}{\cancel{15}} \times \cancel{4}} \text{ gals.} = 56{,}875 \text{ gals. } \textit{Ans.}$$

11. How far can A, who takes 3.1 ft. each step, run, while B, who takes 2.3 ft. each step, runs 220 yds., if A takes 7 steps while B takes 11?

$$\left.\begin{array}{r|l} 2.3 & 3.1 \\ 11 & 7 \end{array}\right. :: 220 \text{ yds.} : \text{what?}$$

$$\frac{3.1 \times 7 \times \overset{20}{\cancel{220}}}{2.3 \times \cancel{11}} \text{ yds.} = \frac{434}{2.3} \text{ yds.} = 188\tfrac{16}{23} \text{ yds. } \textit{Ans.}$$

12. If 6 hours be required for travelling a given distance at a given rate, how long will be required when the distance is diminished by one-fourth and the rate is increased by one-half?

$$\left.\begin{array}{r|l} 1 & \frac{3}{4} \\ 1\frac{1}{2} & 1 \end{array}\right. :: 6 \text{ hrs.} : \text{what?} \qquad \frac{3}{\underset{\cancel{2}}{\cancel{4}}} \times \overset{\cancel{2}}{\cancel{6}} \times \frac{\cancel{2}}{\cancel{3}} \text{ hrs.} = 3 \text{ hrs. } \textit{Ans.}$$

13. How many hours a day must 5 men work to mow the same quantity of grass in 8 days that 7 men can mow in 6 days, working 10 hours a day?

$$\left.\begin{array}{r|l} 5 & 7 \\ 8 & 6 \end{array}\right. :: 10 \text{ hrs.} : \text{what?} \qquad \frac{7 \times \overset{3}{\cancel{6}} \times \overset{\cancel{2}}{\cancel{10}}}{\cancel{5} \times \underset{\substack{\cancel{4} \\ 2}}{\cancel{8}}} \text{ hrs.} = \tfrac{21}{2} \text{ hrs.} = 10\tfrac{1}{2} \text{ hrs.}$$

14. If a bar 10 ft. $6\frac{1}{2}$ in. long, $3\frac{5}{8}$ in. broad, $3\frac{1}{2}$ in. thick weigh 4 cwt. 8.23 lbs., what length must be taken to weigh a long ton when the breadth and thickness are $4\frac{2}{3}$ in. and $4\frac{1}{8}$ in. respectively?

1 l. t. = 2240 lbs. 4 cwt. 8.23 lbs. = 408.23 lbs.

$$\begin{array}{r|l} 40823 & 224000 \\ 4\frac{2}{3} & 3\frac{5}{8} \\ 4\frac{1}{8} & 3\frac{1}{2} \end{array} \;::\; 10\tfrac{13}{24}\text{ ft.} : \text{what?}$$

$$\frac{\overset{\overset{\overset{7000}{\cancel{14000}}}{\cancel{28000}}}{\cancel{224000}} \times \cancel{3} \times \cancel{8} \times 29 \times \cancel{7} \times \overset{23}{\cancel{253}}}{40823 \times \underset{2}{\cancel{14}} \times \underset{\cancel{11}}{\cancel{33}} \times \cancel{8} \times \cancel{2} \times \underset{3}{\cancel{24}}}\text{ ft.} = 38\tfrac{15178}{122469} = 38\text{ ft. } 1.5\text{ in.}$$

15. If 27 men, in 28 days of 10 hours each, dig a trench 126 yds. long, $2\frac{1}{2}$ yds. broad, $1\frac{1}{2}$ yds. deep, how long a trench $2\frac{3}{4}$ yds. broad, $1\frac{3}{4}$ yds. deep, will 56 men dig in 25 days of $8\frac{1}{4}$ hours?

$$\begin{array}{r|l} 27 & 56 \\ 10 & 8\frac{1}{4} \\ 28 & 25 \\ 2\frac{3}{4} & 2\frac{1}{2} \\ 1\frac{3}{4} & 1\frac{1}{2} \end{array} \;::\; 126\text{ yds.} : \text{what?}$$

$$\frac{\cancel{4} \times \overset{\cancel{2}}{\cancel{4}} \times \overset{\cancel{2}}{\cancel{56}} \times \overset{\cancel{3}}{\cancel{33}} \times 25 \times \cancel{5} \times \cancel{3} \times \overset{\overset{6}{\cancel{42}}}{\cancel{126}}}{\cancel{11} \times \cancel{7} \times \underset{\underset{\cancel{3}}{\cancel{9}}}{\cancel{27}} \times \underset{\cancel{2}}{\cancel{10}} \times \cancel{4} \times \cancel{28} \times \cancel{2} \times \cancel{2}}\text{ yds.} = 150\text{ yds. } \textit{Ans.}$$

16. What must be the length of a bar of silver $\frac{3}{4}$ in. square, that it may weigh the same as a bar of gold $\frac{1}{2}$ in. square and $6\frac{3}{4}$ in. long, if the weight of a cubic inch of silver have to that of a cubic inch of gold the ratio 47 : 88?

$$\begin{array}{r|l} (\frac{3}{4})^2 & (\frac{1}{2})^2 \\ 47 & 88 \end{array} \;::\; 6\tfrac{3}{4}\text{ in.} : \text{what?} \qquad \begin{array}{r|l} \frac{9}{16} & \frac{1}{4} \\ 47 & 88 \end{array} \;::\; 6\tfrac{3}{4}\text{ in.} : \text{what?}$$

$$\frac{\overset{4}{\cancel{16}} \times \overset{22}{\cancel{88}} \times \overset{3}{\cancel{27}}}{\cancel{9} \times 47 \times \cancel{4} \times \cancel{4}}\text{ in.} = 5\tfrac{29}{47}\text{ in. } \textit{Ans.}$$

17. If it take 34^{kg} of wool to make 25^{m} of cloth 0.6^{m} wide, how long a piece of cloth 0.8^{m} wide can be made from 108.8^{kg} of wool?

$$\begin{array}{r|l} 34 & 108.8 \\ 0.8 & 0.6 \end{array} \;::\; 25^{m} : \text{what?} \qquad \frac{\overset{\overset{4}{\cancel{136}}}{\cancel{108.8}} \times 0.6 \times 25}{\cancel{34} \times 0.\cancel{8}} = 60^{m}.\ \textit{Ans.}$$

1+.00367 x 27.8 = 1.102026
1.+.00367 x100 = 1.367

18. An oak beam 5.40^{m} long, 0.63^{m} thick, and 0.57^{m} wide weighs 1469.25^{kg}; find the weight of a beam whose dimensions are 4.87^{m}, 0.58^{m}, 0.53^{m}.

$$\left.\begin{array}{r|l} 5.4 & 4.87 \\ 0.63 & 0.58 \\ 0.57 & 0.53 \end{array}\right. :: 1469\tfrac{1}{4}^{kg} : \text{what?}$$

$$\frac{487\times \overset{29}{\cancel{58}}\times 53\times \overset{653}{\cancel{5877}}^{kg}}{\underset{\underset{30}{\cancel{270}}}{\cancel{540}}\times 63\times 57\times 4} = \frac{488782907^{kg}}{430920} = 1134.2776^{kg}. \quad \textit{Ans.}$$

19. A certain quantity of air has a volume of 195.5 cu. ft. at 27.8°. What will be its volume at 100°?

~~100° − 27.8° = 72.2°. 72.2 × 0.00367 = 0.264974.~~

~~1 : 1.264974 :: 195½ cu. ft. : what?~~

$$\cancel{\frac{1.264974\times 195\tfrac{1}{2}}{1}\text{ cu. ft.}} = 247.3 \text{ cu. ft. } \cancel{\textit{Ans}}.$$

20. A quantity of air at a temperature of 15.6° C. has a volume of 4 cu. ft. under a pressure of 12 lbs. to the square inch. What will be its volume at a temperature of 48.7° C., and under a pressure of 14 lbs. the square inch?

$48.7° - 15.6° = 33.1°$. $\quad 33.1 \times 0.00367 = 0.121477$.

$$\left.\begin{array}{r|l} 14 & 12 \\ 1 & 1.121477 \end{array}\right. :: 4 \text{ cu. ft.} : \text{what?}$$

$$\frac{\overset{3}{\cancel{12}}\times 1121477\times \overset{2}{\cancel{4}}}{\underset{7}{\cancel{14}}\times \underset{\underset{125000}{\cancel{500000}}}{\cancel{1000000}}}\text{ cu. ft.} = \frac{3364431}{875000}\text{ cu. ft.} = 3.8 \text{ cu. ft. } \quad \textit{Ans.}$$

Exercise LXV.

1. Divide \$12,000 proportionally to the numbers 3, 4, 5.

$3 + 4 + 5 = 12$.

$$\frac{3}{\cancel{12}}\times\frac{\overset{1000}{\cancel{\$12000}}}{1} = \$3000.$$

$$\frac{4}{\cancel{12}}\times\frac{\overset{1000}{\cancel{\$12000}}}{1} = \$4000.$$

$$\frac{5}{\cancel{12}}\times\frac{\overset{1000}{\cancel{\$12000}}}{1} = \$5000.$$

1.10[illegible] : 1.3[illegible] :: [illegible] : 242[illegible]

2. Divide 815 tons proportionally to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.

$(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}) \times 60 = 30, 40, 45, 48.$

$30 + 40 + 45 + 48 = 163.$

$$\frac{30}{\cancel{163}} \times \frac{\overset{5}{\cancel{815}}}{1} = 150 \text{ t.}$$

$$\frac{40}{\cancel{163}} \times \frac{\overset{5}{\cancel{815}}}{1} = 200 \text{ t.}$$

$$\frac{45}{\cancel{163}} \times \frac{\overset{5}{\cancel{815}}}{1} = 225 \text{ t.}$$

$$\frac{48}{\cancel{163}} \times \frac{\overset{5}{\cancel{815}}}{1} = 240 \text{ t.}$$

3. Divide 6853 lbs. of wool proportionally to $1\frac{3}{4}$, $2\frac{4}{5}$, $5\frac{5}{6}$; and also proportionally to the reciprocals of these numbers.

$(\frac{7}{4}, \frac{14}{5}, \frac{35}{6}) \times 60 = 105, 168, 350.$

$105 + 168 + 350 = 623.$

$$\frac{105}{\cancel{623}} \times \frac{\overset{11}{\cancel{6853}}}{1} = 1155 \text{ lbs.}$$

$$\frac{168}{\cancel{623}} \times \frac{\overset{11}{\cancel{6853}}}{1} = 1848 \text{ lbs.}$$

$$\frac{350}{\cancel{623}} \times \frac{\overset{11}{\cancel{6853}}}{1} = 3850 \text{ lbs.}$$

The reciprocals of

$1\frac{3}{4}, 2\frac{4}{5}, 5\frac{5}{6} = \frac{4}{7}, \frac{5}{14}, \frac{6}{35}.$

$(\frac{4}{7}, \frac{5}{14}, \frac{6}{35}) \times 70 = 40, 25, 12.$

$40 + 25 + 12 = 77.$

$$\frac{40}{\cancel{77}} \times \frac{\overset{89}{\cancel{6853}}}{1} = 3560 \text{ lbs.}$$

$$\frac{25}{\cancel{77}} \times \frac{\overset{89}{\cancel{6853}}}{1} = 2225 \text{ lbs.}$$

$$\frac{12}{\cancel{77}} \times \frac{\overset{89}{\cancel{6853}}}{1} = 1068 \text{ lbs.}$$

4. Two persons join in purchasing some property, one paying \$1250, and the other \$1000. If the property rise in value to \$3600, what will be the value of each one's share?

$\$1000 + \$1250 = \$2250.$

$$\frac{\overset{200}{\cancel{1000}}}{\underset{\cancel{5}}{\cancel{2250}}} \times \frac{\overset{8}{\cancel{\$3600}}}{1} = \$1600.$$

$$\frac{\overset{250}{\cancel{1250}}}{\underset{\cancel{5}}{\cancel{2250}}} \times \frac{\overset{8}{\cancel{\$3600}}}{1} = \$2000.$$

5. Gun-metal is composed of 3 parts (by weight) of tin to 100 parts of copper. What weight of each of these metals will there be in cannon weighing 721 lbs.?

$3 + 100 = 103.$

$$\frac{3}{\cancel{103}} \times \frac{\overset{7}{\cancel{721}}}{1} = 21 \text{ lbs. tin.}$$

$$\frac{100}{\cancel{103}} \times \frac{\overset{7}{\cancel{721}}}{1} = 700 \text{ lbs. cop.}$$

6. Bell-metal contains 78 parts copper and 22 parts tin. What weight of each of these metals will there be in a bell weighing 937 lbs.?

$$78 + 22 = 100.$$

$$\tfrac{78}{100} \times 937 = 730.86 \text{ lbs. copper.}$$

$$\tfrac{22}{100} \times 937 = 206.14 \text{ lbs. tin.}$$

7. It takes 75^{kg} of saltpetre, 12.5^{kg} of charcoal, and 12.5^{kg} of sulphur to make 100^{kg} of powder. How much of each of these substances will be required to make 10,000,000 cartridges, each containing 5^{g} of powder?

$$75 + 12\tfrac{1}{2} + 12\tfrac{1}{2} = 100.$$

$$10{,}000{,}000 \times 5^{g} = 50{,}000{,}000^{g} = 50{,}000^{kg}.$$

$$\frac{75}{\cancel{100}} \times \frac{\overset{500}{\cancel{50000}}}{1} = 37{,}500^{kg} \text{ saltpetre.}$$

$$\frac{125}{\cancel{1000}} \times \frac{\overset{50}{\cancel{50000}}}{1} = 6250^{kg} \begin{cases} \text{charcoal.} \\ \text{sulphur.} \end{cases}$$

8. Yellow copper contains 2 parts of red copper and 1 part zinc. How many ounces of red copper are there in an article weighing 1 lb. made of yellow copper?

$$1 \text{ lb.} = 16 \text{ oz.}$$

$$2 + 1 = 3.$$

$$\tfrac{2}{3} \times \tfrac{16}{1} = \tfrac{32}{3} = 10\tfrac{2}{3} \text{ oz. } \textit{Ans.}$$

9. Type-metal is made of an alloy containing 39 parts of lead to 11 parts antimony. How many pounds of each will be required to make 957 lbs. of type?

$$39 + 11 = 50.$$

$$\tfrac{39}{50} \times \tfrac{957}{1} = \tfrac{37323}{50} = 746.46 \text{ lbs. lead.}$$

$$\tfrac{11}{50} \times \tfrac{957}{1} = \tfrac{10527}{50} = 210.54 \text{ lbs. antimony.}$$

10. Plumber's solder contains 2 parts lead and 1 part tin. How much of each of these in 100 lbs. of solder?

$2 + 1 = 3.$ $\quad \frac{2}{3} \times 100 = 66\frac{2}{3}$ lbs. lead. $\quad \frac{1}{3} \times 100 = 33\frac{1}{3}$ lbs. tin.

11. The air is composed of oxygen and nitrogen. In 100 volumes of air there are 21 volumes of oxygen and 79 of nitrogen. Reckoning the weight of a liter of oxygen to be 1.4295^g, that of a liter of nitrogen 1.2577^g, find the number of grams of each gas in 100^g of air.

$$21 \times 1.4295 = 30.0195.$$
$$79 \times 1.2577 = 99.3583.$$
$$30.0195 + 99.3583 = 129.3778.$$
$$\frac{300195}{1293778} \text{ of } 100^g = \frac{30019500^g}{1293778} = 23.203^g \text{ O.}$$
$$100^g - 23.203^g = 76.797^g \text{ H.}$$

12. What is the value of the gold in a chain weighing 3 oz. 4 dwt., supposing it to be 18 carats fine (that is, 18 parts of pure gold out of 24), at \$19 an ounce?

$$3 \text{ oz. } 4 \text{ dwt.} = 3\tfrac{1}{5} \text{ oz.}$$
$$3\tfrac{1}{5} \times \$19 = \$60\tfrac{4}{5}.$$
$$\tfrac{18}{24} \text{ of } \$60\tfrac{4}{5} = \$45.60. \; \textit{Ans.}$$

Exercise LXVI.

1. Arnold and Baker enter into partnership. Arnold puts in \$6000 for 8 months, and Baker \$4000 for 6 months. Their profits are \$2000. What is each man's share?

$$8 \times \$6000 = \$48,000.$$
$$6 \times \$4000 = \$24,000.$$
$$48,000 + 24,000 = 72,000.$$
$$\frac{\overset{2}{\cancel{48000}}}{\underset{3}{\cancel{72000}}} \times \frac{\$2000}{1} = \$1333.33, \text{ A's.}$$
$$\frac{\cancel{24000}}{\underset{3}{\cancel{72000}}} \times \frac{\$2000}{1} = \$666.67, \text{ B's.}$$

2. Dobson furnishes the firm of Dobson & Fogg with $5000 for 13 months; Fogg furnishes $7000 for 9 months. Their profits are $1700. What is the share of each?

$$13 \times \$5000 = \$65,000. \qquad 9 \times \$7000 = \$63,000.$$

$$65,000 + 63,000 = 128,000.$$

$$\frac{\overset{65}{\cancel{65000}}}{\underset{\underset{32}{\cancel{128}}}{\cancel{128000}}} \times \frac{\overset{425}{\$\cancel{1700}}}{1} = \frac{\$37625}{32} = \$863.28. \text{ Dobson's.}$$

$$\$1700 - \$863.28 = \$836.72. \text{ Fogg's.}$$

3. In a business speculation, A furnishes $800, and after 3 months $250 more; B furnishes $950, and at the end of 2 months withdraws $200; C furnishes $650, and at the end of 6 months $400 more. At the end of a year they realize a profit of $2516. How shall it be divided among them?

A.	B.	C.
$12 \times \$800 = \$9600$	$12 \times \$950 = \$11400$	$12 \times \$650 = \$7800$
$9 \times 250 = 2250$	$10 \times 200 = 2000$	$6 \times 400 = 2400$
$\$11850$	$\$9400$	$\$10200$

$$\$11,850 + \$9400 + \$10,200 = \$31,450.$$

$$\frac{\overset{237}{\cancel{11850}}}{\underset{\cancel{629}}{\cancel{31450}}} \times \frac{\overset{4}{\$\cancel{2516}}}{1} = \$948, \text{ A's.} \qquad \frac{\overset{188}{\cancel{9400}}}{\underset{\cancel{629}}{\cancel{31450}}} \times \frac{\overset{4}{\$\cancel{2516}}}{1} = \$752, \text{ B's.}$$

$$\frac{\overset{204}{\cancel{10200}}}{\underset{\cancel{629}}{\cancel{31450}}} \times \frac{\overset{4}{\$\cancel{2516}}}{1} = \$816, \text{ C's.}$$

4. Two partners, A and B, begin business with capitals of $3500 and $8700, and A is to have 0.12 of the profits for managing the business. How shall a profit of $1906.25 be divided between them?

$$0.12 \text{ of } \$1906.25 = \$228.75. \qquad \$1906.25 - \$228.75 = \$1677.50.$$

$$3500 + 8700 = 12200.$$

$$\frac{\overset{35}{\cancel{3500}}}{\underset{122}{\cancel{12200}}} \times \frac{\$2355}{2} = \$481.25.$$

$$\$481.25 + \$228.75 = \$710, \text{ A's.}$$

$$\$1906.25 - \$710 = \$1196.25, \text{ B's.}$$

5. A puts \$2100 into a business, and B \$1750. At the end of a year each puts in \$700 more, and C joins them with \$2500. At the end of 18 months from this time how shall a profit of \$2166.50 be divided?

A.	B.	C.
$30 \times \$2100 = \$63,000$	$30 \times \$1750 = \$52,500$	$18 \times \$2500 = \$45,000.$
$18 \times 700 = 12,600$	$18 \times 700 = 12,600$	
$\$75,600$	$\$65,100$	

$$7560 + 65,100 + 45,000 = 185,700.$$

$$\frac{\overset{\overset{\overset{63}{\cancel{189}}}{\cancel{756}}}{\cancel{75600}}}{\underset{\underset{\cancel{15475}}{\cancel{46425}}}{\cancel{185700}}} \times \frac{\overset{14}{\cancel{\$216650}}}{\cancel{100}} = \$882, \text{ A's.} \qquad \frac{\overset{\overset{217}{\cancel{651}}}{\cancel{65100}}}{\underset{\underset{2}{\cancel{61900}}}{\cancel{185700}}} \times \frac{\overset{7}{\cancel{\$216650}}}{\cancel{100}} = \$759.50, \text{ B's.}$$

$$\frac{\overset{\overset{\overset{3}{\cancel{15}}}{\cancel{450}}}{\cancel{45000}}}{\underset{\underset{\cancel{1238}}{\cancel{6190}}}{\cancel{185700}}} \times \frac{\overset{175}{\cancel{\$216650}}}{\cancel{100}} = \$525, \text{ C's.}$$

6. Three graziers hire a pasture, for which they pay \$132.50. One puts in 10 oxen for 3 months, another 12 oxen for 4 months, and the third 14 oxen for 2 months. How much of the rent ought each to pay?

$$\begin{aligned} 3 \times 10 &= 30 \\ 4 \times 12 &= 48 \\ 2 \times 14 &= \underline{28} \\ & \ 106 \end{aligned}$$

$$\frac{\overset{12}{\cancel{48}}}{\cancel{106}} \times \frac{\overset{\overset{5}{\cancel{125}}}{\cancel{\$13250}}}{\underset{\cancel{4}}{\cancel{100}}} = \$60. \ (2)$$

$$\frac{\overset{15}{\cancel{30}}}{\cancel{106}} \times \frac{\overset{\overset{5}{\cancel{125}}}{\cancel{\$13250}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{100}}} = \$37.50. \ (1) \qquad \frac{\overset{7}{\cancel{28}}}{\cancel{106}} \times \frac{\overset{\overset{5}{\cancel{125}}}{\cancel{\$13250}}}{\underset{\cancel{4}}{\cancel{100}}} = \$35. \ (3)$$

7. A begins business, with a capital of \$2400, on the 19th of March; and on the 17th of July admits B as a partner, with a capital of \$1800. Dec. 31 the profits are \$943. What is the share of each?

From March 19 to Dec. 31 is 288 dys.

From July 17 to Dec. 31 is 168 dys.

$$\begin{aligned} 288 \times \$2400 &= \$691{,}200 \\ 168 \times 1800 &= \underline{302{,}500} \\ & \$993{,}600 \end{aligned}$$

$$\frac{\overset{16}{\cancel{691200}}}{\underset{23}{\cancel{993600}}} \times \frac{\$\overset{41}{\cancel{943}}}{1} = \$656, \text{ A's.}$$

$$\frac{\overset{7}{\cancel{302400}}}{\underset{23}{\cancel{993600}}} \times \frac{\$\overset{41}{\cancel{943}}}{1} = \$287, \text{ B's.}$$

8. A and B join capitals in the ratio 7 : 11. At the end of 7 months A withdraws $\frac{1}{2}$ of his, and B $\frac{1}{3}$ of his; and, after 11 months more, they divide a profit of \$5148.50. What is the share of each?

$$\begin{aligned} 18 \times 7 &= 126 \\ 11 \times 3\tfrac{1}{2} &= \underline{38\tfrac{1}{2}} \\ & 87\tfrac{1}{2} = \tfrac{525}{6}. \end{aligned} \qquad \begin{aligned} 18 \times 11 &= 198 \\ 11 \times 3\tfrac{2}{3} &= \underline{40\tfrac{1}{3}} \\ & 157\tfrac{2}{3} = \tfrac{946}{6}. \end{aligned}$$

$$525 + 946 = 1471.$$

$$\frac{\overset{21}{\cancel{525}}}{\cancel{1471}} \times \frac{\$\overset{\overset{175}{\cancel{350}}}{\cancel{514850}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{100}}} = \$1837.50, \text{ A's.}$$

$$\$5148.50 - \$1837.50 = \$3311, \text{ B's.}$$

9. Divide £65 9*s.* among three persons, so that the first may have as many half-crowns as the second has shillings; and the second as many guineas as the third has pounds.

1st has $2\frac{1}{2}$ times as much as 2d.

2d has $\frac{21}{20}$ as much as 3d.

3d has 1 part.
2d has $\frac{21}{20}$ part.
1st has $\frac{21}{8}$ part.

3d has 40 parts.
2d has 42 parts.
1st has 105 parts.

All have 187 parts.
£65 9 *s.* = 1309 *s.*

$$\frac{105}{\cancel{187}} \times \frac{\overset{7}{\cancel{1309}}}{1} = 735\,s. = £36\ 15\,s.$$

$$\frac{42}{\cancel{187}} \times \frac{\overset{7}{\cancel{1309}}}{1} = 294\,s. = £14\ 14\,s.$$

$$\frac{40}{\cancel{187}} \times \frac{\overset{7}{\cancel{1309}}}{1} = 280\,s. = £14.$$

10. Two partners begin business each with a capital of \$2000. A adds \$500 at the end of 2 months, and \$500 at the end of 7 months; B adds \$800 at the end of 3 months. What is the share of each, at the year's end, of a profit of \$3605.25?

$$\begin{array}{rrr} 12 \times & \$2000 = & \$24000 \\ 10 \times & 500 = & 5000 \\ 5 \times & 500 = & 2500 \\ \hline & & \$31500 \end{array} \qquad \begin{array}{rrr} 12 \times & \$2000 = & \$24000 \\ 9 \times & 800 = & 7200 \\ \hline & & \$31200 \end{array}$$

$$31{,}500 + 31{,}200 = 62{,}700.$$

$$\frac{\overset{\overset{21}{\cancel{315}}}{\cancel{31500}}}{\underset{\underset{4}{\cancel{4180}}}{\cancel{62700}}} \times \frac{\overset{345}{\$\cancel{360525}}}{\cancel{100}} = \$1811.25, \text{ A's.}$$

$$\$3605.25 - \$1811.25 = \$1794, \text{ B's.}$$

Exercise LXVII.

1. The population of a town in 1870 was 12,275, and it increased 8% in the next ten years. Find its population in 1880.

If 100 represent the population in 1870, then 108 will represent the population in 1880.

$$\frac{\overset{27}{\cancel{108}}}{\underset{4}{\cancel{100}}} \text{ of } \frac{\overset{491}{\cancel{12275}}}{1} = 13{,}257. \textit{ Ans.}$$

2. How much metal will be obtained from 365 tons of ore, if the metal be 7% of the ore?

If 100 represent the ore, then 7 will represent the metal in the ore.

$$\frac{7}{\underset{20}{\cancel{100}}} \text{ of } \frac{\overset{73}{\cancel{365}}}{1} = \frac{511}{20} = 25.55 \text{ tons. } \textit{Ans.}$$

3. If gunpowder contains 75% of saltpetre, 10% of sulphur, 15% of charcoal, how much of each is there in a ton of gunpowder?

If 100 represent the gunpowder, then 75, 10, and 15 will represent respectively the saltpetre, sulphur, and charcoal.

$$\frac{75}{\cancel{100}} \times \frac{\overset{20}{\cancel{2000}}}{1} = 1500 \text{ lbs. saltpetre.}$$

$$\frac{10}{\cancel{100}} \times \frac{\overset{20}{\cancel{2000}}}{1} = 200 \text{ lbs. sulphur.}$$

$$\frac{15}{\cancel{100}} \times \frac{\overset{20}{\cancel{2000}}}{1} = 300 \text{ lbs. charcoal.}$$

4. A manufactory uses 24 tons of coal a day, and 20% of it is lost in smoke. How much coal would be needed if this waste could be prevented?

If 100 represent the number of tons of coal used, then 20 will represent the number of tons lost in smoke.

$$\frac{\cancel{20}}{\underset{5}{\cancel{100}}} \times \frac{24}{1} = \frac{24}{5} = 4.8 \text{ t.} \quad 24 \text{ t.} - 4.8 \text{ t.} = 19.2 \text{ t. } \textit{Ans.}$$

5. Air consists of 20.0265% (by measure) of oxygen gas and 79.9735% of nitrogen. How much oxygen in 1750 cu. ft. of air?

If 100 represent the number of cubic feet of air, then 20.0265 will represent the number of cubic feet of oxygen.

$$\frac{20.0265}{\underset{2}{\cancel{100}}} \times \frac{\overset{35}{\cancel{1750}}}{1} = 350.46 \text{ cu. ft. } \textit{Ans.}$$

6. A town, after decreasing 25%, has 4539 inhabitants. Find its number at first.

If 100 represent the population at first, then 75 will represent the population now.

$$\frac{\overset{4}{\cancel{100}}}{\underset{3}{\cancel{75}}} \times \frac{\overset{1513}{\cancel{4539}}}{1} = 6052. \textit{ Ans.}$$

7. 2% of a regiment of 750 men are killed in an engagement, 6% are wounded, and 4% are missing. What is the number still available for service?

If 100 represent the number of men in the regiment then 2, 6, and 4 will represent the number, killed, wounded, and missing.

$$2 + 4 + 6 = 12$$
$$100 - 12 = 88$$
$$\frac{\overset{22}{\cancel{88}}}{\underset{4}{\cancel{100}}} \times \frac{\overset{30}{\cancel{750}}}{1} = 660 \text{ men. } \textit{Ans.}$$

8. If $3\frac{3}{4}$ tons of sulphur are required to make $31\frac{1}{4}$ tons of gunpowder, what is the per cent of sulphur in gunpowder?

If 100 represent the whole weight, then the number required to represent $3\frac{3}{4}$ tons of sulphur will be $\frac{3\frac{3}{4}}{31\frac{1}{4}}$ of $100 = 12$.

That is, 12%. *Ans.*

9. In a school of 80 children, $17\frac{1}{2}$% are girls. Find the number of boys.

If 100 represent the number of scholars in the school, then $100 - 17\frac{1}{2}$, or $82\frac{1}{2}$, will represent the number of boys in the school.

$$\frac{82\frac{1}{2}}{100} \text{ of } 80 = \frac{\overset{33}{\cancel{165}}}{\cancel{2}} \times \frac{1}{\underset{5}{\cancel{100}}} \times \frac{\overset{\overset{2}{\cancel{4}}}{\cancel{80}}}{1} = 66 \text{ boys. } \textit{Ans.}$$

10. If goods are bought for \$415, and sold for \$500, what is the gain per cent?

\$500 − \$415 = \$85, actual gain.

Since the gain on \$415 is \$85, the gain on 100 is

$$\frac{100}{\underset{83}{\cancel{415}}} \text{ of } \frac{\overset{17}{\cancel{85}}}{1} = 20\tfrac{40}{83}\%. \quad \therefore \text{ the gain is } 20\tfrac{40}{83}\%. \textit{ Ans.}$$

11. If goods are bought for \$415, and sold for \$400, what is the loss per cent?

\$415 – \$400 = \$15, actual loss.

Since the loss on \$415 is \$15, the loss on 100 is

$$\frac{100}{\underset{83}{\cancel{415}}} \text{ of } \frac{\overset{3}{\cancel{15}}}{1} = 3\tfrac{51}{83}. \quad \therefore \text{ the loss is } 3\tfrac{51}{83}\%. \textit{ Ans.}$$

12. \$500 is 4% of what number?

If 4 represent \$500, 100 will represent $\frac{100}{4}$ of \$500 = \$12,500.

13. A farmer buys 24 head of cattle at \$80 a head, and, after losing 6, sells the remainder at \$105 a head. How much does he gain or lose per cent?

24 head of cattle at \$80 per head cost \$1920.

18 head of cattle at \$105 per head cost \$1890.

\$1920 – \$1890 = \$30, actual loss.

Since the loss on \$1920 is \$30, the loss on 100 is

$$\frac{\overset{25}{\cancel{100}}}{\underset{\underset{16}{\cancel{64}}}{\cancel{1920}}} \text{ of } \frac{\cancel{30}}{1} = 1\tfrac{9}{16}. \quad \therefore \text{ he loses } 1\tfrac{9}{16}\%. \textit{ Ans.}$$

14. If a ton (2240 lbs.) of ore in a gold mine yield 5 oz. (troy) of gold, what is the yield per cent?

5 oz. troy = $\frac{5}{12}$ lbs. troy = $\frac{5}{12}$ of $\frac{7000}{5760}$ lbs. av. = $\frac{12}{35}$ lb. av.

If 100 be taken to represent the ore, the number required to represent the metal will be

$$\frac{\frac{12}{35}}{2240} \text{ of } 100 = \frac{1}{\underset{\underset{28}{\cancel{560}}}{\cancel{2240}}} \times \frac{\overset{3}{\cancel{12}}}{\underset{7}{\cancel{35}}} \times \frac{\overset{\cancel{20}}{\cancel{100}}}{1} = \frac{3}{196}. \quad \text{That is, } \tfrac{3}{196} \text{ of } 1\%. \textit{ Ans.}$$

15. If the ore in a mine yields $\frac{3}{80}$ of 1% of pure gold, how many tons (2240 lbs.) of ore must be taken to obtain 7 lbs. (troy) of gold?

7 lbs. troy $= 7 \times \frac{5760}{7000}$ lbs. av. $= 5\frac{19}{25}$ lbs. av.

If 100 be taken to represent the ore, $\frac{3}{80}$ will represent the gold in the ore. If $\frac{3}{80}$ represent $5\frac{19}{25}$ lbs., 100 will represent

$$\left(\frac{100}{\frac{3}{80}} \text{ of } 5\tfrac{19}{25}\right) \text{lbs. or } \frac{\frac{100}{\frac{3}{80}} \text{ of } 5\frac{19}{25}}{2240} \text{ tons} = \frac{\overset{4}{\cancel{100}}}{1} \times \frac{\cancel{80}}{\cancel{3}} \times \frac{\overset{48}{\cancel{144}}}{\cancel{25}} \times \frac{1}{\underset{\underset{7}{\cancel{28}}}{\cancel{2240}}}$$

$$= \tfrac{48}{7} = 6\tfrac{6}{7} \text{ t. } Ans.$$

16. $12\frac{1}{2}$ tons of iron are obtained from 235 tons of ore. What per cent of the ore is iron?

If 100 be taken to represent the whole weight of the ore, the number required to represent $12\frac{1}{2}$ tons will be

$$\frac{12\frac{1}{2}}{235} \text{ of } 100 = \frac{\cancel{25}}{\cancel{2}} \times \frac{1}{\underset{47}{\cancel{235}}} \times \frac{\overset{50}{\cancel{100}}}{1} = 5\tfrac{15}{47}. \text{ That is, } 5\tfrac{15}{47}\%. \; Ans.$$

17. Goods are sold, at a loss of 3%, for \$2667.50. What was the cost?

If 100 be taken to represent the cost, $100 - 3 = 97$ will represent the selling price. Therefore the selling price was $\frac{100}{97}$ of \$2667.50 = \$2750. *Ans.*

18. Teas at 68 cents, 86 cents, and 96 cents a pound, are mixed in equal quantities, and sold at 90 cents a pound. Find the gain per cent.

$\frac{68 + 86 + 96}{3} = 83\frac{1}{3}$ cost per lb. $90 - 83\frac{1}{3} = 6\frac{2}{3}$, actual gain.

Since the gain on $83\frac{1}{3}$ cents is $6\frac{2}{3}$ cents, the gain on 100 is $\frac{100}{83\frac{1}{3}}$ of $6\frac{2}{3} = 8$. That is, 8%. *Ans.*

19. By selling goods for \$1173.92, a merchant gains \$153.12. Find the gain per cent on the cost.

\$1173.92 − \$153.12 = \$1020.80, cost.

Since the gain on \$1020.80 is \$153.12, the gain on 100 is $\frac{100}{1020.80}$ of $153.12 = 15$. That is, 15%. *Ans.*

20. If to 25 gals. of alcohol 2 gals. of water are added, how much per cent of the mixture is water? how much per cent is alcohol?

> $25 + 2 = 27$ gals., the whole amount.
>
> If 100 be taken to represent the whole amount, the number required to represent 2 gals. will be $\frac{2}{27}$ of $100 = 7\frac{11}{27}$. That is, $7\frac{11}{27}\%$ water. *Ans.*
>
> $100 - 7\frac{11}{27} = 92\frac{16}{27}\%$ alcohol. *Ans.*

21. What was the cost when $17\frac{1}{2}\%$ was gained by selling goods for \$253.80?

> If 100 be taken to represent the cost, $100 + 17\frac{1}{2} = 117\frac{1}{2}$ will represent the selling price.
>
> Therefore, the cost was $\frac{100}{117\frac{1}{2}}$ of \$253.80 = \$216. *Ans.*

22. A wine merchant mixes 24 gallons, at \$7 a gallon, with 18 gallons, at \$5 a gallon, and sells the whole at \$7 a gallon. What does he gain per cent?

> $24 \times \$7 + 18 \times \$5 = \$168 + \$90 = \$258$, cost.
>
> $24 + 18 = 42$, whole number of gallons.
>
> $42 \times \$7 = \$294$, selling price.
>
> $\$294 - \$258 = \$36$, actual gain.
>
> Since the gain on \$258 is \$36, the gain on 100 is $\frac{100}{258}$ of $36 = 13\frac{41}{43}$. That is, $13\frac{41}{43}\%$. *Ans.*

23. By selling a horse for \$200, a dealer loses $12\frac{1}{2}\%$. What would he have gained or lost per cent by selling at \$250?

> If 100 be taken to represent the selling price, $100 - 12\frac{1}{2} = 87\frac{1}{2}$ will represent the cost.
>
> Therefore the cost was $\frac{100}{87\frac{1}{2}}$ of \$200 = $\$228\frac{4}{7}$.
>
> $\$250 - \$228\frac{4}{7} = \$21\frac{3}{7}$, gain by selling at \$250.
>
> Since the gain on $\$228\frac{4}{7}$ would have been $\$21\frac{3}{7}$, the gain on 100 would have been $\frac{100}{228\frac{4}{7}}$ of $21\frac{3}{7} = 9\frac{3}{8}$. That is, $9\frac{3}{8}\%$ gain. *Ans.*

24. A spirit merchant buys 75 gals., at \$3.25 a gallon, and, after drawing off 10 gals., sells the remainder so as to gain 5% on the whole. What is the selling price per gallon?

$75 \times \$3.25 = \$243.75$, cost.

If 100 be taken to represent the cost, $100 + 5 = 105$ will represent the selling price.

Therefore, the selling price is $\frac{105}{100}$ of $\$243.75 = \$255.93\frac{3}{4}$.

75 gals. − 10 gals. = 65 gals., number sold.

$\frac{\$255.93\frac{3}{4}}{65} = \$3.93\frac{3}{4}$, selling price per gallon. *Ans.*

25. A person owns two estates worth respectively \$9845 and \$12,155. If the first rise in value 32%, and the second fall 13%, determine the rise or fall per cent in the value of his whole property.

If 100 be taken to represent the value of the first estate at first, $100 + 32 = 132$ will represent its value after rising 32%.

Therefore, the value after rising is $\frac{132}{100}$ of $\$9845 = \$12,995.40$.

If 100 be taken to represent the value of the second estate at first, $100 - 13 = 87$ will represent its value after falling 13%.

Therefore, the value after falling is $\frac{87}{100}$ of $\$12,155 = \$10,574.85$.

$\$9845 + \$12,155 = \$22,000$, value of the estates at first.

$\$12,995.40 + \$10,574.85 = \$23,570.25$, value afterwards.

$\$23,570.25 - \$22,000 = \$1570.25$, actual gain.

Since the gain on \$22,000 is \$1570.25, the gain on 100 is $\frac{100}{22000}$ of $\$1570.25 = 7\frac{11}{80}$. That is, $7\frac{11}{80}$% gain. *Ans.*

26. A tradesman marks an article \$5, but takes off 5% for cash. If his profit is 14%, what was the cost of the article?

If 100 be taken to represent the marked price, $100 - 5 = 95$ will represent the actual price.

Therefore, the actual price was $\frac{100}{95}$ of $\$5 = \$4.75$.

If 100 be taken to represent the cost, $100 + 14 = 114$ will represent the price.

Therefore, the cost was $\frac{100}{114}$ of $\$4.75 = \$4.16\frac{2}{3}$. *Ans.*

27. What would a dishonest dealer gain per cent by using a false weight of 15 oz. instead of a pound?

16 oz. − 15 oz. = 1 oz., actual gain.

Since the gain on 15 oz. is 1 oz., the gain on 100 is $\frac{1}{15}$ of $100 = 6\frac{2}{3}$. That is, $6\frac{2}{3}$%. *Ans.*

28. A dishonest dealer gains 12% by using false weights. What is the real weight of his pound?

> If 100 be taken to represent the false weight of a pound, $100 + 12 = 112$ will represent the true weight of a pound.
> Therefore, the false weight is $\frac{100}{112}$ of 16 oz. $= 14\frac{2}{7}$ oz. *Ans.*

29. A tradesman, in selling goods, deducts from the marked price 5% for cash. What is the marked price of some goods for which he receives \$7.12$\frac{1}{2}$?

> If 100 be taken to represent the marked price, $100 - 5 = 95$ will represent the actual price.
> Therefore, the marked price is $\frac{100}{95}$ of \$7.12$\frac{1}{2}$ = \$7.50. *Ans.*

30. The lead ore from a certain mine yields 60% of metal, and of the metal $\frac{3}{4}$ of 1% is silver. How much silver and lead will be obtained from 1200 tons of ore?

> If 100 be taken to represent the ore, 60 will represent the metal.
> Therefore, the amount of metal is $\frac{60}{100}$ of 1200 t. = 720 t.
> If 100 be taken to represent the metal, $\frac{3}{4}$ will represent the silver.
> Therefore, the amount of silver is $\dfrac{\frac{3}{4}}{100}$ of 720 = 5.4 t. *Ans.*
> 720 t. − 5.4 t. = 714.6 t., lead. *Ans.*

31. If ore loses $41\frac{1}{2}$% of its weight in roasting, and $43\frac{3}{4}$% of the remainder in smelting, how much ore will be required to yield 1000 tons of metal?

> If 100 be taken to represent the ore, $100 - 41\frac{1}{2} = 58\frac{1}{2}$ will represent the ore after roasting.
> If $58\frac{1}{2}$ represent the ore after roasting, $58\frac{1}{2} - 43\frac{3}{4}\%$ of $58\frac{1}{2} = 32\frac{29}{32}$ will represent the ore after smelting.
> Therefore, $\dfrac{32\frac{29}{32}}{100}$ t. will be the amount of metal in 1 ton of ore,
> and $\dfrac{1000}{\dfrac{32\frac{29}{32}}{100}}$ t. = 3038.936 t. will be the amount of ore required to make 1000 tons of metal. *Ans.*

32. How many per cent above cost must a man mark his goods in order to take off 10%, and still make a profit of 17%?

If 100 be taken to represent the cost, the selling price will be represented by 117.

As the selling price is to be 10% below the marked price, the selling price (117) will be $\frac{90}{100}$ of the marked price.

Therefore, the marked price will be $\frac{100}{90}$ of $117 = 130$.

That is, the goods must be marked 30% above cost. *Ans.*

33. How many per cent above cost must a man mark his goods in order to take off $12\frac{1}{2}$%, and still make a profit of $12\frac{1}{2}$%?

If 100 be taken to represent the cost, the selling price will be represented by $112\frac{1}{2}$.

As the selling price is to be $12\frac{1}{2}$% below the marked price, the selling price ($112\frac{1}{2}$) will be $\frac{87\frac{1}{2}}{100}$ of the marked price.

Therefore, the marked price will be $\frac{100}{87\frac{1}{2}}$ of $112\frac{1}{2} = 128\frac{4}{7}$.

That is, the goods must be marked $28\frac{4}{7}$% above cost. *Ans.*

34. How many per cent above cost must a man mark his goods in order to take off 15%, and still make a profit of 15%?

If 100 be taken to represent the cost, the selling price will be represented by 115.

As the selling price is to be 15% below the marked price, the selling price (115) will be $\frac{85}{100}$ of the marked price.

Therefore, the marked price will be $\frac{100}{85}$ of $115 = 135\frac{5}{17}$.

That is, the goods must be marked $35\frac{5}{17}$% above cost. *Ans.*

35. How many per cent above cost must a man mark his goods in order to take off $33\frac{1}{3}$%, and still make a profit of $33\frac{1}{3}$%?

If 100 be taken to represent the cost, the selling price will be represented by $133\frac{1}{3}$.

As the selling price is to be $33\frac{1}{3}$% below the marked price, the selling price ($133\frac{1}{3}$) will be $\frac{66\frac{2}{3}}{100}$ of the marked price.

Therefore, the marked price will be $\frac{100}{66\frac{2}{3}}$ of $133\frac{1}{3} = 200$.

That is, the goods must be marked 100% above cost. *Ans.*

36. If 5% of the population of a town has been the increase in the preceding ten years, what per cent of the population ten years ago has been added?

If 100 be taken to represent the population, the population 10 years ago will be represented by 95.
Since the gain on 95 is 5, the gain on 100 is $\frac{100}{95}$ of $5 = 5\frac{5}{19}$.
That is, $5\frac{5}{19}\%$. *Ans.*

37. If, in a population of 27,000,000, 13% are foreign-born, how many foreign-born are there? What is the ratio of the foreign-born to the native?

If 100 be taken to represent the population, 13 will represent the foreign-born population.
Therefore, the foreign-born population is $\frac{13}{100}$ of $27{,}000{,}000 = 3{,}510{,}000$ (1). $13\% : 87\% = 13 : 87$. (2) *Ans.*

38. A man bought a horse for \$70, and sold him for \$80. What per cent did he gain? What per cent of the money received for the horse was gained?

$\$80 - \$70 = \$10$ actual gain.
Since the gain on \$70 is \$10, the gain on 100 is $\frac{10}{70}$ of $100 = 14\frac{2}{7}$.
That is, $14\frac{2}{7}\%$. (1) *Ans.*
Since the gain on \$80 is \$10, the gain on 100 is $\frac{10}{80}$ of $100 = 12\frac{1}{2}$.
That is, $12\frac{1}{2}\%$. (2) *Ans.*

39. If, by selling goods for $12\frac{1}{2}$ per cent profit, a merchant clears \$800, what was the cost of the goods, and for how much were they sold?

If $12\frac{1}{2}$ represent \$800, 100 will represent $\frac{100}{12\frac{1}{2}}$ of $\$800 = \$6400$, cost. $\$6400 + \$800 = \$7200$, selling price. *Ans.*

40. A man selling eggs at 40 cents a dozen clears $33\frac{1}{3}\%$ on the cost; what was the cost? Another, selling at the same price, clears $33\frac{1}{3}\%$ of his receipts; what did his eggs cost?

If 100 be taken to represent the cost, $133\frac{1}{3}$ will represent the selling price.

Therefore the cost was $\frac{100}{133\frac{1}{3}}$ of 40 cents = 30 cents. (1) *Ans.*

If 100 be taken to represent the selling price, $66\frac{2}{3}$ will represent the cost.

Therefore, the cost was $\frac{66\frac{2}{3}}{100}$ of 40 cents = $26\frac{2}{3}$ cents. (2) *Ans.*

41. By selling a carriage for $117, a carriage-maker lost 10% of the cost. What ought he to have sold it for to make 10%?

If 100 be taken to represent the cost, 90 will represent the selling price.

Therefore, the cost was $\frac{100}{90}$ of $117 = $130.

If 100 be taken to represent the cost, 110 will represent the price at which he ought to have sold it.

Therefore, the selling price ought to have been $\frac{110}{100}$ of $130 = $143.

42. A man gained in January 3% in weight, and in February lost 3%. What per cent of his weight on the first day of January is his weight on the first day of March?

If 100 be taken to represent his weight Jan. 1, 103 will represent it Feb. 1.

If 100 be taken to represent his weight Feb. 1, 97 will represent it March 1.

Therefore, his weight March 1 was $\frac{97}{100}$ of 103 = $99\frac{91}{100}$.

$99\frac{91}{100}$ is $99\frac{91}{100}$% of 100. *Ans.*

43. 7 lbs. of a certain article lose 3 oz. in weight by drying. What per cent of the original weight is water?

7 lbs. = 112 oz.

If 100 be taken to represent the whole weight, the number required to represent 3 oz. is $\frac{3}{112}$ of 100 = $2\frac{19}{28}$.

That is, $2\frac{19}{28}$%. *Ans.*

44. 7 lbs. of a dry article have lost 3 oz. by drying. What per cent of the original weight was water?

7 lbs. = 112 oz. 112 oz. + 3 oz. = 115 oz., whole weight.

If 100 be taken to represent the whole weight, the number required to represent 3 oz. is $\frac{3}{115}$ of 100 = $2\frac{14}{23}$.

That is, $2\frac{14}{23}$%. *Ans.*

45. A dry article was exposed to damp air, and absorbed 3 oz. of water; it then weighed 7 lbs. What per cent of its present weight is water?

7 lbs. = 112 oz.

If 100 be taken to represent the whole weight, the number required to represent 3 oz. is $\frac{3}{112}$ of $100 = 2\frac{19}{28}$.

That is, $2\frac{19}{28}\%$. *Ans.*

46. If rosin is melted with 20% of its weight of tallow, what per cent of tallow does the mixture contain?

If 100 be taken to represent the rosin, 120 will represent the mixture.

Therefore, the tallow will be represented by $\frac{20}{120}$ of $100 = 16\frac{2}{3}$.

That is, $16\frac{2}{3}\%$. *Ans.*

47. If 20% of a mixture of tallow and rosin is tallow, what per cent of the weight of the rosin is the weight of the tallow?

If 100 be taken to represent the mixture, $100 - 20 = 80$ will represent the rosin.

If 80 is 100%, 20 is $\frac{20}{80}$ of $100\% = 25\%$. *Ans.*

48. How many pounds of tallow must be mixed with $8\frac{1}{2}$ lbs. of rosin in order that the mixture may contain 15% of tallow?

If 100 be taken to represent the mixture, $100 - 15 = 85$ will represent the rosin.

If 85 represent $8\frac{1}{2}$ lbs., 15 will represent $\frac{15}{85}$ of $8\frac{1}{2}$ lbs. $= 1\frac{1}{2}$ lb.

49. Nitrogen gas, under standard pressure and temperature, is $\frac{1}{8}$ of 1% of the weight of an equal volume of water. What is its specific gravity?

If 100 be taken to represent the water, $\frac{1}{8}$ will represent nitrogen.

Therefore, the specific gravity of nitrogen is $\frac{\frac{1}{8}}{100} = 0.00125$. *Ans.*

50. Oxygen gas is $\frac{1}{7}$ of 1% of the weight of an equal volume of water; what is its specific gravity? How many gallons of oxygen will it take to weigh as much as a pint of water? How many of nitrogen?

If 100 be taken to represent water, $\frac{1}{7}$ will represent oxygen.

Therefore, the specific gravity of oxygen is $\frac{\frac{1}{7}}{100} = \frac{1}{700}$. (1) *Ans.*

As 1 gal. of oxygen weighs $\frac{1}{700}$ of a gallon of water, it will take as many gallons of oxygen to weigh as much as a pint of water as $\frac{\frac{1}{8}}{\frac{1}{700}} = 87\frac{1}{2}$ gals. (2) *Ans.*

As 1 gal. of nitrogen weighs 0.00125 of a gallon of water, it will take as many gallons of nitrogen to weigh as much as a pint of water as $\frac{\frac{1}{8}}{0.00125} = 100$ gals. (3) *Ans.*

51. If common air consist of 4 volumes of oxygen to 13 of nitrogen, what is its specific gravity?

4 volumes of oxygen $= 4 \times \frac{1}{700} = \frac{1}{175}$.
13 volumes of nitrogen $= 13 \times \frac{1}{800} = \frac{13}{800}$.
$\frac{1}{175} + \frac{13}{800} = \frac{123}{5600}$, for 17 volumes.
Therefore, the specific gravity is $\frac{123}{5600} \div 17 = 0.00129$. *Ans.*

52. How many gallons of air weigh as much as a pint of water?

As 1 gallon of air weighs 0.00129 of a gallon of water, it will take as many gallons of air to weigh as much as a pint of water as $\frac{\frac{1}{8}}{0.00129} = 96\frac{116}{129}$ gals. *Ans.*

53. If by heating iron 185° F. it expands $\frac{1}{8}$ of 1%, what will be the expansion of iron in passing from $-20°$ F. to $+120°$ F.?

The difference between $-20°$ and $+120°$ is 140°.

If $\frac{1}{8}$ of 1% is the expansion for 185°, the expansion for 140° is $\frac{140}{185}$ of $\frac{1}{8}$ of 1% $= \frac{7}{74}$ of 1%. *Ans.*

54. A tubular iron bridge is 450 ft. long, and one end is fast to a pier. How much play must be allowed at the other end, if the iron expands at the above rate, and if the climate varies from $-30°$ F. in winter to $+130°$ F. in a July sun?

The difference between $-30°$ and $+130°$ is 160°.

If $\frac{1}{8}$ of 1% is the expansion for 185°, the expansion for 160° is $\frac{160}{185}$ of $\frac{1}{8}$ of 1% $= \frac{4}{37}$ of 1%.

$\frac{4}{37}$ of 1% of 450 ft. is $\frac{18}{37}$ ft. $= 5\frac{31}{37}$ in. *Ans.*

55. How much longer is 100 miles of iron rail at 118° F. than at 20° below zero?

The difference between $-20°$ and $+118°$ is 138°.

If $\frac{1}{8}$ of 1% is the expansion for 185°, the expansion for 138° is $\frac{138}{185}$ of $\frac{1}{8}$ of $1\% = \frac{69}{740}$ of 1%.

$\frac{69}{740}$ of 1% of 100 mi. is $\frac{69}{740}$ mi. $= 492\frac{12}{37}$ ft. *Ans.*

Exercise LXVIII.

1. Find the brokerage, at $\frac{1}{8}$ of 1%, to be paid on $10,450.

$$\$10{,}450 \times 0.00\tfrac{1}{8} = \$13.06. \textit{ Ans.}$$

2. Find the commission on $2595, at $2\frac{1}{2}\%$.

$$\$2595 \times 0.025 = \$64.88. \textit{ Ans.}$$

3. An agent sells 200 bbls. of flour, at $6.25; 600 gals. molasses, at 65 cents; and charges a commission of $1\frac{3}{4}\%$. What are the net proceeds?

$$\begin{array}{rr} 200 \times \$6.25 = & \$1250.00 \\ 600 \times \ \ 0.65 = & \underline{390.00} \\ & \$1640.00 \\ 1640 \times 0.01\tfrac{3}{4} = & \underline{28.70} \\ & \$1611.30. \textit{ Ans.} \end{array}$$

4. A commission merchant received $1640 with which to buy corn, after deducting a commission of $2\frac{1}{2}\%$. What is the amount of his commission, and how many bushels of corn at $62\frac{1}{2}$ cents a bushel can he buy?

If 100 be taken to represent the amount to be paid for the goods, $102\frac{1}{2}$ will represent the $1640. Therefore, the amount expended for goods will be $\frac{100}{102\frac{1}{2}}$ of $\$1640 = \$1600$.

And $2\frac{1}{2}\%$ of $\$1600 = \$40$, commissions. (1) *Ans.*

$\frac{1600}{0.62\frac{1}{2}}$ bu. = 2560 bu. can be bought for $1600. (2) *Ans.*

5. A commission merchant sells a consignment of cotton for \$5216. He pays \$51 for freight and storage, and charges a commission of $2\frac{1}{4}\%$. What are the net proceeds?

Consignment	=		\$5216.00	
$\$5216 \times 0.02\frac{1}{4}$	=	\$117.36		= commission.
Storage	=	51.00		
Total expenses	=		168.36	
			\$5047.64	= net proceeds. *Ans.*

6. A consignment of butter was sold for \$1570, of which \$1546.45 were the net proceeds. What was the rate per cent of commission?

\$1570 − \$1546.45 = \$23.55, commission.

If 100 be taken to represent the consignment, the number required to represent \$23.55 will be $\frac{23.55}{1570}$ of 100 = $1\frac{1}{2}$.

That is, $1\frac{1}{2}\%$. *Ans.*

7. What are the net proceeds from the sale of 2250 bbls. of flour, at \$6.25 a barrel, if the charges for freight and storage be 50 cents a barrel, commission for selling 2%, for guaranteeing payment $1\frac{1}{2}\%$?

$2250 \times \$6.25$	=		\$14062.50	
$\$14062.50 \times 0.035$	=	\$492.19		
$2250 \times \$0.50$	=	1125.00		
Total expenses	=		1617.19	
			\$12445.31	= net proceeds. *Ans.*

8. A commission merchant sells 350 crates of peaches, at \$2.60. If the commission be $4\frac{1}{2}\%$, find the net proceeds.

$$350 \times \$2.60 = \$910.00$$
$$\$910 \times 0.045 = 40.95$$
$$\$869.05. \textit{ Ans.}$$

9. A man sells 420 acres of land, at \$40 an acre, and charges $1\frac{1}{4}\%$ commission. What is his commission?

$$420 \times \$40 = \$16{,}800$$
$$\$16{,}800 \times 0.01\tfrac{1}{4} = \$210. \textit{ Ans.}$$

10. An agent, charging $4\frac{1}{2}\%$ commission, receives for his services \$313. Find the amount of his sales.

If $4\frac{1}{2}$ represent \$313, 100 will represent $\frac{100}{4\frac{1}{2}}$ of \$313 = \$6955.56.

11. A merchant buys, through an agent, 730 yds. of carpeting, at \$1.25 a yard, and pays the agent $\frac{3}{4}$ of 1% commission; the freight amounted to \$7.37. At what price per yard must the carpeting be sold to realize a profit of 20%?

$730 \times \$1.25 = \$912.50$.
$0.0075 \times \$912.50 = \$6.84$, commission.
$\$912.50 + \$6.84 + \$7.37 = \$926.71$, total expenses.
If 100 represent the cost, 120 will represent selling price.
Therefore, the selling price must be $\frac{120}{100}$ of \$926.71 = \$1112.05.
$\$1112.05 \div 730 = \$1.523$. *Ans.*

12. An agent sells a consignment of goods for \$2100. He pays \$33.50 for freight, and, reserving his commission, remits \$2024.77. Find the rate of his commission.

$\$2024.77 + \$33.50 = \$2058.27$, consignment less commission.
$\$2100 - \$2058.27 = \$41.73$, commission.
If 100 be taken to represent the consignment, the number required to represent \$41.73 will be $\frac{41.73}{2100}$ of $100 = 1\frac{691}{700}$.
That is, the commission was $1\frac{691}{700}\%$. *Ans.*

13. A commission merchant has consigned to him 5000 lbs. of cotton, which he sells at 14 cents a pound, and charges 2% commission. With the net proceeds he buys cotton cloth, at 10 cents a yard, charging $1\frac{1}{2}\%$ commission for buying. How many yards of cloth does he buy?

$$
\begin{array}{ll}
5000 \times \$0.14 = & \$700 \\
\$700 \times \ \ 0.02 = & \underline{\ \ \ 14} \\
 & \$686 = \text{net proceeds.}
\end{array}
$$

If 100 be taken to represent the amount to be paid for the goods, 101.5 will represent the \$686.

Therefore, the amount expended for goods will be $\frac{100}{101.5}$ of $\$686 = \$675.86$.

$\frac{675.86}{0.10} = 6758.6$ yds. can be bought for $675.86, at $0.10. *Ans.*

14. A commission merchant has consigned to him 500 bbls. of flour, which he sells at $5.50 a barrel, and charges $2\frac{1}{2}\%$ commission; the expenses for freight, etc., amounted to $250. With the net proceeds he buys sugar, at $6\frac{1}{4}$ cents a pound, charging $2\frac{1}{2}\%$ commission for buying. How much sugar does he buy, and what is the amount of his commissions?

$500 \times \$5.50$ =		$2750.00	
$\$2750 \times 0.025$ =	$68.75		= commission.
Freight, etc. =	250.00		
Total expenses =		318.75	
		$2431.25 net proceeds.	

If 100 be taken to represent the amount to be paid for the sugar, $2\frac{1}{2}$ will represent the commission, and $102\frac{1}{2}$ will represent the $2431.25. Therefore, the amount expended for sugar will be $\frac{100}{102\frac{1}{2}}$ of $\$2431.25 = \$2371.95$.

And $2\frac{1}{2}\%$ of $\$2371.95 = \$59.30$, commission.

Therefore, $\frac{2371.95}{0.06\frac{1}{4}}$ lbs. = 37,951.2 lbs. will be bought. (1) *Ans.*

$\$68.75 + \$59.30 = \$128.05$, whole commission. (2) *Ans.*

15. A collector's commission for collecting taxes, at $1\frac{1}{2}\%$, is $206.55. What was the sum collected?

If $1\frac{1}{2}$ represent $206.55, 100 will represent $\frac{100}{1\frac{1}{2}}$ of $\$206.55 = \$13,770$.

16. An agent received $2961 with which to purchase goods after deducting his commission at 5%. How much was his commission?

If 100 be taken to represent the amount to be paid for the goods, 5 will represent the commission, and 105 will represent the $2961. Therefore, the amount expended for the goods will be $\frac{100}{105}$ of $\$2961 = \$2820$.

And 5% of $\$2820 = \$141$, commission. *Ans.*

17. An agent buys 3100 bbls. of flour, at $4.50 a barrel, and charges $1\frac{1}{2}\%$ commission. What is the amount of the bill, including the commission?

$$\begin{array}{rr} 3100 \times \$4.50 = & \$13,950.00 \\ \$13,950 \times 0.015 = & 209.25 \\ \hline & \$14,159.25. \end{array} \textit{ Ans.}$$

18. A broker receives $6150 to invest in cotton, at $10\frac{1}{4}$ cents a pound. His commission is $2\frac{1}{2}\%$. How many pounds of cotton can he buy?

If 100 be taken to represent the amount to be paid for the cotton, $102\frac{1}{2}$ will represent the $6150. Therefore, the amount expended for cotton will be $\frac{100}{102\frac{1}{2}}$ of $\$6150 = \$6000$.

$\frac{6000}{0.10\frac{1}{4}}$ lbs. $= 58,536\frac{24}{41}$ lbs. of cotton will be bought. *Ans.*

19. An agent sells 1100 bbls. of flour, at $4.50 a barrel, and charges $2\frac{1}{2}\%$ commission. He invests the proceeds in steel, at $1\frac{1}{2}$ cents a pound, charging $1\frac{1}{2}\%$ commission. What is his entire commission, and how many tons of steel (2240 lbs. to a ton) does he buy?

$$\begin{array}{rr} 1100 \times \$4.50 = & \$4950.00 \\ \$4950 \times 0.025 = & 123.75 = \text{commission.} \\ \hline & \$4826.25 \end{array}$$

If 100 be taken to represent the amount to be paid for the steel, $1\frac{1}{2}$ will represent the commission, and $101\frac{1}{2}$ will represent the $4826.25. Therefore, the amount expended for steel will be $\frac{100}{101\frac{1}{2}}$ of $\$4826.25 = \$4754.93$.

And $1\frac{1}{2}\%$ of $\$4754.93 = \$71.32$, commission.

$\$123.75 + \$71.32 = \$195.07$, total commission. (1) *Ans.*

Therefore, $\frac{4754.93}{0.015}$ lbs. $= 316,995\frac{1}{3}$ lbs. $= 141.51562$ t. of steel will be bought. (2) *Ans.*

EXERCISE LXIX.

1. Find the premium of fire insurance for \$2650, at $\frac{1}{2}$ of 1%.

$$\$2650 \times 0.005 = \$13.25. \textit{ Ans.}$$

2. Find the premium to be paid for insuring a person's life for \$2500, at an age for which the rate is $2\frac{1}{4}$%.

$$\$2500 \times 0.02\tfrac{1}{4} = \$56.25. \textit{ Ans.}$$

3. At $2\frac{3}{4}$%, what premium of insurance will be paid on a vessel worth \$36,400?

$$\$36{,}400 \times 0.02\tfrac{3}{4} = \$1001.00. \textit{ Ans.}$$

4. A vessel is worth \$12,052. Determine the sum to be insured, and the premium to be paid at $1\frac{3}{4}$%, so that in the event of loss the owner may receive both the value of the vessel and the premium.

If 100 be taken to represent the sum to be insured, $1\frac{3}{4}$ will represent the premium, and $100 - 1\frac{3}{4} = 98\frac{1}{4}$ will represent the value of the vessel.

Hence, the sum to be insured will be

$\$12{,}052 \div 0.98\frac{1}{4} = \$12{,}266.67$.

And $1\frac{3}{4}$% of \$12,266.67 = \$214.67, premium. *Ans.*

5. The premium for insurance at $1\frac{1}{4}$% is \$150. What is the amount insured?

If $1\frac{1}{4}$ represent \$150, 100 will represent $\frac{100}{1\frac{1}{4}}$ of $\$150 = \$12{,}000$.

6. If a premium of insurance at $2\frac{3}{4}$% amount to \$28.60, what is the sum insured?

If $2\frac{3}{4}$ represent \$28.60, 100 will represent $\frac{100}{2\frac{3}{4}}$ of $\$28.60 = \$1040$.

7. A vessel is so insured that if lost the owner may receive both the value of the vessel and the premium. The value of the vessel is \$96,084, and the rate of insurance $1\frac{7}{8}$%. Find the premium.

If 100 be taken to represent the sum to be insured, $1\frac{7}{8}$ will represent the premium, and $100 - 1\frac{7}{8} = 98\frac{1}{8}$ will represent the value of the vessel.

Hence, the sum to be insured will be $\$96,084 \div 0.98\frac{1}{8} = \$97,920$.

And $1\frac{7}{8}\%$ of $\$97,920 = \$1836$, premium. *Ans.*

8. A building worth \$8000 is insured at $\frac{5}{8}$ of its value, at $\frac{1}{8}$ of 1% per annum. What is the annual premium?

$$\tfrac{5}{8} \text{ of } \$8000 = \$5000.$$
$$\$5000 \times 0.00\tfrac{1}{8} = \$6.25. \ \textit{Ans.}$$

9. Four companies join in insuring a ship and cargo for \$60,000. One company takes $\frac{1}{3}$, at $\frac{3}{5}$ of 1%; a second takes \$10,000, at $\frac{3}{4}$ of 1%; a third, \$15,000, at $\frac{5}{8}$ of 1%; a fourth, the remainder, at $\frac{1}{2}$ of 1%. How much is paid for insurance?

(1)	(2)	(3)	(4)
$\frac{1}{3}$ of \$60,000 = \$20,000.	\$10000	\$15000	The remainder is
\$20000	$0.00\frac{3}{4}$	$0.00\frac{5}{8}$	\$15000
$0.00\frac{3}{5}$	\$75.00	\$93.75	$0.00\frac{1}{2}$
\$120.00			\$75.00

$\$120 + \$75 + \$93.75 + \$75 = \$363.75$, total premium. *Ans.*

10. If the ship in the last problem receive damage to the amount of \$4500, what ought each company to pay?

$\frac{1}{3}$ of $\$4500 = \$1500$ (1).

$\frac{10000}{60000} = \frac{1}{6}$. $\frac{1}{6}$ of $\$4500 = \$750$ (2).

$\frac{15000}{60000} = \frac{1}{4}$. $\frac{1}{4}$ of $\$4500 = \$1125$ (3), (4).

11. A man insures his life for \$10,000, paying \$350 a year in advance. He dies the day before the fifth premium was due. The company pay his widow \$10,000. How much have they lost by him, if the interest gained on the premiums paid amount to \$175?

$4 \times \$350 + \$175 = \$1575$, total premiums with interest.

$\$10,000 - \$1575 = \$8425$, company's loss. *Ans.*

12. A merchant shipped a cargo to London; and to cover both the cargo and the premium, he took out a policy of $100,800, at $3\frac{1}{2}$%. What was the value of the cargo?

If 100 be taken to represent the sum insured, $100 - 3\frac{1}{2} = 96\frac{1}{2}$ will represent the value of the cargo.

Hence, the value of the cargo is $0.96\frac{1}{2}$ of \$100,800 = \$97,272.

13. Three companies insure, at $\frac{3}{4}$ of its value, a building worth $16,000. The first company takes $\frac{1}{3}$ the risk, at $\frac{3}{4}$ of 1%; the second, $\frac{2}{5}$ of it, at $\frac{7}{8}$ of 1%; and the third, the remainder, at $\frac{3}{4}$ of 1%. Find the total premium.

$$\frac{3}{4} \text{ of } \$16{,}000 = \$12{,}000.$$

$\frac{1}{3}$ of \$12,000 = \$4000. $\frac{2}{5}$ of \$12,000 = \$4800.

$$\begin{array}{r} \$4000 \\ 0.00\frac{3}{4} \\ \hline \$30.00 \end{array} \qquad \begin{array}{r} \$4800 \\ 0.00\frac{7}{8} \\ \hline \$42.00 \end{array}$$

$$\$12{,}000 - \$4000 - \$4800 = \$3200.$$

$$\begin{array}{r} \$3200 \\ 0.00\frac{3}{4} \\ \hline \$24.00 \end{array}$$

\$30 + \$42 + \$24 = \$96, total premium. *Ans.*

14. S. Williams pays $18.40 premium for insuring his house for $\frac{2}{3}$ of its value, at $1\frac{1}{2}$%. What is the value of his house?

If $1\frac{1}{2}$ represent \$18.40, 100 will represent $\frac{100}{1\frac{1}{2}}$ of \$18.40 = \$$1226\frac{2}{3}$.

If \$$1226\frac{2}{3}$ is $\frac{2}{3}$, the whole is $\frac{3}{2}$ of \$$1226\frac{2}{3}$ = \$1840. *Ans.*

Exercise LXX.

1. If James Brown be assessed $2500 on his house, and $5200 on personal property, and pays for 2 polls at $1.50 each, how much will his tax be, the rate being $12.18 on $1000?

$$\$2500 + \$5200 = \$7700.$$
$$0.01218 \text{ of } \$7700 = \$93.79.$$
$$\$93.79 + \$3.00 = \$96.79. \textit{ Ans.}$$

2. If the rate of tax be $12.25 on $1000, and the tax be $11,788.50, what is the valuation?

$12.25 on $1000 = 1.225%.

If 1.225 represent $11,788.50, 100 will represent $\frac{100}{1.225}$ of $11,788.50 = $962,326.53. *Ans.*

3. If the assessed valuation of a town be $1,777,000, and the property-tax be $6870, what is the rate on $1000?

$6870 ÷ $1,777,000 = 0.003866.

That is, the rate is 0.3866 of 1%, or $3.866 on $1000. *Ans.*

4. What sum must be assessed, in order that $15,000 shall remain after paying a commission of 2% for collecting the taxes?

If 100 be taken to represent the sum to be assessed, 98 will represent the $15,000.

Therefore, the sum to be assessed is $\frac{100}{98}$ of $15,000 = $15,306.12.

5. A tax of $1857.60 is levied upon a school district for building a school-house. The assessed valuation of the district is $1,935,000. What is the tax on property valued at $6250?

$1857.60 ÷ $1,935,000 = 0.00096.

Therefore, tax on the property is 0.00096 of $6250 = $6.00. *Ans.*

6. In a certain town there are 1350 polls. The assessed value of the real estate is $713,250; of the personal property is $738,954; the poll-tax is $2. The tax on property is $1\frac{1}{8}$%. But only 96% of the tax can be collected, and the collector is paid $2\frac{1}{2}$% of the amount collected. How much does the town receive from the taxes?

The amount of poll-taxes = 1350 × $2 = $2700.

$713,250 + $738,954 = $1,452,204, assessed value.

$0.01\frac{1}{8}$ of $1,452,204 = $16,337.31, amount to be raised on property.

$16,337.31 + $2700 = $19,037.31, amount to be raised.

0.96 of $19,037.31 = $18,275.82, amount collected.

$0.02\frac{1}{2}$ of $18,275.82 = $456.90, collector's pay.

$18,275.82 − $456.90 = $17,818.92, amount town receives. *Ans.*

7. What is the duty, at 20% *ad valorem* (that is, 20% of the cost), on 320 boxes of raisins, each containing 40 lbs., and costing 8 cents a pound?

$$320 \times 40 \times \$0.08 = \$1024.00.$$
$$0.20 \text{ of } \$1024 = \$204.80. \quad \textit{Ans.}$$

8. What is the duty, at 6 cents a gallon, on 420 hhds. of molasses, 63 gals. in a hogshead?

$$420 \times 63 \times \$0.06 = \$1587.60. \quad \textit{Ans.}$$

9. At 40%, what is the duty on 300 tons of iron (2240 lbs. to a ton) invoiced at $1\frac{1}{2}$ cents a pound?

$$300 \times 2240 \times \$0.01\tfrac{1}{2} = \$10,080.$$
$$0.40 \text{ of } \$10,080 = \$4032. \quad \textit{Ans.}$$

10. Paid $1360.80 duty on 300 hhds. of molasses, each containing 63 gals., at 25 cents a gallon. What was the rate per cent of duty?

$$300 \times 63 \times \$0.25 = \$4725.$$

If 100 be taken to represent $4725, the number required to represent $1360.80 is $\frac{\$1360.80}{\$4725}$ of $100 = 28\frac{4}{5}$.

That is, $28\frac{4}{5}$%. *Ans.*

11. A sugar refiner imports 50 hhds. of sugar weighing 480 lbs. each, and 120 hhds. of molasses containing 63 gals. each. What is the amount of the duties, if the sugar pay 3 cents a pound, and the molasses 8 cents a gallon, an allowance being made on the sugar of 10%, and 2% on the molasses?

$$50 \times 480 \times \$0.03 = \$720.$$
$$0.90 \text{ of } \$720 = \$648.$$
$$120 \times 63 \times \$0.08 = \$604.80$$
$$0.98 \text{ of } \$604.80 = \$592.70.$$
$$\$648 + \$592.70 = \$1240.70. \quad \textit{Ans.}$$

12. An importer paid $825 duty on an invoice of silks, the duty being 24%. But damages of 15% were allowed at the custom-house. What was the entire cost of the goods?

If 24 represent \$825, 100 will represent $\frac{100}{24}$ of \$825 = \$3437.50.
If 100 be taken to represent the cost, 100 − 15 = 85 will represent the invoice price.
$\frac{100}{85}$ of \$3437.50 = \$4044.12, cost.
\$4044.12 + \$825 = \$4869.12. *Ans.*

13. Paid \$325 duty on goods which had been damaged; allowance for damage is 24%, and the duty was 24%. What was the invoice price of the goods?

If 24 represent \$325, 100 will represent $\frac{100}{24}$ of \$325 = \$1354.17.
If 100 be taken to represent the cost, 100 − 24 = 76 will represent the invoice price.
Therefore, the invoice price was $\frac{100}{76}$ of \$1354.17 = \$1781.80.

Exercise LXXI.

1. Find the interest of \$680.40 for 2 yrs. 4 mos. 6 dys., at 6%.

$$\begin{aligned} 2\text{ yrs.} &= 0.12 \\ 4\text{ mos.} &= 0.02 \\ 6\text{ dys.} &= \underline{0.001} \\ & 0.141 \end{aligned}$$

$$\begin{array}{r} \$680.40 \\ \underline{\times\, 0.141} \\ \$95.94.\ \textit{Ans.} \end{array}$$

2. Find the interest of \$25.625 for 30 dys., at 6%.

30 dys. = 0.005.

$$\begin{array}{r} \$25.625 \\ \underline{\times\, 0.005} \\ \$0.13.\ \textit{Ans.} \end{array}$$

3. Find the interest of \$85.85 for 1 yr. 7 mos. 21 dys., at 6%.

$$\begin{aligned} 1\text{ yr.} &= 0.06 \\ 7\text{ mos.} &= 0.035 \\ 21\text{ dys.} &= \underline{0.0035} \\ & 0.0985 \end{aligned}$$

$$\begin{array}{r} \$85.85 \\ \underline{\times\, 0.0985} \\ \$8.46.\ \textit{Ans.} \end{array}$$

4. Find the interest of \$1100 for 3 yrs. 4 mos., at 5%.

$$\begin{aligned} 3\text{ yrs.} &= 0.18 \\ 4\text{ mos.} &= \underline{0.02} \\ & 0.20 \\ & \underline{5} \\ & 6\overline{)1.00} \\ & 0.1\tfrac{2}{3} \end{aligned}$$

$$\begin{array}{r} \$1100 \\ \underline{\times\, 0.1\frac{2}{3}} \\ \$183.33.\ \textit{Ans.} \end{array}$$

5. Find the interest of \$1275 for 3 yrs. 2 mos. 15 dys., at 8%.

$$\begin{array}{rr} 3\text{ yrs.} = & 0.18 \\ 2\text{ mos.} = & 0.01 \\ 15\text{ dys.} = & \underline{0.0025} \\ & 0.1925 \\ & \underline{4} \\ & 3\overline{)0.7700} \\ & 0.25\tfrac{2}{3} \end{array}$$

$$\begin{array}{r} \$1275 \\ \underline{\times 0.25\tfrac{2}{3}} \\ \$327.25.\ \textit{Ans.} \end{array}$$

6. Find the interest of \$475.16 for 27 dys., at $4\frac{1}{2}$%.

$$\begin{array}{rr} 27\text{ dys.} = & 0.0045 \\ & \underline{\times 3} \\ & 4\overline{)0.0135} \\ & 0.0033\tfrac{3}{4} \end{array}$$

$$\begin{array}{r} \$475.16 \\ \underline{\times 0.0033\tfrac{3}{4}} \\ \$1.60.\ \textit{Ans.} \end{array}$$

7. Find the interest of \$1290.50 for 60 dys., at 6%.

60 dys. = 0.01.

$$\begin{array}{r} \$1290.50 \\ \underline{\times 0.01} \\ \$12.91.\ \textit{Ans.} \end{array}$$

8. Find the interest of \$125 for 1 yr. 2 mos. 2 dys., at 9%.

$$\begin{array}{rr} 1\text{ yr.} = & 0.06 \\ 2\text{ mos.} = & 0.01 \\ 2\text{ dys.} = & \underline{0.000\tfrac{1}{3}} \\ & 0.070\tfrac{1}{3} \\ & \underline{\times 3} \\ & 2\overline{)0.211} \\ & 0.1055 \\ & \underline{\times \$125} \\ & \$13.19.\ \textit{Ans.} \end{array}$$

9. Find the interest of \$250.80 for 10 mos. 10 dys., at $3\frac{1}{2}$%.

$$\begin{array}{rr} 10\text{ mos.} = & 0.05 \\ 10\text{ dys.} = & \underline{0.001\tfrac{2}{3}} \\ & 0.051\tfrac{2}{3} \\ & \underline{7} \\ & 12\overline{)0.361\tfrac{2}{3}} \\ & 0.030\tfrac{5}{36} \end{array}$$

$$\begin{array}{r} \$250.80 \\ \underline{\times 0.030\tfrac{5}{36}} \\ \$7.56.\ \textit{Ans.} \end{array}$$

10. Find the interest of \$258.85 from Mar. 6 to June 24, at 5%.

mos.	dys.
6	24
3	6
3	18

$$\begin{array}{rr} 3\text{ mos.} = & 0.015 \\ 18\text{ dys.} = & \underline{0.003} \\ & 6\overline{)0.018} \\ & 0.003 \\ & \underline{\times 5} \\ & 0.015 \end{array}$$

$$\begin{array}{r} \$258.85 \\ \underline{\times 0.015} \\ \$3.88.\ \textit{Ans.} \end{array}$$

11. Find the interest of \$380 for 2 yrs. 11 mos. 27 dys., at $4\frac{1}{2}$%.

$$\begin{aligned} 2 \text{ yrs.} &= 0.12 \\ 11 \text{ mos.} &= 0.055 \\ 27 \text{ dys.} &= \underline{0.0045} \\ & \quad 0.1795 \\ & \quad \underline{\times 3} \\ 4)&\underline{0.5385} \\ & \quad 0.1346\tfrac{1}{4} \\ & \quad \underline{\times \$380} \\ & \quad \$51.16. \ \textit{Ans.} \end{aligned}$$

12. Find the interest of \$475.05 for 1 yr. 9 mos. 14 dys., at $7\frac{3}{10}$%.

$$\begin{aligned} 1 \text{ yr.} &= 0.073 \\ 9 \text{ mos.} &= 0.05475 \\ 14 \text{ dys.} &= \underline{0.002838} \\ & \quad 0.130588 \\ & \quad \underline{\times \$475.05} \\ & \quad \$62.04. \ \textit{Ans.} \end{aligned}$$

13. Find the interest of \$725.40 for 11 mos. 24 dys., at $5\frac{1}{4}$%.

$$\begin{aligned} 11 \text{ mos.} &= 0.0481\tfrac{1}{4} \\ 24 \text{ dys.} &= \underline{0.0035} \\ & \quad 0.0516\tfrac{1}{4} \end{aligned}$$

$$\begin{aligned} &\$725.40 \\ &\underline{\times 0.0516\tfrac{1}{4}} \\ &\$37.45. \ \textit{Ans.} \end{aligned}$$

14. Find the interest of \$680.50 for 2 yrs. 6 days, at 5%.

$$\begin{aligned} &\$680.50 \\ &\underline{\times 0.05} \\ &\$34.0250 \text{ int. 1 yr.} \\ &\underline{\times 2\tfrac{1}{60}} \\ &\$68.62. \ \textit{Ans.} \end{aligned}$$

15. Find the interest of \$630.50 for 90 dys., at 6%.

$$90 \text{ dys.} = 0.01\tfrac{1}{2}.$$

$$\begin{aligned} &\$630.50 \\ &\underline{\times 0.01\tfrac{1}{2}} \\ &\$9.46. \ \textit{Ans.} \end{aligned}$$

16. Find the interest of \$547.60 from Feb. 20 to Dec. 5, at $6\frac{1}{2}$%.

mos.	dys.
12	5
2	20
9	15

$$\begin{aligned} 9 \text{ mos.} &= 0.045 \\ 15 \text{ dys.} &= \underline{0.0025} \\ 6)&\underline{0.0475} \\ & \quad 0.0079\tfrac{1}{6} \\ & \quad \underline{\times 6\tfrac{1}{2}} \\ & \quad 0.05145\tfrac{5}{6} \end{aligned}$$

$$\begin{aligned} &\$547.60 \\ &\underline{\times 0.05145\tfrac{5}{6}} \\ &\$28.18. \ \textit{Ans.} \end{aligned}$$

17. Find the interest of \$875 from May 5, 1880, to June 21, 1881, at $5\frac{1}{2}$%.

yrs.	mos.	dys.
1881	6	21
1880	5	5
1	1	16

1 yr. $= 0.06$
1 mo. $= 0.005$
16 dys. $= 0.002\frac{2}{3}$

$12)0.067\frac{2}{3}$
$0.005\frac{23}{36}$
$0.062\frac{1}{36}$

$\$875$
$\times 0.062\frac{1}{36}$
$\$54.27$. *Ans.*

18. Find the interest of $758.50 from Jan. 5 to July 1, at $4\frac{1}{2}$%.

mos.	dys.
7	1
1	5
5	26

5 mos. $= 0.025$
26 dys. $= 0.004\frac{1}{3}$

$4)0.029\frac{1}{3}$
$0.007\frac{1}{3}$
0.022

$\$758.50$
$\times 0.022$
$\$16.69$. *Ans.*

19. Find the interest of $342.42 from Feb. 5, 1879, to Mar. 15, 1881, at 7%.

yrs.	mos.	dys.
1881	3	15
1879	2	5
2	1	10

2 yrs. $= 0.12$
1 mo. $= 0.005$
10 dys. $= 0.001\frac{2}{3}$

$6)0.12\frac{2}{3}$
$0.02\frac{1}{9}$
$0.14\frac{7}{9}$

$\$342.42$
$\times 0.14\frac{7}{9}$
$\$50.60$. *Ans.*

20. Find the interest of $540 from Mar. 5 to Sept. 21, at $3\frac{1}{2}$%.

mos.	dys.
9	21
3	5
6	16

6 mos. $= 0.03$
16 dys. $= 0.002\frac{2}{3}$
$0.032\frac{2}{3}$
7

$12)0.228\frac{2}{3}$
$0.019\frac{1}{18}$

$\$540$
$\times 0.019\frac{1}{18}$
$\$10.29$. *Ans.*

21. Find the amount of $431.50 for 2 yrs. 8 mos., at $4\frac{1}{2}$%.

$\$431.50$
$\times 0.04\frac{1}{2}$
$\$19.4175$ = Int. for 1 yr.
$\times 2\frac{2}{3}$ = 2 yrs. 8 mos.
$\$51.78$
431.50
$\$483.28$. *Ans.*

22. Find the amount of \$476.50 from July 5, 1880, to Feb. 9, 1881, at 4%.

yrs.	mos.	dys.
1881	2	9
1880	7	5
	7	4

7 mos. = 0.035
4 dys. = $0.000\frac{2}{3}$

3)$0.035\frac{2}{3}$
$0.011\frac{8}{9}$
$0.023\frac{7}{9}$

\$476.50
× $0.023\frac{7}{9}$
\$11.33
476.50
\$487.83. *Ans.*

23. Find the amount of \$319.20 from April 7 to Aug. 31, at $3\frac{1}{4}$%.

mos.	dys.
8	31
4	7
4	24

4 mos. = 0.02
24 dys. = 0.004

6)0.024
0.004
$3\frac{1}{4}$
0.013

\$319.20
× 0.013
\$4.15
319.20
\$323.35. *Ans.*

24. Find the amount of \$6460 from June 15, 1878, to May 7, 1880, at $4\frac{1}{4}$%.

yrs.	mos.	dys.
1880	5	7
1878	6	15
1	10	22

1 yr. = 0.06
10 mos. = 0.05
22 dys. = $0.003\frac{2}{3}$

6)$0.113\frac{2}{3}$
$0.018\frac{17}{18}$
× 4.25
$0.08051\frac{7}{18}$
× \$6460
\$520.12
6460.00
\$6980.12. *Ans.*

25. Find the amount of \$150 from Aug. 5, 1879, to Mar. 17, 1881, at 7%.

yrs.	mos.	dys.
1881	3	17
1879	8	5
1	7	12

1 yr. = 0.06
7 mos. = 0.035
12 dys. = 0.002

6)0.097
$0.016\frac{1}{6}$
$0.113\frac{1}{6}$
× \$150
\$16.98
150.00
\$166.98. *Ans.*

26. Find the amount of \$527.20 from Jan. 1 to Nov. 20, at $4\frac{1}{2}\%$.

mos.	dys.
11	20
1	1
10	19

10 mos. = 0.05
19 dys. = $0.003\frac{1}{6}$

$4)\,0.053\frac{1}{6}$
$0.013\frac{7}{24}$
$0.039\frac{7}{8}$

\$527.20
$\times\,0.039\frac{7}{8}$
\$21.02
527.20
\$548.22. *Ans.*

27. Find the amount of \$1250 from Nov. 15, 1880, to Mar. 1, 1881, at 5%.

yrs.	mos.	dys.
1881	3	1
1880	11	15
	3	16

3 mos. = 0.015
16 dys. = $0.002\frac{2}{3}$

$6)\,0.017\frac{2}{3}$
$0.002\frac{17}{18}$
$0.014\frac{13}{18}$

\$1250
$\times\,0.014\frac{13}{18}$
\$18.40
1250.00
\$1268.40. *Ans.*

28. Find the amount of \$624.36 from Mar. 5 to Dec. 20, at $7\frac{3}{10}\%$.

mos.	dys.
12	20
3	5
9	15

$= 9\frac{1}{2}$ mos.

\$624.36
$\times\,0.073$
12)\$45.57828. Int. for 1 yr.
\$3.79819. Int. for 1 mo.
$\times\,9\frac{1}{2}$
\$36.08
624.36
\$660.44. *Ans.*

29. Find the amount of \$12,260 from May 6 to Oct. 24, at $3\frac{3}{4}\%$.

mos.	dys.
10	24
5	6
5	18

5 mos. = 0.025
18 dys. = 0.003
0.028
$\times\,3\frac{3}{4}$
6)0.105
0.0175

\$12260
$\times\,0.0175$
\$214.55
12260.00
\$12,474.55. *Ans.*

30. Find the amount of $11,216 from Oct. 20 to Dec. 31, at 1% a month.

mos.	dys.
12	31
10	20
2	11

2 mos. = 0.02
11 dys. = $0.003\frac{2}{3}$
$0.023\frac{2}{3}$

$11216
$\times\ 0.023\frac{2}{3}$
$265.45
11216.00
$11,481.45. *Ans.*

31. Find the rate per cent when the interest on $326 for 15 yrs. is $220.05.

Interest on $326 for 15 yrs. is $202.05;
on $326 for 1 yr. is $\frac{1}{15}$ of $202.05;
on $1 for 1 yr. is $\frac{1}{326}$ of $\frac{1}{15}$ of $202.05 = $$0.04\frac{1}{2}$.
Therefore, the rate required is $4\frac{1}{2}$%. *Ans.*

32. Find the rate per cent when the interest on $372.50 for 18 yrs. is $301.725.

Interest on $372.50 for 18 yrs. is $301.725;
on $372.50 for 1 yr. is $\frac{\$301.725}{18}$;
on $1 for 1 yr. is $\frac{\$301.725}{18 \times 372.50} = \$0.04\frac{1}{2}$.
Therefore, the rate required is $4\frac{1}{2}$%. *Ans.*

33. Find the rate per cent when $245 amount to $$252.96\frac{1}{4}$ for 9 mos.

The interest is $\$252.96\frac{1}{4} - \$245 = \$7.96\frac{1}{4}$.
The time is 9 mos. = 0.75 yr.
Interest on $245 for 0.75 yr. is $$7.96\frac{1}{4}$;
on $245 for 1 yr. is $\frac{\$7.96\frac{1}{4}}{0.75}$;
on $1 for 1 yr. is $\frac{\$7.96\frac{1}{4}}{0.75 \times 245} = \$0.04\frac{1}{3}$.
Therefore, the rate required is $4\frac{1}{3}$%. *Ans.*

34. Find the rate per cent when the interest on \$235.25 is \$70.575 for 5 yrs.

Interest on \$235.25 for 5 yrs. is \$70.575;

$$\text{on } \$235.25 \text{ for 1 yr. is } \frac{\$70.575}{5};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$70.575}{5 \times 235.25} = \$0.06.$$

Therefore, the rate required is 6%. *Ans.*

35. Find the rate per cent when \$363.125 amount to \$371.598 for 7 mos.

The interest is \$371.598 − \$363.125 = \$8.473.
The time is 7 mos. = $\frac{7}{12}$ yr.
Interest on \$363.125 for $\frac{7}{12}$ yr. is \$8.473;

$$\text{on } \$363.125 \text{ for 1 yr. is } \frac{\$8.473}{\frac{7}{12}};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$8.473}{\frac{7}{12} \times \$363.125} = \$0.04, \text{ nearly.}$$

Therefore, the rate required is 4%, nearly. *Ans.*

36. Find the rate per cent when the interest on \$249.43$\frac{3}{4}$ is \$49.88$\frac{3}{4}$ for 5 yrs. 4 mos.

The time is 5 yrs. 4 mos. = 5$\frac{1}{3}$ yrs.
Interest on \$249.43$\frac{3}{4}$ for 5$\frac{1}{3}$ yrs. is \$49.88$\frac{3}{4}$;

$$\text{on } \$249.43\tfrac{3}{4} \text{ for 1 yr. is } \frac{\$49.88\frac{3}{4}}{5\frac{1}{3}};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$49.88\frac{3}{4}}{5\frac{1}{3} \times 249.43\frac{3}{4}} = \$0.03\tfrac{3}{4}.$$

Therefore, the rate required is 3$\frac{3}{4}$%. *Ans.*

37. Find the rate per cent when \$350 amount to \$406.70 for 3 yrs. 7 mos. 6 dys.

The time is 3 yrs. 7 mos. 6 dys. = 3.6 yrs.
The interest is \$406.70 − \$350 = \$56.30.
Interest on \$350 for 3.6 yrs. is \$56.30;

$$\text{on } \$350 \text{ for 1 yr. is } \frac{\$56.30}{3.6};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$56.30}{3.6 \times 350} = \$0.04\tfrac{1}{2}.$$

Therefore, the rate required is 4$\frac{1}{2}$%. *Ans.*

38. Find the rate per cent when the interest on \$6875 is \$72.05 for 90 dys.

The time is 90 dys. = 0.25 yr.

Interest on \$6875 for 0.25 yr. is \$72.05;

$$\text{on } \$6875 \text{ for 1 yr. is } \frac{\$72.05}{0.25};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$72.05}{0.25 \times 6875} = \$0.04\tfrac{24}{125}.$$

Therefore, the rate required is $4\frac{1}{5}$%, nearly. *Ans.*

39. Find the rate per cent when the interest on \$642 is \$10.70 for 5 mos.

The time is 5 mos. = $\frac{5}{12}$ yr.

Interest on \$642 for $\frac{5}{12}$ yr. is \$10.70;

$$\text{on } \$642 \text{ for 1 yr. is } \frac{\$10.70}{\frac{5}{12}};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$10.70}{\frac{5}{12} \times 642} = \$0.04.$$

Therefore, the rate required is 4%. *Ans.*

40. Find the rate per cent when the interest on \$8432 for 2 yrs. 7 mos. 23 dys. is \$1339.28.

The time is 2 yrs. 7 mos. 23 dys. = $2\frac{233}{360}$ yrs.

Interest on \$8432 for $2\frac{233}{360}$ yrs. is \$1339.28;

$$\text{on } \$8432 \text{ for 1 yr. is } \frac{\$1339.28}{2\frac{233}{360}};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$1339.28}{2\frac{233}{360} \times 8432} = \$0.06, \text{ nearly.}$$

Therefore, the rate required is 6%, nearly. *Ans.*

41. Find the rate per cent when a sum of money is doubled in 14 yrs.

Interest on \$1 for 14 yrs. is \$1;

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$1}{14} = \$0.07\tfrac{1}{7}.$$

Therefore, the rate required is $7\frac{1}{7}$%. *Ans.*

42. Find the rate per cent when an investment for 5 yrs. 2 mos. produces a sum equal to $\frac{2}{5}$ of the capital.

The time is 5 yrs. 2 mos. = $5\frac{1}{6}$ years.

Interest on \$1 for $5\frac{1}{6}$ yrs. is $\frac{\$2}{5}$;

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$\frac{2}{5}}{5\frac{1}{6}} = \$0.07\tfrac{23}{31}.$$

Therefore, the rate required is $7\frac{23}{31}\%$. *Ans.*

43. Find the rate per cent when an investment for 3 yrs. 1 mo. 15 dys. produces a sum equal to $\frac{1}{8}$ of the capital.

The time is 3 yrs. 1 mo. 15 dys. = $3\frac{1}{8}$ yrs.

Interest on \$1 for $3\frac{1}{8}$ yrs. is $\frac{\$1}{8}$;

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$\frac{1}{8}}{3\frac{1}{8}} = \$0.04.$$

Therefore, the rate required is 4%. *Ans.*

44. Find the time in which the interest on \$450 will amount to \$72, at 4%.

Interest on \$450, at 4%, for 1 yr. is 0.04 of \$450 = \$18.

Therefore, the number of years will be $\frac{\$72}{\$18} = 4$ yrs. *Ans.*

45. Find the time in which the interest on \$487.50 will amount to \$39, at 4%.

Interest on \$487.50, at 4%, for 1 yr. is 0.04 of \$487.50 = \$19.50.

Therefore, the number of years will be $\frac{\$39}{\$19.50} = 2$ yrs. *Ans.*

46. Find the time in which the interest on \$238.75 will amount to \$64.46, at $4\frac{1}{2}\%$.

Interest on \$238.75, at $4\frac{1}{2}\%$, for 1 yr. is 0.045 of \$238.75 = \$10.74.

Therefore, the number of years will be $\frac{\$64.46}{\$10.74} = 6$ yrs. *Ans.*

47. Find the time in which the sum of \$793.875 will amount to \$805.84, at $5\frac{1}{2}\%$.

The interest is \$805.84 − \$793.875 = \$11.965.
Interest on \$793.875, for 1 yr. at $5\frac{1}{2}\%$ is 0.055 of \$793.875 = \$43.663.

Therefore, the number of years will be $\frac{\$11.965}{\$43.663} = 0.274$.

0.274 yr. = 3 mos. 9 dys. *Ans.*

48. Find the time in which a sum of money will double itself at 4%.

Interest on \$1, at 4%, for 1 yr. is 0.04 of \$1 = \$0.04.

Therefore, the number of years will be $\frac{\$1}{\$0.04} = 25$. *Ans.*

49. Find the time in which the sum of \$10 will amount to \$17, at 6%.

The interest is \$17 − \$10 = \$7.
Interest on \$10, at 6%, for 1 yr. is 0.06 of \$10 = \$0.60.

Therefore, the number of years will be $\frac{\$7}{\$0.60} = 11\frac{2}{3}$.

$11\frac{2}{3}$ yrs. = 11 yrs. 8 mos. *Ans.*

50. Find the time in which the sum of \$502.67 will amount to \$578.07, at $4\frac{1}{2}\%$.

The interest is \$587.07 − \$502.67 = \$75.40.
Interest on \$502.67, at $4\frac{1}{2}\%$, for 1 yr. is 0.045 of \$502.67 = \$22.62.

Therefore, the number of years will be $\frac{\$75.40}{\$22.62} = 3\frac{1}{3}$.

$3\frac{1}{3}$ yrs. = 3 yrs. 4 mos. *Ans.*

51. Find the time in which the interest on \$537.50 will amount to \$80.625, at 4%.

Interest on \$537.50, at 4%, for 1 yr. is 0.04 of \$537.50 = \$21.50.

Therefore, the number of years will be $\frac{\$80.625}{\$21.50} = 3.75$.

3.75 yrs. = 3 yrs. 9 mos. *Ans.*

52. Find the time in which the interest on \$6875 will amount to \$75.05, at $4\frac{1}{4}\%$

Interest on \$6875, at $4\frac{1}{4}\%$, for 1 yr. is 0.0425 of \$6875 = \$292.19.

Therefore, the number of years will be $\frac{\$75.05}{\$292.19} = 0.256.$

0.256 yrs. = 3 mos. 2 dys. *Ans.*

53. Find the time in which the interest on \$8520 will amount to \$1746.60, at 6%.

Interest on \$8520, at 6%, for 1 yr. is 0.06 of \$8520 = \$511.20.

Therefore, the number of years will be $\frac{\$1746.60}{\$511.20} = 3\frac{5}{12}.$

$3\frac{5}{12}$ yrs. = 3 yrs. 5 mos. *Ans.*

54. Find the principal that will produce \$90 interest in 3 yrs., at 4%.

Interest for 1 yr. is $\frac{\$90}{3} = \$30.$

Interest on \$1 for 1 yr., at 4% = \$0.04.

Hence, principal required $= \frac{30}{0.04}$ of \$1 = \$750. *Ans.*

55. Find the principal that will produce \$63 interest in 3 yrs., at $6\frac{1}{4}\%$.

Interest for 1 yr. is $\frac{\$63}{3} = \$21.$

Interest on \$1 for 1 yr., at $6\frac{1}{4}\%$ = \$0.0625.

Hence, principal required $= \frac{21}{0.0625}$ of \$1 = \$336. *Ans.*

56. Find the principal that will produce \$100 interest in 8 yrs. 6 mos., at 5%.

8 yrs. 6 mos. = 8.5 yrs.

Interest for 1 yr. $= \frac{\$100}{8.5} = \$11.7647.$

Interest on \$1 for 1 yr., at 5% = \$0.05.

Hence, principal required $= \frac{11.7647}{0.05}$ of \$1 = \$235.29. *Ans.*

57. Find the principal that will produce \$1746.60 interest in 3 yrs. 5 mos., at 6%.

3 yrs. 5 mos. $= 3\frac{5}{12}$ yrs.

Interest for 1 yr. $= \dfrac{\$1746.60}{3\frac{5}{12}} = \$511.20.$

Interest on \$1 for 1 yr., at 6%, is \$0.06.

Hence, principal required $= \dfrac{511.20}{0.06}$ of \$1 = \$8520. *Ans.*

58. Find the principal that will produce \$12 interest in 7 mos., at 5%.

7 mos. $= \frac{7}{12}$ yr.

Interest for 1 yr. $= \dfrac{\$12}{\frac{7}{12}} = \$20.5714.$

Interest on \$1 for 1 yr., at 5% = \$0.05.

Hence, principal required $= \dfrac{20.5714}{0.05}$ of \$1 = \$411.43. *Ans.*

59. Find the principal that will produce \$50 interest in 228 dys., at $4\frac{1}{2}$%.

228 dys. $= \frac{19}{30}$ yr.

Interest for 1 yr. $= \dfrac{\$50}{\frac{19}{30}} = \$78.9474.$

Interest on \$1 for 1 yr., at $4\frac{1}{2}$% = \$0.045.

Hence, principal required $= \dfrac{78.9474}{0.045}$ of \$1 = \$1754.39. *Ans.*

60. Find the principal that will produce \$1339.28 interest in 2 yrs. 7 mos. 24 dys., at 6%.

2 yrs. 7 mos. 24 dys. = 2.65 yrs.

Interest for 1 yr. $= \dfrac{\$1339.28}{2.65} = \$505.3887.$

Interest on \$1 for 1 yr., at 6% = \$0.06.

Hence, principal required $= \dfrac{505.3887}{0.06}$ of \$1 = \$8423.14. *Ans.*

61. Find the principal that will produce $1312.65 interest in 2 yrs. 3 mos., at 6%.

2 yrs. 3 mos. = 2.25 yrs.

Interest for 1 yr. $= \frac{\$1312.65}{2.25} = \$583.40.$

Interest on $1 for 1 yr., at 6% = $0.06.

Hence, principal required $= \frac{583.40}{0.06}$ of $1 = $9723.33. *Ans.*

62. Find the principal that will produce $750 interest in 3 yrs. 8 mos., at 5%.

3 yrs. 8 mos. = $3\frac{2}{3}$ yrs.

Interest for 1 yr. $= \frac{\$750}{3\frac{2}{3}} = \$204.5455.$

Interest on $1 for 1 yr., at 5% = $0.05.

Hence, principal required $= \frac{204.5455}{0.05}$ of $1 = $4090.91. *Ans.*

63. Find the principal that will amount to $840 in 3 yrs., at 4%.

If the principal be represented by 100, the interest will be represented by $3 \times 4 = 12$, and the amount will be represented by 112. Hence, the principal $= \frac{100}{112}$ of $840 = $750. *Ans.*

64. Find the principal that will amount to $901.1384 in 2 yrs. 6 mos., at $4\frac{1}{8}$%.

2 yrs. 6 mos. = $2\frac{1}{2}$ yrs.

If the principal be represented by 100, the interest will be represented by $2\frac{1}{2} \times 4\frac{1}{8} = 10\frac{5}{16}$, and the amount will be represented by $110\frac{5}{16}$.

Hence, the principal $= \frac{100}{110\frac{5}{16}}$ of $901.1384 = $816.896. *Ans.*

65. Find the principal that will amount to $6000 in 21 dys., at 5%.

21 days. $= \frac{7}{60}$ yr.

If the principal be represented by 100, the interest will be represented by $\frac{7}{60} \times 5 = \frac{7}{12}$, and the amount will be represented by $100\frac{7}{12}$.

Hence, the principal $= \frac{100}{100\frac{7}{12}}$ of $6000 = $5982.55. *Ans.*

66. Find the principal that will amount to $\$297.60$ in 8 mos., at 6%.

> 8 mos. $= \frac{2}{3}$ yr.
>
> If the principal be represented by 100, the interest will be represented by $\frac{2}{3} \times 6 = 4$, and the amount will be represented by 104.
>
> Hence, the principal $= \frac{100}{104}$ of $\$297.60 = \$286.15$. *Ans.*

67. Find the principal that will amount to $\$6378.75$ in 1 yr. 1 mo., at 5%.

> 1 yr. 1 mo. $= 1\frac{1}{12}$ yr.
>
> If the principal be represented by 100, the interest will be represented by $1\frac{1}{12} \times 5 = 5\frac{5}{12}$, and the amount will be represented by $105\frac{5}{12}$.
>
> Hence, the principal $= \dfrac{100}{105\frac{5}{12}}$ of $\$6378.75 = \$6050.99$. *Ans.*

68. Find the principal that will amount to $\$21,047.95$ in 1 yr. 7 mos. 21 dys., at $4\frac{1}{2}$%.

> 1 yr. 7 mos. 21 dys. $= 1\frac{77}{120}$ yr.
>
> If the principal be represented by 100, the interest will be represented by $1\frac{77}{120} \times 4\frac{1}{2} = 7\frac{31}{80}$, and the amount will be represented by $107\frac{31}{80}$.
>
> Hence, the principal $= \dfrac{100}{107\frac{31}{80}}$ of $\$21,047.95 = \$19,600$. *Ans.*

69. Find the principal that will amount to $\$185.09$ in 2 yrs. 3 mos. 18 dys., at 5%.

> 2 yrs. 3 mos. 18 dys. $= 2\frac{17}{60}$ yrs.
>
> If the principal be represented by 100, the interest will be represented by $2\frac{17}{60} \times 5 = 11\frac{5}{12}$, and the amount will be represented by $111\frac{5}{12}$.
>
> Hence, the principal $= \dfrac{100}{111\frac{5}{12}}$ of $\$185.09 = \$166$. *Ans.*

70. Find the principal that will amount to $\$659.40$ in 2 yrs. 11 mos. 15 dys., at 6%.

2 yrs. 11 mos. 15 dys. $= 2\frac{23}{24}$ yrs.

If the principal be represented by 100, the interest will be represented by $2\frac{23}{24} \times 6 = 17\frac{3}{4}$, and the amount will be represented by $117\frac{3}{4}$.

Hence, the principal $= \frac{100}{117\frac{3}{4}}$ of \$659.40 = \$560. *Ans.*

71. Find the principal that will amount to \$9437.516 in 2 yrs. 7 mos. 24 dys., at $4\frac{1}{2}\%$.

2 yrs. 7 mos. 24 dys. $= 2\frac{13}{20}$ yrs.

If the principal be represented by 100, the interest will be represented by $2\frac{13}{20} \times 4\frac{1}{2} = 11\frac{37}{40}$, and the amount will be represented by $111\frac{37}{40}$.

Hence, the principal $= \frac{100}{111\frac{37}{40}}$ of \$9437.516 = \$8432. *Ans.*

72. Find the principal that will amount to \$10,266.60 in 3 yrs. 5 mos., at 6%.

3 yrs. 5 mos. $= 3\frac{5}{12}$ yrs.

If the principal be represented by 100, the interest will be represented by $3\frac{5}{12} \times 6 = 20\frac{1}{2}$, and the amount will be represented by $120\frac{1}{2}$.

Hence, the principal $= \frac{100}{120\frac{1}{2}}$ of \$10,266.60 = \$8520. *Ans.*

73. What is the interest of \$195 for 2 yrs. 2 mos. 2 dys., at $6\frac{1}{2}\%$?

$$\begin{array}{lr}
\text{2 yrs.} = & 0.12 \\
\text{2 mos.} = & 0.01 \\
\text{2 dys.} = & \underline{0.000\frac{1}{3}} \\
12) & 0.130\frac{1}{3} \\
 & \underline{0.010\frac{31}{36}} \\
 & 0.141\frac{1}{4}
\end{array}
\qquad\qquad
\begin{array}{r}
\$195 \\
\underline{\times\, 0.141\frac{1}{4}} \\
\$27.53.\ \textit{Ans.}
\end{array}$$

74. At what rate per cent will \$1025.20 produce \$25.72 in 4 mos. 9 dys.?

4 mos. 9 days. = 0.3583 yr.

Interest on \$1025.20 for 0.3583 yr. is \$25.72;

$$\text{on } \$1025.20 \text{ for 1 yr. is } \frac{\$25.72}{0.3583};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$25.72}{0.3583 \times 1025.20} = \$0.07, \text{ nearly.}$$

Hence, the rate required is 7%, nearly. *Ans.*

75. The principal is \$653; the interest \$5.52; the rate 8%. Find the time.

Interest on \$653, at 8%, for 1 yr. is 0.08 of \$653 = \$52.24.

Therefore, the number of years will be $\frac{\$5.52}{52.24} = 0.1056.$

0.1056 yr. = 1 mo. 8 dys. *Ans.*

76. Find the amount of \$520 for 2 mos. 3 dys., at $4\frac{1}{2}$%.

2 mos. = 0.01	\$520
3 dys. = 0.0005	× $0.0078\frac{3}{4}$
4) 0.0105	\$4.10
$0.0026\frac{1}{4}$	520.00
$0.0078\frac{3}{4}$	\$524.10. *Ans.*

77. What sum bearing interest at $4\frac{1}{2}$% will yield an annual income of \$1000?

Interest on \$1 for 1 yr., at $4\frac{1}{2}$% = \$0.045.

Hence, principal required $= \frac{1000}{0.045}$ of \$1 = \$22,222.22. *Ans.*

78. How long will it take \$4000 to produce \$625 interest, at $5\frac{1}{2}$%?

Interest on \$4000, at $5\frac{1}{2}$%, for 1 yr. is 0.055 of \$4000 = \$220.

Therefore, the number of years will be $\frac{\$625}{\$220} = 2.841.$

And 2.841 yrs. = 2 yrs. 10 mos. 3 dys. *Ans.*

79. At what rate per cent will \$3000 produce \$250 interest in 1 yr. 2 mos. 24 dys.?

1 yr. 2 mos. 24 dys. $= 1\frac{7}{30}$ yrs.

Interest on \$3000 for $1\frac{7}{30}$ yrs. is \$250;

$$\text{on } \$3000 \text{ for 1 yr. is } \frac{\$250}{1\frac{7}{30}};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$250}{1\frac{7}{30} \times 3000} = \$0.06\tfrac{28}{37}.$$

Hence, the rate required is $6\frac{28}{37}\%$. *Ans.*

80. Find the interest of \$1721.84 from April 1 to Nov. 12 at $4\frac{1}{2}\%$.

mos.	dys.
11	12
4	1
7	11

$$\begin{aligned} 7 \text{ mos.} &= 0.035 \\ 11 \text{ dys.} &= 0.001\tfrac{5}{6} \\ 4)&\,0.036\tfrac{5}{6} \\ &\,0.009\tfrac{5}{24} \\ &\,0.027\tfrac{5}{8} \end{aligned}$$

$$\begin{array}{r} \$1721.84 \\ \times\, 0.027\tfrac{5}{8} \\ \hline \$47.57. \end{array}$$ *Ans.*

81. How long must \$3904.92 be on interest to amount to \$4568.76, at 5%.

The interest $= \$4568.76 - \$3904.92 = \$663.84$.

Interest on \$3904.92, at 5%, for 1 yr. is 0.05 of \$3904.92 $= \$195.246$.

Therefore, the number of years will be $\frac{\$663.84}{\$195.246} = 3.4$.

And 3.4 yrs. = 3 yrs. 4 mos. 24 dys. *Ans.*

82. Find the interest of \$137.60 from July 3 to Dec. 12, at $7\frac{3}{10}\%$.

mos.	dys.
12	12
7	3
5	9

= 5.3 mos.

$$\begin{array}{rl} \$137.60 & \\ \times\, 0.073 & \\ \hline 12)\$10.04480. & \text{Int. for 1 yr.} \\ \$0.837075. & \text{Int. for 1 mo.} \\ \times\, 5.3 & \\ \hline \$4.44. & \end{array}$$ *Ans.*

83. Find the interest of \$680.20, at $7\frac{1}{2}\%$, for 73 dys., reckoning 365 dys. for a year.

$$73 \text{ dys.} = \tfrac{73}{365} \text{ yr.} = \tfrac{1}{5} \text{ yr.}$$

$$\begin{array}{r} \$680.20 \\ \times\, 0.07\frac{1}{2} \\ \hline 5)\$51.0150 \\ \hline \$10.20. \end{array} \quad \textit{Ans.}$$

Exercise LXXII.

Find *day of maturity*, the *time to run*, the *discount*, and *proceeds* of the following notes:

1. \$750. New York, Jan. 1, 1881.

Four months from date I promise to pay to the order of James Fay seven hundred and fifty dollars, value received.

Discounted at 7%, Jan. 12. John Pray.

The note becomes due 4 mos. from Jan. 1 = May $^{1}/_{4}$.

The time to run is 19 dys. in January, 28 in February, 31 in March, 30 in April, and 4 in May = 112 dys.

The discount is the interest on \$750, for 112 dys., at 7%.

Therefore, the discount is $0.021\frac{7}{9}$ of \$750 = \$16.33.

And the proceeds is \$750 − \$16.33 = \$733.67. *Ans.*

2. \$4325.50. Boston, Jan. 3, 1881.

Sixty days from date I promise to pay to James Finn, or order, four thousand three hundred twenty-five and $\frac{50}{100}$ dollars, value received.

Discounted at $6\frac{1}{2}\%$, Jan. 6. George Bellows.

Counting 60 days, from Jan. 3, there are 28 in January, 28 in February, and 4 in March.

Therefore, the note becomes due March $^{4}/_{7}$.

The time to run is 25 dys. in January, 28 in February, and 7 in March = 60 dys.

The discount is the interest on \$4325.50, for 60 dys., at $6\frac{1}{2}\%$.

Therefore, the discount is $0.010\frac{5}{6}$ of \$4325.50 = \$46.86.

And the proceeds is \$4325.50 − \$46.86 = \$4278.64. *Ans.*

3. $1340.70. RICHMOND, Va., Jan. 6, 1881.

Ninety days from date I promise to pay to the order of Peter Bright thirteen hundred forty and $\frac{70}{100}$ dollars, value received.

Discounted at 7%, Jan. 26. GEORGE WRIGHT.

Counting 90 dys., from Jan. 6, there are 25 in January, 28 in February, 31 in March, and 6 in April.

Therefore, the note becomes due April $^6/_9$.

The time to run is 5 dys. in January, 28 in February, 31 in March, and 9 in April = 73 dys.

The discount is the interest on $1340.70, for 73 dys., at 7%.

Therefore, the discount is $0.014\frac{7}{36}$ of $1340.70 = $19.03.

And the proceeds is $1340.70 – $19.03 = $1321.67. *Ans.*

4. $1456.30. CHARLESTON, S.C., Jan. 19, 1881.

Three months after date I promise to pay to the order of John George fourteen hundred fifty-six and $\frac{30}{100}$ dollars, value received.

Discounted at 5%, Feb. 1. JOHN WALDORF.

The note becomes due 3 mos. from Jan. 19 = April $^{19}/_{22}$.

The time to run is 2 mos. 21 dys.

The discount is the interest on $1456.30, for 2 mos. 21 dys., at 5%.

Therefore, the discount is 0.01125 of $1456.30 = $16.38.

And the proceeds is $1456.30 – $16.38 = $1439.92. *Ans.*

5. $4550.36. DETROIT, Mich., Feb. 2, 1881.

Four months after date I promise to pay to the order of John Callender four thousand five hundred fifty and $\frac{36}{100}$ dollars, value received.

Discounted at $5\frac{1}{2}$%, Feb. 16. JAMES BARTON.

The note becomes due 4 mos. from Feb. 2 = June $^2/_5$.

The time to run is 3 mos. 19 dys.

The discount is the interest on $4550.36, for 3 mos. 19 dys., at $5\frac{1}{2}$%.

Therefore, the discount is $0.0166\frac{1}{3}\frac{1}{8}$ of $4550.36 = $75.78.

And the proceeds is $4550.36 – $75.78 = $4474.58.

6. $5000. CHICAGO, Ill., Oct. 4, 1880.

Six months after date I promise to pay to John Adams, or order, five thousand dollars, value received, with interest at seven per cent.

Discounted at 8%, Dec. 31. WILLIAM DUNN.

Interest on note for 6 mos. 3 dys. = \$177.92.
Amount of note \$5000 + \$177.92 = \$5177.92.
Day of maturity, April $^4/_7$.
Time to run, 3 mos. 6 dys.
Discount on \$5177.92, at 8%, for 3 mos. 6 dys. = \$110.46.
Proceeds is \$5177.92 − \$110.46 = \$5067.46. *Ans.*

7. \$4760. MILWAUKEE, Wis., Jan. 1, 1881.

Ninety days after date I promise to pay to the order of James Pike four thousand seven hundred and sixty dollars, value received.

Discounted at $7\frac{1}{2}$%, Feb. 15. WILLIAM CLEMENT.

Counting 90 dys. from Jan. 1, there are 30 in January, 28 in February, 31 in March, and 1 in April.
Therefore, the note becomes due April $^1/_4$.
The time to run is 13 dys. in February, 31 in March, and 4 in April = 48 dys.
The discount is the interest on \$4760 for 48 dys., at $7\frac{1}{2}$%.
Therefore, the discount is 0.01 of \$4760 = \$47.60.
And the proceeds is \$4760 − \$47.60 = \$4712.40. *Ans.*

8. \$2017.85. KANSAS CITY, Mo., Jan. 14, 1881.

Three months after date I promise to pay to the order of John Brown two thousand seventeen and $\frac{85}{100}$ dollars, value received.

Discounted at 10%, Mar. 1. TIMOTHY BRUCE.

The note becomes due 3 mos. from Jan. 14 = April $^{14}/_{17}$.
The time to run is 1 mo. 16 dys.
The discount is the interest on \$2017.85 for 1 mo. 16 dys., at 10%.
Therefore, the discount is $0.012\frac{7}{9}$ of \$2017.85 = \$25.78.
And the proceeds is \$2017.85 − \$25.78 = \$1992.07. *Ans.*

9. \$652.45. CONCORD, N.H., Jan. 25, 1881.

Five months after date I promise to pay to the order of Charles Barrett six hundred fifty-two and $\frac{45}{100}$ dollars, value received, with interest at six per cent.

Discounted at $4\frac{1}{2}$%, Mar. 15. WILLIAM KIMBALL.

Interest on note for 5 mos. 3 dys. = \$16.64.
Amount of note \$652.45 + \$16.64 = \$669.09.

Day of maturity, June $^{25}/_{28}$.
Time to run, 3 mos. 13 dys.
Discount on \$669.09, at $4\frac{1}{2}\%$, for 3 mos. 13 dys. = \$8.61.
Proceeds is \$669.09 − \$8.61 = \$660.48. *Ans.*

10. \$9040. BALTIMORE, Md., Jan. 19, 1881.

Sixty days from date I promise to pay to the order of Charles Carroll nine thousand and forty dollars, value received.

Discounted at $5\frac{1}{2}\%$, Feb. 16. JAMES MONROE.

Counting 60 dys., from Jan. 19, there are 12 in January, 28 in February, and 20 in March.
Therefore, the note becomes due March $^{20}/_{23}$.
The time to run is 12 dys. in February and 23 in March = 35 dys.
The discount is the interest on \$9040, for 35 dys., at $5\frac{1}{2}\%$.
Therefore, the discount is $0.00\frac{77}{144}$ of \$9040 = \$48.34.
And the proceeds is \$9040 − \$48.34 = \$8991.66. *Ans.*

11. \$215. AUGUSTA, Me., Jan. 28, 1881.

Thirty days after date I promise to pay to the order of James Fogg two hundred and fifteen dollars, value received.

Discounted at 6%, Feb. 3. JOHN MOSES.

Counting 30 dys., from Jan. 28, there are 3 in January and 27 in February.
Therefore, the note becomes due Feb. $^{27}/$March $_{2}$.
The time to run is 25 dys. in February and 2 in March = 27 dys.
The discount is the interest on \$215, for 27 dys., at 6%.
Therefore, the discount is 0.0045 of \$215 = \$0.97.
And the proceeds is \$215 − 0.97 = \$214.03. *Ans.*

12. Find the face of a note at 90 dys. that will realize \$850 when discounted at 7%.

The discount on \$1 for 93 dys. is $\$0.0180\frac{5}{6}$.
Proceeds on \$1 is $\$1 - \$0.0180\frac{5}{6} = \$0.9819\frac{1}{6}$.
Therefore, the face required for \$850 is $\dfrac{\$850}{0.9819\frac{1}{6}} = \$865.65$. *Ans.*

13. Find the face of a note at 4 mos. that will realize \$1600 when discounted at $5\frac{1}{2}\%$.

The discount on \$1 for 4 mos. 3 dys. is $\$0.0187\frac{11}{12}$.
Proceeds on \$1 is $\$1 - \$0.0187\frac{11}{12} = \$0.9812\frac{1}{12}$.
Therefore, the face required for \$1600 is $\dfrac{\$1600}{0.9812\frac{1}{12}} = \$1630.64$.

14. Find the face of a note at 30 dys. that will realize \$1200 when discounted at $6\frac{1}{2}\%$.

The discount on \$1 for 33 dys. is $\$0.0059\frac{7}{12}$.
Proceeds on \$1 is $\$1 - \$0.0059\frac{7}{12} = \$0.9940\frac{5}{12}$.
Therefore, the face required for \$1200 is $\dfrac{\$1200}{0.9940\frac{5}{12}} = \$1207.19$.

15. Find the face of a note at 60 dys. that will realize \$4000 when discounted at 8%.

The discount on \$1 for 63 dys. is \$0.0140.
Proceeds on \$1 is \$1 – \$0.014 = \$0.986.
Therefore, the face required for \$4000 is $\dfrac{\$4000}{0.986} = \$4056.80$. *Ans.*

16. Find the face of a note at 2 mos. that will realize \$4500 when discounted at $7\frac{3}{10}\%$.

The discount on \$1 for 63 dys. is \$0.012775.
Proceeds on \$1 is \$1 – \$0.012775 = \$0.987225.
Therefore, the face required for \$4500 is $\dfrac{\$4500}{0.987225} = \$4558.23$.

17. Find the face of a note at 3 mos. that will realize \$1100 when discounted at 7%.

The discount on \$1 for 3 mos. 3 dys. is $\$0.01808\frac{2}{3}$.
Proceeds on \$1 is $\$1 - \$0.01808\frac{2}{3} = \$0.98191\frac{1}{3}$.
Therefore, the face required for $\$1100 = \dfrac{\$1100}{0.98191\frac{1}{3}} = \$1220.26$.

18. Find the present worth of \$500 due in 11 mos. at 5%.

If 100 be taken to represent the present worth, the discount will be represented by $\frac{11}{12} \times 5 = 4\frac{7}{12}$.

The given sum will be represented by $104\frac{7}{12}$.

Hence, the present worth is $\frac{100}{104\frac{7}{12}}$ of \$500 = \$478.09. *Ans.*

19. Find the present worth and discount of \$3334.62 due in 2 yrs., at $4\frac{1}{2}\%$.

If 100 be taken to represent the present worth, the discount will be represented by $2 \times 4\frac{1}{2} = 9$.

The given sum will be represented by 109.

Hence, the present worth is $\frac{100}{109}$ of \$3334.62 = \$3059.28; and the discount is \$3334.62 − \$3059.28 = \$275.34. *Ans.*

20. Find the present worth and discount of \$4261.33 due at the end of 1 yr. 6 mos., at 6%.

If 100 be taken to represent the present worth, the discount will be represented by $1\frac{1}{2} \times 6 = 9$.

The given sum will be represented by 109.

Hence, the present worth is $\frac{100}{109}$ of \$4261.33 = \$3909.48.

The discount is \$4261.33 − \$3909.48 = \$351.85. *Ans.*

21. Find the present worth and discount of \$2416.50 due in 7 mos., at 5%.

If 100 be taken to represent the present worth, the discount will be represented by $\frac{7}{12} \times 5 = 2\frac{11}{12}$.

The given sum will be represented by $102\frac{11}{12}$.

Hence, the present worth is $\frac{100}{102\frac{11}{12}}$ of \$2416.50 = \$2348.02, and the discount is \$2416.50 − \$2348.02 = \$68.48. *Ans.*

22. Find the present worth of \$678.40 due in 16 mos., at $4\frac{1}{2}\%$.

If 100 be taken to represent the present worth, the discount will be represented by $1\frac{1}{3} \times 4\frac{1}{2} = 6$.

The given sum will be represented by 106.

Hence, the present worth is $\frac{100}{106}$ of \$678.40 = \$640. *Ans.*

23. Find the present worth and discount of \$574.17 due in 2 yrs. 3 mos., at $5\frac{1}{2}\%$.

If 100 be taken to represent the present worth, the discount will be represented by $2\frac{1}{4} \times 5\frac{1}{3} = 12$.

The given sum will be represented by 112.

Hence, the present worth is $\frac{100}{112}$ of $\$574.17 = \$512.65$; and the discount is $\$574.17 - \$512.65 = \$61.52$. *Ans.*

24. Find the present worth and discount of $625.13 due in 8 mos., at $7\frac{3}{10}\%$.

If 100 be taken to represent the present worth, the discount will be represented by $\frac{2}{3} \times 7\frac{3}{10} = 4\frac{13}{15}$.

The given sum will be represented by $104\frac{13}{15}$.

Hence, the present worth is $\frac{100}{104\frac{13}{15}}$ of $\$625.13 = \$596.12$; and the discount is $\$625.13 - \$596.12 = \$29.01$. *Ans.*

25. Find the present worth and discount of $715.20 due in 1 yr. 4 mos., at $3\frac{1}{2}\%$.

If 100 be taken to represent the present worth, the discount will be represented by $1\frac{1}{3} \times 3\frac{1}{2} = 4\frac{2}{3}$.

The given sum will be represented by $104\frac{2}{3}$.

Hence, the present worth is $\frac{100}{104\frac{2}{3}}$ of $\$715.20 = \$683.31$; and the discount is $\$715.20 - \$683.31 = \$31.89$. *Ans.*

26. If I buy goods Jan. 10, at 30 dys., for $218; Feb. 5, at 60 dys., for $421; and pay Feb. 10, $200, March 17, $50, what is due April 25, interest at 6%?

$218 + 2 mos. 16 dys. interest =		$220.80
$421 + 19 dys. interest =		422.33
		$643.13
$200 + 2 mos. 15 dys. interest =	$202.50	
$50 + 1 mo. 8 dys. interest =	50.32	
		$252.82
Balance due,		$390.31. *Ans.*

27. A note for $618.75, dated April 17, 1880, payable on demand, bears the following endorsements: June 5, $126.50; Aug. 20, $137.25; Nov. 17, $210. What is due Jan. 1, 1881, reckoning interest at 6%?

Amount of $618.75 for 8 mos. 14 dys. =		$644.94
Amount of $126.50 for 6 mos. 26 dys. =	$130 84	
Amount of $137.25 for 4 mos. 11 dys. =	140.25	
Amount of $210 for 1 mo. 14 dys. =	211.54	
		$482.63
Balance due,		$162.31. *Ans.*

28. A note for $1000, dated April 1, 1880, payable on demand, with interest at 7%, bears the following endorsements: May 6, $200; July 5, $225.37; Oct. 18, $322. What is due Jan. 1, 1881?

Amount of $1000 for 9 mos. =		$1052.50
Amount of $200 for 7 mos. 25 dys. =	$209.14	
Amount of $225.37 for 5 mos. 26 dys. =	233.08	
Amount of $322 for 2 mos. 13 dys. =	326.57	
		$768.79
Balance due,		$283.71. *Ans.*

29. A note for $835.25, dated July 1, 1880, payable on demand, with interest at 6½%, bears the following endorsements: Aug. 20, $157.50; Sept. 21, $180.25; Oct. 5, $200; Dec. 1, $80. What is due Jan. 1, 1881?

Amount of $835.25 for 6 mos. =		$862.39
Amount of $157.50 for 4 mos. 11 dys. =	$161.23	
Amount of $180.25 for 3 mos. 10 dys. =	183.49	
Amount of $200 for 2 mos. 26 dys. =	203.11	
Amount of $80 for 1 mo. =	80.43	
		$628.26
Balance due,		$234.13. *Ans.*

30. A note of $2000, dated Jan. 22, 1880, and drawing interest at 6%, had payments endorsed upon it as follows: May 20, 1880, $100; July 20, 1880, $325; Nov. 2. 1880, $20; Dec. 23, 1880, $125. Find the balance due March 1, 1881.

yr.	mos.	dys.				
					$2000	
1880	5	20			0.019⅔	
1880	1	22			$39.33	1st interest.
	3	28	0.019⅔		2000.00	
					$2039.33	
	$100.				100.00	1st payment.
1880	7	20			$1939.33	2d principal.
1880	5	20			0.01	
	2		0.01		$19.40	2d interest.
					1939.33	
					$1958.73	
	$325.				325.00	2d payment.
1880	11	2			$1633.73	3d principal.
1880	7	20			0.017	
	3	12	0.017	$20	$27.78	3d interest.
					$1633.73	3d principal.
	$20.				0.0085	
1880	12	23		$125	$13.88	4th interest.
1880	11	2			27.78	3d interest.
					1633.73	
	1	21	0.0085		$1675.39	
					145.00	3d & 4th payments.
	$570.				$1530.39	4th principal.
1881	3	1			0.011⅓	
1880	12	23			$17.34	5th interest.
					1530.39	
	2	8	0.011⅓		$1547.73	*Ans.*

31. A note of $1662.50, dated Jan. 15, 1880, and drawing interest at $6\frac{1}{2}\%$, had payments endorsed upon it as follows: April 30, 1880, $25; June 24, 1880, $25; Sept. 2, 1880, $625; Jan. 31, 1881, $700. Find the balance due May 12, 1881.

yr.	mos.	dys.	
1880	4	30	
1880	1	15	
	3	15	$0.0189\frac{7}{12}$
	$25.		
1880	6	24	
1880	4	30	
	1	24	$0.0097\frac{1}{2}$
	$25.		
1880	9	2	
1880	6	24	
	2	8	$0.012\frac{5}{18}$
	$625.		
1881	1	31	
1880	9	2	
	4	29	$0.026\frac{65}{72}$
	$700.		
1881	5	12	
1881	1	31	
	3	11	$0.018\frac{17}{72}$

	$1662.50	
	$0.0189\frac{7}{12}$	
$25	$31.52	1st interest.
	$1662.50	1st principal.
	$0.0097\frac{1}{2}$	
$25	$16.21	2d interest.
	31.52	1st interest.
	1662.50	
	$1710.23	
	50.00	1st & 2d payments.
	$1660.23	2d principal.
	$0.012\frac{5}{18}$	
	$20.38	3d interest.
	1660.23	
	$1680.61	
	625.00	3d payment.
	$1055.61	
	$0.026\frac{65}{72}$	
	$28.40	4th interest.
	1055.61	
	$1084.01	
	700.00	4th payment.
	$384.01	4th principal.
	$0.018\frac{17}{72}$	
	$7.00	
	384.01	
	$391.01	*Ans.*

32. A note of $4560, dated Jan. 22, 1879, and drawing interest at 7%, had payments endorsed upon it as follows: Jan. 10, 1880, $2000; Aug. 31, 1880, $500; Jan. 15, 1881, $1200; March 4, 1881, $860. Find the balance due June 15, 1881.

yr.	mos.	dys.	
1880	1	10	
1879	1	22	
	11	18	$0.067\frac{2}{3}$

$4560
$0.067\frac{2}{3}$
$308.56 1st interest.
4560.00
$4868.56
2000.00 1st payment.
$2868.56 2d principal.

$2000.

1880	8	31	
1880	1	10	
	7	21	$0.0449\frac{1}{6}$

$2868.56 2d principal.
$0.0449\frac{1}{6}$
$128.85 2d interest.
2868.56
$2997.41
500.00 2d payment.
$2497.41 3d principal.

$500.

1881	1	15	
1880	8	31	
	4	14	$0.026\frac{1}{18}$

$2497.41 3d principal.
$0.026\frac{1}{18}$
$65.07 3d interest.
2497.41
$2562.48
1200.00 3d payment.
$1362.48 4th principal.

$1200.

1881	3	4	
1881	1	15	
	1	19	$0.009\frac{19}{36}$

$1362.48 4th principal.
$0.009\frac{19}{36}$
$12.98 4th interest.
1362.48
$1375.46
860.00 4th payment.
$515.46 5th principal.

$860.

1881	6	15	
1881	3	4	
	3	11	$0.018\frac{23}{36}$

$515.46 5th principal.
$0.018\frac{23}{36}$
$10.12 5th interest.
$515.46
$525.58 *Ans.*

33. A note of $785.50, dated Jan. 30, 1879, and drawing interest at 5%, had payments endorsed upon it as follows: July 17, 1879, $100; Jan. 29, 1880, $100; Dec. 31, 1880, $20; Mar. 16, 1881, $300; June 14, 1881, $50. Find the balance due July 23, 1881.

yr.	mos.	dys.	
1879	7	17	
1879	1	30	
	5	17	$0.023\frac{7}{36}$
	$100.		
1880	1	29	
1879	7	17	
	6	12	$0.02\frac{2}{3}$
	$100.		
1880	12	31	
1880	1	29	
	11	2	$0.046\frac{1}{9}$
	$20.		
1881	3	16	
1880	12	31	
	2	15	$0.010\frac{5}{12}$
	$300.		
1881	6	14	
1881	3	16	
	2	28	$0.01\frac{2}{9}$
	$50.		
1881	7	23	
1881	6	14	
	1	9	$0.005\frac{5}{12}$

	$785.50	
	$0.023\frac{7}{36}$	
	$18.22	1st interest.
	785.50	
	$803.72	
	100.00	1st payment.
	$703.72	2d principal.
	$0.02\frac{2}{3}$	
	$18.77	2d interest.
	703.72	
	$722.49	
	100.00	2d payment.
	$622.49	3d principal.
	$0.046\frac{1}{9}$	
$20	$28.71	3d interest.
	$622.49	3d principal.
	$0.010\frac{5}{12}$	
$300	$6.48	4th interest.
	28.71	3d interest.
	622.49	
	$657.68	
	320.00	3d & 4th payments.
	$337.68	4th principal.
	$0.01\frac{2}{9}$	
	$4.13	5th interest.
	337.68	
	$341.81	
	50.00	5th payment.
	$291.81	5th principal.
	$0.005\frac{5}{12}$	
	$1.58	6th interest.
	291.81	
	$293.39	*Ans.*

34. A note of $300.25, dated Aug. 4, 1879, and drawing interest at $6\frac{1}{2}\%$, had payments endorsed upon it as follows: Oct. 14, 1879, $100; July 21, 1880, $100; Oct. 11, 1880, $50; Jan. 18, 1881, $50. Find the amount due July 22, 1881.

yr.	mos.	dys.	
1879	10	14	
1879	8	4	
	2	10	$0.012\frac{23}{36}$
	$100.		
1880	7	21	
1879	10	14	
	9	7	$0.049\frac{27}{40}$
	$100.		
1880	10	14	
1880	7	21	
	2	23	$0.014\frac{71}{72}$
	$50.		
1881	1	18	
1880	10	11	
	3	7	$0.017\frac{17}{72}$
	$50.		
1881	7	22	
1881	1	18	
	6	4	$0.033\frac{2}{9}$

$300.25	
$0.012\frac{23}{36}$	
$3.79	1st interest.
300.25	
$304.04	
100.00	1st payment.
$204.04	2d principal.
$0.049\frac{27}{40}$	
$10.14	2d interest.
204.04	
$214.18	
100.00	2d payment.
$114.18	3d principal.
$0.014\frac{71}{72}$	
$1.71	3d interest.
114.18	
$115.89	
50.00	3d payment.
$65.89	4th principal.
$0.017\frac{37}{72}$	
$1.15	4th interest.
65.89	
$67.04	
50.00	4th payment.
$17.04	5th principal.
$0.033\frac{2}{9}$	
$0.57	5th interest.
17.04	
$17.61	*Ans.*

35. Find the amount of $356.25 in 4 yrs., at 5%, compound interest.

$356.25
0.05
$17.81
365.25
$374.06
0.05
$18.70
374.06
$392.76
0.05
$19.64
392.76
$412.40
0.05
$20.62
$412.40
$433.02 *Ans.*

36. Find the amount of $637.50 in 2 yrs. 6 mos., at 4%, compound interest.

$637.50
0.04
$25.50
637.50
$663.00
0.04
$26.52
663.00
$689.52
0.02
$13.79
689.52
$703.31 *Ans.*

37. Find the compound interest of $800 in 3 yrs. 9 mos., at 6%.

$800.00
0.06
$48.00
800.00
$848.00
0.06
$50.88
848.00
$898.88
0.06
$53.93
898.88
$952.81
0.04½
$42.88
952.81
$995.69
800.00
$195.69 *Ans.*

38. Find the compound interest of $39.35 in 1 yr. 9 mos., at 5%.

$39.35
0.05
$1.97
39.35
$41.32
0.03¾
$1.55
41.32
$42.87
39.35
$3.52 *Ans.*

39. Find the compound interest of \$300 in 2 yrs., at 4%, interest being payable semi-annually.

$$
\begin{array}{r}
\$300 \\
0.02 \\ \hline
\$6.00 \\
300.00 \\ \hline
\$306.00 \\
0.02 \\ \hline
\$6.12 \\
306.00 \\ \hline
\$312.12
\end{array}
\qquad\qquad
\begin{array}{r}
\$312.12 \\
0.02 \\ \hline
\$6.24 \\
312.12 \\ \hline
\$318.36 \\
0.02 \\ \hline
\$6.37 \\
318.36 \\ \hline
\$324.73 \\
300.00 \\ \hline
\$24.73 \text{ \textit{Ans.}}
\end{array}
$$

40. Find the compound interest of \$525 in 1 yr. 6 mos., at 5%, interest being payable semi-annually.

$$
\begin{array}{r}
\$525 \\
0.01\frac{1}{4} \\ \hline
\$6.56 \\
525.00 \\ \hline
\$531.56 \\
0.01\frac{1}{4} \\ \hline
\$6.64 \\
531.56 \\ \hline
\$538.20 \\
0.01\frac{1}{4} \\ \hline
\$6.73 \\
538.20 \\ \hline
\$544.93
\end{array}
\qquad\qquad
\begin{array}{r}
\$544.93 \\
0.01\frac{1}{4} \\ \hline
\$6.81 \\
544.93 \\ \hline
\$551.74 \\
0.01\frac{1}{4} \\ \hline
\$6.90 \\
551.74 \\ \hline
\$558.64 \\
0.01\frac{1}{4} \\ \hline
\$6.98 \\
558.64 \\ \hline
\$565.62 \\
525.00 \\ \hline
\$40.62 \text{ \textit{Ans.}}
\end{array}
$$

41. Find the compound interest of \$10,000 in 6 mos., at 6%, interest being paid monthly.

\$10000	\$10150.75
0.005	0.005
\$50.00	\$50.75
10000.00	10150.75
\$10050.00	\$10201.50
0.005	0.005
\$50.25	\$51.01
10050.00	10201.50
\$10100.25	\$10252.51
0.005	0.005
\$50.50	\$51.26
10100.25	10252.51
\$10150.75	\$10303.77
	10000.00
	\$303.77 *Ans.*

42. What principal will amount to \$137.81 in 2 yrs., at 5% compound interest?

The amount of \$1 for 2 yrs. at 5% is \$1 $\times 1.05^2 =$ \$1.1025.
The principal is \$137.81 $\div$ 1.1025 = \$125. *Ans.*

43. What principal will amount to \$1860.96 in 3 yrs., at 6% compound interest?

The amount of \$1 for 3 yrs. at 6% is \$1 $\times 1.06^3 =$ \$1.191016.
The principal is \$1860.96 $\div$ 1.191016 = \$1562.50. *Ans.*

44. What principal will amount to \$1500 in 1 yr., at 4% compound interest, payable quarterly?

The amount of \$1 for 1 yr. at 4%, payable quarterly, is
\$1 $\times 1.01^3 =$ \$1.04060401.
The principal is \$1500 $\div$ 1.04060401 = \$1441.47. *Ans.*

45. What principal will produce \$100 in 1 yr. 6 mos., at 6% compound interest, payable semi-annually?

The amount of \$1 for 1 yr. 6 mos., payable semi-annually, is $\$1 \times 1.03^3 = \$1.092727$.
The interest is $\$1.092727 - \$1 = \$0.092727$.
The principal is $\$100 \div 0.092727 = \$1078.43$. *Ans.*

46. Find the interest due May 19, 1881, on a note dated Dec. 26, 1877, for \$1224.60, with interest payable annually at 5%, when no interest has been paid.

yrs.	mos.	dys.	
1881	5	19	
1877	12	26	
3	4	23	$0.203\frac{5}{6}$

$$\begin{array}{rl} \$1224.60 & \\ 0.203\frac{5}{6} & \\ \hline 6)249.61 & \\ 41.60 & \\ \hline \$208.01 & \text{interest for 3 yrs. 4 mos. 23 dys.} \end{array}$$

yrs.	mos.	dys.
2	4	23
1	4	23
	4	23
4	2	9 $= 4\frac{23}{120}$ yrs.

$$\begin{array}{rl} \$1224.60 & \\ 0.05 & \\ \hline \$61.23 & \text{annual interest.} \\ 0.05 & \\ \hline \$3.0615 & \text{interest on annual interest.} \\ 4\frac{23}{120} & \\ \hline \$12.83 & \\ 208.01 & \\ \hline \$220.84 & \text{total interest. } \textit{Ans.} \end{array}$$

47. Find the amount due May 27, 1881, on a note dated Jan. 4, 1879, for \$215.50, with interest payable annually at 5½%, when no interest has been paid.

yrs.	mos.	dys.	
1881	5	27	
1879	1	4	
2	4	23	$0.143\frac{5}{6}$

$$\begin{array}{rl} \$215.50 & \\ 0.143\frac{5}{6} & \\ \hline 12)30.996 & \\ 2.583 & \\ \hline \$28.41 & \text{interest for 2 yrs. 4 mos. 23 dys.} \end{array}$$

yrs.	mos.	dys.
1	4	23
	4	23
1	9	16 $= 1\frac{143}{180}$ yrs.

\$215.50	
$0.05\frac{1}{2}$	
\$11.85	annual interest.
$0.05\frac{1}{2}$	
\$0.6518	
$1\frac{143}{180}$	
\$1.17	interest on annual interest.
28.41	
\$29.58	total interest.
215.50	
\$245.08	*Ans.*

48. Find the amount due Jan. 16, 1881, on a note dated Jan. 8, 1879, for \$3115.20, with interest payable annually at 5%, when no interest has been paid.

yrs.	mos.	dys.	
1881	1	16	
1879	1	8	
2	0	8	$0.10\frac{1}{9}$

yrs.	mos.	dys.
1	0	8
		8
1	0	16 $= 1\frac{2}{45}$ yrs.

\$3115.20	
$0.10\frac{1}{9}$	
\$314.981	interest for 2 yrs. 8 dys.
\$3115.20	
0.05	
\$155.76	annual interest.
0.05	
\$7.788	
$1\frac{2}{45}$	
\$8.134	interest on annual interest.
314.981	
\$323.12	interest.
3115.20	
\$3438.32	*Ans.*

49. Find the amount due Jan. 18, 1881, on a note dated Jan. 8, 1877, for \$2875, at 6%: (1) simple interest; (2) annual interest; (3) compound interest.

(1)

yrs.	mos.	dys.
1881	1	18
1877	1	8
4	0	10

$0.241\frac{2}{3}$

$$\begin{array}{r} \$2875 \\ 0.241\frac{2}{3} \\ \hline \$694.79 \\ 2875.00 \\ \hline \$3569.79 \end{array}$$ *Ans.*

(2)

yrs.	mos.	dys.
3	0	10
2	0	10
1	0	10
		10
6	1	10

$= 6\frac{1}{9}$ yrs.

Simple interest $= \$694.79$.

$$\begin{array}{r} \$2875 \\ 0.06 \\ \hline \$172.50 \\ 0.06 \\ \hline \$10.35 \\ 6\frac{1}{9} \\ \hline \$63.25 \\ 694.79 \\ 2875.00 \\ \hline \$3633.04 \end{array}$$ *Ans.*

(3)

$$\begin{array}{r} \$2875 \\ 0.06 \\ \hline \$172.50 \\ 2875.00 \\ \hline \$3047.50 \\ 0.06 \\ \hline \$182.85 \\ 3047.50 \\ \hline \$3230.35 \\ 0.06 \\ \hline \$193.82 \\ 3230.35 \\ \hline \$3424.17 \\ 0.06 \\ \hline \$205.45 \\ 3424.17 \\ \hline \$3629.62 \\ 0.001\frac{2}{3} \\ \hline \$6.05 \\ 3629.62 \\ \hline \$3635.67 \end{array}$$ *Ans.*

Exercise LXXIII.

1. Find the cost of \$4000 stock, at $109\frac{7}{8}$.

If \$1 stock cost $\$1.09\frac{7}{8}$, \$4000 stock will cost $4000 \times \$1.09\frac{7}{8} = \$4395$. *Ans.*

2. Find the cost of \$2500 stock, at 98.

If \$1 stock cost \$0.98, \$2500 stock will cost $2500 \times \$0.98 = \$2450$. *Ans.*

3. Find the cost of \$3900 stock, at $78\frac{1}{8}$.

If \$1 stock cost $\$0.78\frac{1}{8}$, \$3900 stock will cost
$3900 \times \$0.78\frac{1}{8} = \$3046.88$. *Ans.*

4. Find the cost of \$4700 stock, at $100\frac{1}{2}$.

If \$1 stock cost $\$1.00\frac{1}{2}$, \$4700 stock will cost
$4700 \times \$1.00\frac{1}{2} = \$4723.50$. *Ans.*

5. Find the cost of \$1250 stock, at $87\frac{5}{8}$, brokerage $\frac{1}{8}$.

If $\frac{1}{8}$ be the brokerage, $87\frac{5}{8} + \frac{1}{8} = 87\frac{3}{4}$ is total cost.
If \$1 stock cost $\$0.87\frac{3}{4}$, \$1250 stock will cost
$1250 \times \$0.87\frac{3}{4} = \$1096.88$. *Ans.*

6. How much bank stock, at $75\frac{1}{4}$, may be bought for \$8729?

If $\$0.75\frac{1}{4}$ buys \$1 stock, \$8729 will buy $\frac{\$8729}{0.75\frac{1}{4}} = \$11600$. *Ans.*

7. How much railroad stock, at $91\frac{1}{8}$, may be bought for $\$4237\frac{5}{16}$?

If $\$0.91\frac{1}{8}$ buy \$1 stock, $\$4237\frac{5}{16}$ will buy $\frac{\$4237\frac{5}{16}}{0.91\frac{1}{8}} = \$4650$.

8. How much railroad stock may be bought for \$6305, at $121\frac{1}{4}$?

If $\$1.21\frac{1}{4}$ buy \$1 stock, \$6305 will buy $\frac{\$6305}{1.21\frac{1}{4}} = \$5200$. *Ans.*

9. How much railroad stock may be bought for \$5137.50, at $102\frac{3}{4}$?

If $\$102\frac{3}{4}$ buy \$1 stock, \$5137.50 will buy $\frac{\$5137.50}{1.02\frac{3}{4}} = \$5000$.

10. How many \$100 railroad shares, at $68\frac{3}{4}$, may be bought for \$1650?

If 1 share cost $\$68\frac{3}{4}$, $\frac{\$1650}{68\frac{3}{4}} = 24$ shares can be bought for \$1650.

11. What must be the price of stock, in order that \$9200 stock may be bought for \$8970?

If \$9200 stock cost \$8970,
then \$1 stock costs $\$8970 \div 9200 = 0.97\frac{1}{2}$.
Hence, the stock is quoted at $97\frac{1}{2}$. *Ans.*

12. If \$1500 stock be bought for \$1374.375, what is the price of the stock?

If \$1500 stock cost \$1374.375,
then \$1 stock costs $\$1374.375 \div 1500 = \$0.91\frac{5}{8}$.
Hence, the stock is quoted at $91\frac{5}{8}$. *Ans.*

13. What income will be derived from \$29,700 4% stock?

0.04 of $\$29,700 = \$1188$. *Ans.*

14. Find the income from \$4500 6% stock.

0.06 of $\$4500 = \$270$. *Ans.*

15. How much will a person receive from \$9400 railroad stock if a dividend of $3\frac{1}{2}\%$ be declared?

0.035 of $\$9400 = \$329$. *Ans.*

16. How much 8% stock must be bought to give an income of \$2400?

Since \$0.08 is derived from \$1 stock, \$2400 will be derived from $\$2400 \div 0.08 = \$30,000$. *Ans.*

17. A person receives \$343 as his semi-annual dividend from a 7% stock. How much stock does he hold?

Since \$0.07 is derived from \$1 stock, $2 \times \$343 = \$686$ will be derived from $\$686 \div 0.07 = \$9800$. *Ans.*

18. Find the total income of a person whose property consists of \$3000 6% stocks and \$8200 7% stocks.

0.06 of $\$3000 = \$180$, income from 6% stocks.
0.07 of $\$8200 = \$574$, income from 7% stocks.
$\$180 + \$574 = \$754$, total income. *Ans.*

19. Find the rate of dividend paid by some stock, when a holder of \$24,600 receives \$924.50.

If 100 be taken to represent \$24,600, the number required to represent \$924.50 will be $\frac{\$924.50}{\$24600}$ of $100 = 3\frac{373}{492}$.

That is, $3\frac{373}{492}\%$. *Ans.*

20. Find the rate per cent at which \$11,100 will yield a semi-annual return of \$499.50.

If 100 be taken to represent \$11,100, the number required to represent $2 \times \$499.50 = \$999$ will be $\frac{\$999}{\$11100}$ of $100 = 9$.

That is, 9%. *Ans.*

21. If \$19,500 be invested in 4% stock, at 91, what income will be received?

\$91 is the cost of \$100 stock.

Hence, $\$19{,}500 = \text{cost of } \frac{\$19500}{0.91} = \$21{,}428.57$ stock.

And 0.04 of $\$21{,}428.57 = \$857.14$. *Ans.*

22. Find the income on \$7000 when invested in 4% stock, at $103\frac{1}{4}$.

$\$103\frac{1}{4}$ is the cost of \$100 stock.
Hence, $\$7000 = \text{cost of } \$7000 \div 1.03\frac{1}{4} = \$6779.66$.
And 0.04 of $\$6779.66 = \$271.19$. *Ans.*

23. What income will be derived from \$6800 if it be invested in 7% stocks, at 130?

\$130 is the cost of \$100 stock.
Hence, $\$6800 = \text{cost of } \$6800 \div 1.30 = \$5230.77$.
And 0.07 of $\$5230.77 = \$366.15$. *Ans.*

24. A person invests \$7650 in railroad stock, at $63\frac{3}{4}$. What will he receive if a dividend of $3\frac{1}{4}\%$ be declared?

$\$63\frac{3}{4}$ is the cost of \$100 stock.
Hence, $\$7650 = \text{cost of } \$7650 \div 0.63\frac{3}{4} = \$12{,}000$.
And $0.03\frac{1}{4}$ of $\$12{,}000 = \$390$. *Ans.*

25. If 3% stocks are at $88\frac{1}{8}$, what rate per cent interest will a purchaser receive on his money?

\$100 stock costs $\$88\frac{1}{8}$. \$100 stock pays \$3.
Hence, the $\$88\frac{1}{8}$ invested pays \$3.
Therefore, the rate of interest is $3 \div 88\frac{1}{8} = 0.03\frac{19}{47}$ or $3\frac{19}{47}\%$. *Ans.*

26. If an 8% stock is at 150, what rate per cent interest will a purchaser receive on his money?

$100 stock costs $150. $100 stock pays $8.
Hence, the $150 invested pays $8.
Therefore, the rate of interest is $8 \div 150 = 0.05\frac{1}{3}$ or $5\frac{1}{3}\%$. *Ans.*

27. If a 10% stock is at 175, what rate per cent interest will an investor receive on his money?

$100 stock costs $175. $100 stock pays $10.
Hence, the $175 invested pays $10.
Therefore, the rate of interest is $10 \div 175 = 0.05\frac{5}{7}$ or $5\frac{5}{7}\%$. *Ans.*

28. If a $4\frac{1}{2}\%$ stock is at 85, what rate per cent interest will a purchaser receive on his money?

$100 stock costs $85. $100 stock pays $$4\frac{1}{2}$.
Hence, the $85 invested pays $$4\frac{1}{2}$.
Therefore, the rate of interest is $4\frac{1}{2} \div 85 = 0.05\frac{5}{17} = 5\frac{5}{17}\%$. *Ans.*

29. If a 7% stock is at 114, what rate per cent interest will a purchaser receive on his money?

$100 stock costs $114. $100 stock pays $7.
Hence, the $114 invested pays $7.
Therefore, the rate of interest is $7 \div 114 = 0.06\frac{8}{57}$ or $6\frac{8}{57}\%$. *Ans.*

30. If a 6% stock is at 130, what rate per cent interest will a purchaser receive on his money?

$100 stock costs $130. $100 stock pays $6.
Hence, the $130 invested pays $6.
Therefore, the rate of interest is $6 \div 130 = 0.04\frac{8}{13}$ or $4\frac{8}{13}\%$. *Ans.*

31. If an 8% stock is at 140, what rate per cent interest will a purchaser receive on his money?

$100 stock costs $140. $100 stock pays $8.
Hence, the $140 invested pays $8.
Therefore, the rate of interest is $8 \div 140 = 0.05\frac{5}{7}$ or $5\frac{5}{7}\%$ *Ans.*

32. How much money must be invested in 4% stock, at 92, to receive \$245 income?

> \$100 stock costs \$92. \$100 stock pays \$4.
> Hence, the \$92 invested pays \$4.
> Therefore, the rate of interest is $4 \div 92 = 0.04\frac{8}{23}$ or $4\frac{8}{23}$%.
> Since $\$0.04\frac{8}{23}$ is derived from \$1 stock, \$245 will be derived from $\$245 \div 0.04\frac{8}{23} = \$5635$. *Ans.*

33. Find the sum required for an investment in a 4% stock, at $98\frac{1}{2}$, to produce an income of \$200 a year.

> \$100 stock cost $\$98\frac{1}{2}$. \$100 stock pays \$4.
> Hence, the $\$98\frac{1}{2}$ invested pays \$4.
> Therefore, the rate of interest is $4 \div 98\frac{1}{2} = 0.04\frac{12}{197}$ or $4\frac{12}{197}$%.
> Since $\$0.04\frac{12}{197}$ is derived from \$1 stock, \$200 will be derived from $\$200 \div 0.04\frac{12}{197} = \$4925$. *Ans.*

34. A person bought some bank stock at 107, and received \$384.25 when a dividend of $7\frac{1}{4}$% was paid. How much had he invested?

> Since $\$0.07\frac{1}{4}$ is derived from \$1 stock, \$384.25 will be derived from $\$384.25 \div 0.07\frac{1}{4} = \$5300$.
> \$5300 is the cost of 53 shares at 100.
> Therefore, the cost of 53 shares at 107 will be $53 \times \$107 = \$5671$.

35. What must be the price of a 5% stock, in order that a buyer may receive 6% on his money?

> If \$100 pay \$5 at 5%, the number of dollars required to pay \$5 at 6% will be $\frac{5}{6}$ of $\$100 = \$83\frac{1}{3}$.
> That is, the stock is quoted at $83\frac{1}{3}$. *Ans.*

36. What must be the price of a 7% stock, in order that a buyer may receive 6% interest on his money?

> If \$100 pay \$7 at 7%, the number of dollars required to pay \$7 at 6% will be $\frac{7}{6}$ of $\$100 = \$116\frac{2}{3}$.
> That is, the price of the stock will be $116\frac{2}{3}$. *Ans.*

37. What may be paid for an 8% stock, in order that a buyer may receive 6% interest on his money? for a 9% stock? for a 10% stock?

If $100 pay $8 at 8%, the number of dollars required to pay $8 at 6% will be $\frac{8}{6}$ of $100 = 133\frac{1}{3}$.

That is, the price of the stock will be 133$\frac{1}{3}$. (1) *Ans.*

If $100 pay $9 at 9%, the number of dollars required to pay $9 at 6% will be $\frac{9}{6}$ of $100 = $150.

That is, the price of the stock will be 150. (2) *Ans.*

If $100 pay $10 at 10%, the number of dollars required to pay $10 at 6% will be $\frac{10}{6}$ of $100 = 166\frac{2}{3}$.

That is, the price of the stock will be 166$\frac{2}{3}$. (3) *Ans.*

38. A person invested $2855 in a bank, when the stock was at 142$\frac{3}{4}$. What is the rate per cent of the dividend when he receives $150?

The number of shares is $2855 ÷ 142\frac{3}{4}$ = 20.

If 20 shares pay $150, 1 share will pay $\frac{\$150}{20} = \$7\frac{1}{2}$.

That is, 7$\frac{1}{2}$%. *Ans.*

39. How much will be received for some 3% stock, from which an income of $250 has been derived, if sold at 87$\frac{1}{2}$?

Since $0.03 is derived from $1 stock, $250 will be derived from $250 ÷ 0.03 = $8333.33 = 83$\frac{1}{3}$ shares.

83$\frac{1}{3}$ shares at 87\frac{1}{2}$ will cost 83$\frac{1}{3}$ × 87\frac{1}{2}$ = $7291.67. *Ans.*

40. If a 5% stock pays $340 income, and is sold out for $7990, at what price is it sold?

Since $0.05 is derived from $1 stock, $340 will be derived from $340 ÷ 0.05 = $6800.

If $6800 stock be sold for $7990, then $1 is sold for $7990 ÷ 6800 = 1.17\frac{1}{2}$.

Hence, the stock is sold at 117$\frac{1}{2}$. *Ans.*

41. On what per cent stock must an investment have been made from which $185.50 was derived yearly, and which, when sold out at 97, brought $5141?

If 97 represent $5141, 100 will represent $\frac{100}{97}$ of $5141 = $5300.

The interest on $5300 for 1 year is $185.50;

$$\text{on } \$1 \text{ for 1 year is } \frac{\$185.50}{\$5300} = \$0.035.$$

Hence, the rate is 3$\frac{1}{2}$%. *Ans.*

42. A person receives $4\frac{1}{6}\%$ interest on his money by investing it in some 6% stock. At what price did he buy it?

If \$100 pay \$6 at 6%, the number of dollars required to pay \$6 at $4\frac{1}{6}\%$ is $\frac{6}{4\frac{1}{6}}$ of \$100 = \$144.

Therefore, the price of the stock was 144. *Ans.*

43. If \$4800 3% stocks be sold at 88, and the proceeds be invested in 5% stocks at $105\frac{3}{5}$, what additional income will be obtained?

0.03 of \$4800 = \$144, income from the 3% stock.
0.88 of \$4800 = \$4224, amount from the 3% stock.
\$1.056 is paid for \$1 worth of 5% stock.
Hence, \$4224 is paid for \$4224 ÷ 1.056 = \$4000 stock.
0.05 of \$4000 = \$200, income from 5% stock.
\$200 − \$144 = \$56, increase in income. *Ans.*

44. If \$7800 $3\frac{1}{2}\%$ stocks be sold at 60, and the proceeds be invested in 5% stocks at 90, find the alteration in income.

$0.03\frac{1}{2}$ of \$7800 = \$273, income from the $3\frac{1}{2}\%$ stock.
0.60 of \$7800 = \$4680, amount from the $3\frac{1}{2}\%$ stock.
\$0.90 is paid for \$1 worth of 5% stock.
Hence, \$4680 is paid for \$4680 ÷ 0.90 = \$5200.
0.05 of \$5200 = \$260, income from 5% stock.
\$273 − \$260 = \$13, decrease in income. *Ans.*

45. If \$10,000 3% stocks be sold at 88, and the proceeds be invested in $3\frac{1}{2}\%$ stock at par, find the alteration in income.

0.03 of \$10,000 = \$300, income from the 3% stock.
0.88 of \$10,000 = \$8800, amount from the 3% stock.
$0.03\frac{1}{2}$ of \$8800 = \$308, income from the $3\frac{1}{2}\%$ stock.
\$308 − \$300 = \$8, increase in income. *Ans.*

46. If \$10,000 8% stocks be sold at 150, and the proceeds be invested in 6% stocks at par, find the alteration in income.

0.08 of \$10,000 = \$800, income from the 8% stock.
1.50 of \$10,000 = \$15,000, amount from the 8% stock.
0.06 of \$15,000 = \$900, income from the 6% stock.
\$900 − \$800 = \$100, increase in income. *Ans.*

47. If $8000 10% stocks be sold at 170, and the proceeds be invested in 5% stocks at 68, find the alteration in income.

0.10 of $8000 = $800, income from the 10% stock.
1.70 of $8000 = $13,600, amount from the 10% stock.
$0.68 is paid for $1 worth of 5% stock. Hence, $13,600 is paid for $13,600 ÷ 0.68 = $20,000 stock.
0.05 of $20,000 = $1000, income from the 5% stock.
$1000 − $800 = $200, increase in income. *Ans.*

48. If $7000 8% stocks be sold at 150, and the proceeds be invested in 6% stocks, at 105, find the alteration in income.

0.08 of $7000 = $560, income from the 8% stock.
1.50 of $7000 = $10,500, amount from the 8% stock.
$1.05 is paid for $1 worth of 6% stock. Hence, $10,500 is paid for $10,500 ÷ 1.05 = $10,000 stock.
0.06 of $10,000 = $600, income from the 6% stock.
$600 − $560 = $40, increase in income. *Ans.*

49. If $1000 8% stocks be sold at 170, and the proceeds be invested in 5% stocks at par, find the alteration in income.

0.08 of $1000 = $80, income from the 8% stock.
1.70 of $1000 = $1700, amount from the 8% stock.
0.05 of $1700 = $85, income from the 5% stock.
$85 − $80 = $5, increase in income. *Ans.*

50. If $8000 5% stocks be sold at 90, and the proceeds be invested in 3½% stocks at 60, find the alteration in income.

0.05 of $8000 = $400, income from the 5% stock.
0.90 of $8000 = $7200, amount from the 5% stock.
$0.60 is paid for $1 worth of 3½% stock. Hence, $7200 is paid for $7200 ÷ 0.60 = $12,000 stock.
0.03½ of $12,000 = $420, income from the 3½% stock.
$420 − $400 = $20, increase in income. *Ans.*

51. If $10,000 3½% stocks be sold at 65, and the proceeds be invested in 8% stocks at 130, find the alteration in income.

$0.03\frac{1}{2}$ of \$10,000 = \$350, income from the $3\frac{1}{2}\%$ stock.
0.65 of \$10,000 = \$6500, amount from the $3\frac{1}{2}\%$ stock.
\$1.30 is paid for \$1 worth of 8% stock. Hence, \$6500 is paid for \$6500 ÷ 1.30 = \$5000 stock.
0.08 of \$5000 = \$400, income from the 8% stock.
\$400 − \$350 = \$50, increase in income. *Ans.*

52. If \$8000 $4\frac{1}{2}\%$ stocks be sold at 70, and the proceeds be invested in 10% stocks at 160, find the alteration in income.

$0.04\frac{1}{2}$ of \$8000 = \$360, income from the $4\frac{1}{2}\%$ stock.
0.70 of \$8000 = \$5600, amount from the $4\frac{1}{2}\%$ stock.
\$1.60 is paid for \$1 worth of 10% stock. Hence, \$5600 is paid for \$5600 ÷ 1.60 = \$3500 stock.
0.10 of \$3500 = \$350, income from the 10% stock.
\$360 − \$350 = \$10, decrease in income. *Ans.*

53. If \$6000 6% stocks be sold at 90, and the proceeds be invested in 10% stocks at 135, find the alteration in income.

0.06 of \$6000 = \$360, income from the 6% stock.
0.90 of \$6000 = \$5400, amount from the 6% stock.
\$1.35 is paid for \$1 worth of 10% stock. Hence, \$5400 is paid for \$5400 ÷ 1.35 = \$4000 stock.
0.10 of \$4000 = \$400, income from the 10% stock.
\$400 − \$360 = \$40, increase in income. *Ans.*

54. Find the rate of interest obtained by investing in a stock, at 124, paying $6\frac{1}{4}$ per cent per annum.

\$100 stock costs \$124. \$100 stock pays \$$6\frac{1}{4}$.
Hence, the \$124 invested pays \$$6\frac{1}{4}$.
Therefore, the rate of interest is $6\frac{1}{4} \div 124 = 0.05\frac{5}{124}$ or $5\frac{5}{124}\%$.

55. What is the price of stock if \$7000 stock can be bought for \$5880?

If \$7000 stock cost \$5880, \$1 stock costs \$5880 ÷ 7000 = \$0.84.
Hence, the stock is quoted at 84. *Ans.*

56. Find the price of mining shares issued at \$15 a share and sold at $2\frac{1}{4}\%$ discount.

\$15 − $0.02\frac{1}{4}$ of \$15 = \$15 − \$$0.33\frac{3}{4}$ = \$$14.66\frac{1}{4}$. *Ans.*

57. How much $3\frac{1}{2}\%$ stock must be sold at $81\frac{7}{8}$, in order to buy \$5000 4% stock at $94\frac{1}{2}$; brokerage, $\frac{1}{8}$ in each transaction?

If $\frac{1}{8}$ be the brokerage, $94\frac{1}{2} + \frac{1}{8} = 94\frac{5}{8}$, price to buyer.
If $\frac{1}{8}$ be the brokerage, $81\frac{7}{8} - \frac{1}{8} = 81\frac{3}{4}$, price to seller.
Therefore, as much stock must be sold as

$$\frac{94\frac{5}{8}}{81\frac{3}{4}} \text{ of } \$5000 = \$5787.46. \quad \textit{Ans.}$$

58. How much stock must be sold at $96\frac{7}{8}$ to raise a sufficient sum for discounting a note for \$1000, due 49 days hence, and discounted at $5\frac{1}{2}\%$?

The discount is the interest on \$1000 for 52 days.
Therefore, the discount is $\$1000 \times 0.00794 = \$7.94$.
And the proceeds is $\$1000 - \$7.94 = \$992.06$.
If \$0.96875 buy \$1 stock, \$992.06 will buy
$\$992.06 \div 0.96875 = \$1024.06$. *Ans.*

59. A broker bought \$5000 stock at $88\frac{1}{2}$. At what price must he sell it to gain \$100?

He paid $\$5000 \times 0.885 = \$4425$.
To make \$100, he must sell for $\$4425 + \$100 = \$4525$.
Therefore, he must sell at $\frac{\$4525}{\$5000} = 0.905$ or $90\frac{1}{2}$. *Ans.*

60. If a broker buy stock at 85, at what price must he sell it to make $12\frac{1}{4}\%$ profit; brokerage, $\frac{1}{8}$ on each transaction.

If the brokerage be $\frac{1}{8}$, the price to the buyer is $85\frac{1}{8}$.
If the brokerage be $\frac{1}{8}$, the selling price must be
$12\frac{1}{4} + \frac{1}{8} = 12\frac{3}{8}\%$ above the cost.
Therefore, the selling price must be
$1.12\frac{3}{8}$ of $85.125 = 95\frac{3}{4}$ nearly. *Ans.*

61. Which is the more profitable stock for investment, a 4% at 85 or a 3% at 63? a $3\frac{1}{2}\%$ stock at $67\frac{1}{4}$ or a 4% stock at $81\frac{1}{2}$?

4% at $85 = 4\frac{12}{17}\%$ at 100; and 3% at $63 = 4\frac{16}{21}\%$ at 100.
$4\frac{16}{21}$ is greater than $4\frac{12}{17}$.
Therefore, 3% at 63 is the better investment. (1) *Ans.*
$3\frac{1}{2}\%$ at $67\frac{1}{4} = 5\frac{55}{269}\%$ at 100; and 4% at $81\frac{1}{2} = 4\frac{148}{163}$.
$5\frac{55}{269}$ is greater than $4\frac{148}{163}$.
Therefore, $3\frac{1}{2}\%$ at $67\frac{1}{4}$ is the better investment. (2) *Ans.*

62. Find the price of a $4\frac{1}{2}$% stock to equal a $3\frac{1}{2}$% stock at $88\frac{1}{2}$.

$100 stock costs $88\frac{1}{2}$. $100 stock pays $3\frac{1}{2}$.
Therefore, the $88\frac{1}{2}$ invested pays $3\frac{1}{2}$.
Hence, the rate of interest is $3\frac{1}{2} \div 88\frac{1}{2} = 0.03\frac{169}{177}$, or $3\frac{169}{177}$%.
If $100 pay $$3\frac{169}{177}$ at $3\frac{169}{177}$%, the number of dollars required to pay $$3\frac{169}{177}$ at $4\frac{1}{2}$% will be $\dfrac{4\frac{1}{2}}{3\frac{169}{177}} = 1.13\frac{11}{14}$.
That is, the stock will cost $113\frac{11}{14}$. *Ans.*

63. Find the price of a 5% stock to equal a 3% stock at $89\frac{1}{2}$.

$100 stock costs $89\frac{1}{2}$. $100 stock pays $3.
Therefore, the $89\frac{1}{2}$ invested pays $3.
Hence, the rate of interest is $3 \div 89\frac{1}{2} = 0.03\frac{63}{179}$, or $3\frac{63}{179}$%.
If $100 pay $$3\frac{63}{179}$ at $3\frac{63}{179}$%, the number of dollars required to pay $$3\frac{63}{179}$ at 5% will be $\dfrac{5}{3\frac{63}{179}} = 1.49\frac{1}{6}$.
That is, the 5% stock will cost $149\frac{1}{6}$. *Ans.*

64. Find the price of a $3\frac{1}{2}$% stock to equal a 6% stock at par.

If $100 pay $6 at 6%, the number of dollars required to pay $6 at $3\frac{1}{2}$% will be $\dfrac{3\frac{1}{2}}{6} = 0.58\frac{1}{3}$.

That is, the $3\frac{1}{2}$% stock will cost $58\frac{1}{3}$. *Ans.*

65. Find the profit or loss in buying $80,000 stock, at $91\frac{5}{8}$, and selling at 90; brokerage, $\frac{1}{8}$ on each transaction.

The loss on $1 is $$0.91\frac{5}{8}$ + $$0.00\frac{1}{8}$ – ($0.90 – $$0.00\frac{1}{8}$) = $$0.01\frac{7}{8}$.
The loss on $80,000 is $0.01\frac{7}{8}$ of $80,000 = $1500. *Ans.*

66. Which is the better investment, a 5% stock at $137\frac{1}{4}$, or a $3\frac{1}{2}$% stock at $91\frac{1}{2}$? What rate of interest would be received from each investment?

5% at $137\frac{1}{4} = 3\frac{353}{549}$% at 100; and $3\frac{1}{2}$% at $91\frac{1}{2} = 3\frac{151}{183}$% at 100.
$3\frac{151}{183}$ is greater than $3\frac{353}{549}$.
Therefore, $3\frac{1}{2}$% stock at $91\frac{1}{2}$ is the better investment. *Ans.*

67. A person invests \$7370 in the purchase of a stock at 92. What loss will he sustain if he sell at 90, brokerage being $\frac{1}{8}$ in each transaction?

If $\frac{1}{8}$ be the brokerage, $92 + \frac{1}{8} = 92\frac{1}{8}$, price to the buyer.

Hence, \$0.92$\frac{1}{8}$ buys \$1 stock; \$7370 buys $\frac{\$7370}{0.92\frac{1}{8}} = \$8000$.

If $\frac{1}{8}$ be the brokerage, $90 - \frac{1}{8} = 89\frac{7}{8}$, price to the seller.
Hence, \$1 will bring \$0.89$\frac{7}{8}$, and \$8000, $8000 \times \$0.89\frac{7}{8} = \$7190$.
$\$7370 - \$7190 = \$180$ loss. *Ans.*

68. How much stock must be sold at 90$\frac{5}{8}$ so that when the proceeds are invested in a mortgage, at 6%, \$543.75 a year may be received?

Since \$0.06 is derived from \$1 stock, \$543.75 will be derived from $\$543.75 \div 0.06 = \$9062.50$.

If \$1 costs \$0.90$\frac{5}{8}$, \$9062.50 will cost $\frac{\$9062.50}{0.90\frac{5}{8}} = \$10,000$. *Ans.*

69. A person invests $\frac{2}{7}$ of his money at 6%, $\frac{2}{5}$ at $4\frac{1}{2}$%, and the rest at $3\frac{1}{2}$%. What per cent will he receive on the whole amount?

$$
\begin{aligned}
\tfrac{2}{7} \times 6 &= 1\tfrac{5}{7} \\
\tfrac{2}{5} \times 4\tfrac{1}{2} &= 1\tfrac{4}{5} \\
\tfrac{11}{35} \times 3\tfrac{1}{2} &= 1\tfrac{1}{10} \\
\hline
& 4\tfrac{43}{70}\%. \ \textit{Ans.}
\end{aligned}
$$

Exercise LXXIV.

1. Find the cost of a draft on New York for \$1100, at $\frac{1}{4}$ of 1% premium.

$\$1100 \times 1.00\frac{1}{4} = \$1102.75$. *Ans.*

2. Find the cost of a draft on New Orleans for \$1350, at $\frac{1}{4}$ of 1% discount.

$\$1350 \times 1.00\frac{1}{4} = \$1346.62$. *Ans.*

3. Find the cost of a draft for $1600, payable 30 days after sight, when exchange is $\frac{1}{4}$ of 1% premium, and interest 6%.

	$1600.00	
0.0055 of $1600 =	8.80	discount at 6% for 33 days.
	$1591.20	proceeds of draft.
0.0025 of $1600 =	4.00	premium at $\frac{1}{4}$ of 1%.
	$1595.20	cost of draft. *Ans.*

4. Find the cost of a draft for $500, payable 60 days after sight, when exchange is $\frac{1}{2}$ of 1% discount, and interest 7%.

	$500.000	
0.01225 of $500 =	6.125	discount at 7% for 63 days.
	$493.875	proceeds of draft.
0.005 of $500 =	2.500	discount at $\frac{1}{2}$ of 1%.
	$491.38	cost of draft. *Ans.*

5. Find the cost of a draft for $1200, payable in 90 days after sight, when exchange is $\frac{1}{2}$ of 1% premium, and interest 7%.

	$1200.00	
0.0180$\frac{5}{6}$ of $1200 =	21.70	discount at 7% for 93 days.
	$1178.30	proceeds of draft.
0.005 of $1200 =	6.00	premium at $\frac{1}{2}$ of 1%.
	$1184.30	cost of draft. *Ans.*

6. Find the cost of a draft for $950, payable in 30 days, when exchange is at par, and interest $4\frac{1}{2}$%.

	$950.00	
0.004125 of $950 =	3.92	discount at $4\frac{1}{2}$% for 33 days.
	$946.08	cost of draft. *Ans.*

7. Find the cost of a draft for $725, payable in 60 days, when exchange is at $\frac{1}{4}$ of 1% discount, and interest 5%.

	$725.00	
0.00875 of $725 =	6.34	discount at 5% for 63 days.
	$718.66	proceeds of draft.
0.0025 of $725 =	1.81	discount at $\frac{1}{4}$ of 1%.
	$716.85	cost of draft. *Ans.*

8. Find the cost of a draft for $810, payable in 90 days, when exchange is at $\frac{1}{4}$ of 1% premium, and interest $5\frac{1}{2}$%.

	$810.00	
$0.0142\frac{1}{12}$ of $810 =	11.51	discount at $5\frac{1}{2}$% for 93 days.
	$798.49	proceeds of draft.
0.0025 of $810 =	2.03	premium at $\frac{1}{4}$ of 1%.
	$800.52	cost of draft. *Ans.*

9. Find the face of a draft, payable 30 days after sight, that can be bought for $274, when exchange is at par, and interest 6%.

$0.0055 = Discount on $1 for 33 days, at 6%.
$1 − $0.0055 = $0.9945, proceeds of $1.
Therefore, $274 ÷ 0.9945 = $275.52, face of draft. *Ans.*

10. Find the face of a draft, payable 60 days after sight, that can be bought for $1250, when exchange is at $\frac{1}{4}$ of 1% premium, and interest 7%.

$0.01225 =		Discount on $1 for 63 days, at 7%.
$1 − 0.01225 =	$0.98775	proceeds of $1.
	0.0025	premium on $1.
	$0.99025	cost of $1.

Therefore, $1250 ÷ 0.99025 = $1262.31, face of draft. *Ans.*

11. Find the face of a draft, payable 60 days after date, that can be bought for $1125, when exchange is at $\frac{1}{4}$ of 1% discount, and interest $5\frac{1}{2}$%.

$$
\begin{array}{rll}
\$0.009625 = & \text{Discount on } \$1 \text{ for 63 days, at } 5\tfrac{1}{2}\%. & \\
\$1 - \$0.009625 = & \$0.990375 & \text{proceeds of } \$1. \\
& \underline{0.0025} & \text{discount on } \$1. \\
& \$0.987875 & \text{cost of } \$1.
\end{array}
$$

Therefore, $\$1125 \div 0.987875 = \$1138.81$, face of draft. *Ans.*

12. Find the face of a draft, payable 30 days after date, that can be bought for \$520, when exchange is at $\frac{1}{2}$ of 1% premium, and interest 4%.

$$
\begin{array}{rll}
\$0.0036\tfrac{2}{3} = & \text{Discount on } \$1 \text{ for 33 days, at } 4\%. & \\
\$1 - \$0.0036\tfrac{2}{3} = & \$0.9963\tfrac{1}{3} & \text{proceeds of } \$1. \\
& \underline{0.005} & \text{discount on } \$1. \\
& \$1.0013\tfrac{1}{3} & \text{cost of } \$1.
\end{array}
$$

Therefore, $\$520 \div 1.0013\tfrac{1}{3} = \$519.31$, face of draft. *Ans.*

13. Find the face of a draft, payable 90 days after date, that can be bought for \$10,000, when exchange is at par, and interest $4\frac{1}{2}\%$.

$\$0.011625 =$ Discount on \$1 for 93 days, at $4\frac{1}{2}\%$.

$\$1 - 0.011625 = \$0.988375$, proceeds of \$1.

Therefore, $\$10,000 \div 0.988375 = \$10,117.63$, face of draft. *Ans.*

Exercise LXXV.

1. Find the cost of a draft on London for £320 10*s.* 6*d.*, when sterling exchange is quoted at 4.83.

£320 10*s.* 6*d.* = £320.525.

$\$4.83 \times 320.525 = \$1548.14$. *Ans.*

2. Find the cost of a thirty-day draft on London for £150, when thirty-day bills are quoted at 4.82, and the broker's commission is $\frac{1}{8}$ of 1% of cost of draft.

$\$4.82 \times 150 = \$723.$

$\frac{1}{8}\%$ of $\$723 = \$0.90.$

$\$723 + \$0.90 = \$723.90$. *Ans.*

3. Find the cost of the following draft, when sixty-day bills are quoted at 4.81, and the broker's commission is $\frac{1}{8}$ of 1% of cost of draft:

£500. NEW YORK, Feb. 17, 1881.

Sixty days after sight of this *First of Exchange* (Second and Third of the same tenor and date unpaid), pay to the order of James Wilson five hundred pounds, value received, and charge to account of

To JAMES SAGE & Co., London. SIMON MORTON & Co.

$$\$4.81 \times 500 = \$2405.$$
$$\tfrac{1}{8}\% \text{ of } \$2405 = \$3.01.$$
$$\$2405 + \$3.01 = \$2408.01. \textit{ Ans.}$$

4. Find the face of a draft on Glasgow that can be bought for \$2000, when sterling exchange is quoted at 4.84.

$$\$2000 \div \$4.84 = 413\tfrac{27}{121}.$$
$$£413\tfrac{27}{121} = £413\ 4\,s.\ 5.5\,d. \textit{ Ans.}$$

5. Find the face of a draft on Dublin that can be bought for \$135.24, when sterling exchange is quoted at 4.83.

$$\$135.24 \div \$4.83 = 28.$$
$$\text{That is, } £28. \textit{ Ans.}$$

6. How large a draft on London can be bought for \$4000, when exchange is quoted at $4.86\frac{2}{3}$?

$$\$4000 \div \$4.86\tfrac{2}{3} = 821\tfrac{67}{73}.$$
$$£821\tfrac{67}{73} = £821\ 18\,s.\ 4.3\,d. \textit{ Ans.}$$

7. Find the cost of a draft on Paris for 8000 fr., when Paris exchange is quoted at $5.12\frac{3}{8}$, and brokerage is $\frac{1}{8}$ of 1%.

$$0.00\tfrac{1}{8} \text{ of } 8000 \text{ fr.} = 10 \text{ fr.}$$
$$8000 \text{ fr.} + 10 \text{ fr.} = 8010 \text{ fr.}$$
$$8000 \div 5.12\tfrac{3}{8} = 1563.31.$$
$$\text{Hence, draft of } 8000 \text{ fr.} = \$1563.31. \textit{ Ans.}$$

8. Find the cost of a draft on Paris for 10,000 fr., when Paris exchange is quoted at 5.14.

$$10{,}000 \div 5.14 = 1945.53.$$
Hence, draft of 10,000 fr. = \$1945.53. *Ans.*

9. How large a sixty-day draft on Paris can be bought for \$1500, when sixty-day bills are quoted at $5.11\frac{1}{2}$?

$$1500 \times 5.11\tfrac{1}{2} = 7672.5.$$
That is, 7672.5 fr. *Ans.*

10. How large a sight draft on Paris can be bought for \$2840, when Paris exchange is quoted at $5.13\frac{1}{2}$?

$$2840 \times 5.13\tfrac{1}{2} = 14{,}583.4.$$
That is, 14,583.4 fr. *Ans.*

11. Find the cost of a draft on Hamburg for 2876 marks, when German exchange is quoted at $0.95\frac{1}{2}$.

$$\tfrac{2876}{4} \text{ of } \$0.95\tfrac{1}{2} = \$686.65. \textit{ Ans.}$$

12. Find the cost of a draft on Munich for 12,000 marks, when German exchange is quoted at $0.94\frac{5}{8}$.

$$\tfrac{12000}{4} \text{ of } \$0.94\tfrac{5}{8} = \$2838.75. \textit{ Ans.}$$

13. How large a draft on Frankfort can be bought for \$1200, when German exchange is quoted at $0.95\frac{1}{2}$?

$$0.95\tfrac{1}{2} \div 4 = 0.23\tfrac{7}{8}.$$
$$1200 \div 0.23\tfrac{7}{8} = 5026.17 \text{ marks. } \textit{Ans.}$$

EXERCISE LXXVI.

1. If a dozen eggs weigh 692^g, what is the mean weight of an egg?

$$12\overline{)692^g}$$
$$57\tfrac{2}{3}^g. \textit{ Ans.}$$

2. Seven boys weigh respectively 119.7 lbs., 105 lbs., 178.3 lbs., 165.3 lbs., 142.8 lbs., 109 lbs., 154.2 lbs. What is their average weight?

$$\begin{array}{r} 119.7 \\ 105.0 \\ 178.3 \\ 165.3 \\ 142.8 \\ 109.0 \\ 154.2 \\ \hline 7)\underline{974.3} \text{ lbs.} \\ 139\tfrac{1}{7}\tfrac{8}{0}\text{ lbs. } \textit{Ans.} \end{array}$$

3. A merchant mixes 1 lb. of coffee worth 27 cents, 1 lb. worth 35 cents, and 1 lb. worth 40 cents. What are the 3 lbs. together worth? How much a pound is the mixture worth?

$$\begin{array}{r} \$0.27 \\ 0.35 \\ 0.40 \\ \hline 3)\underline{1.02} \\ \$0.34.\ \textit{Ans.} \end{array}$$

4. A merchant mixes 2 lbs. of coffee worth 27 cents a pound, 3 lbs. worth 35 cents a pound, and 1 lb. worth 40 cents. What is a pound of the mixture worth?

$$\begin{array}{rrr} 2 \times & \$0.27 = & \$0.54 \\ 3 \times & 0.35 = & 1.05 \\ 1 \times & 0.40 = & 0.40 \\ \hline & & 6)\underline{\$1.99} \\ & & \$0.33\tfrac{1}{6}.\ \textit{Ans.} \end{array}$$

5. What is the value per pound of a mixture of coffee containing 7 lbs. worth 26 cents a pound, 4 lbs. worth 31 cents a pound, and 10 lbs. worth 34 cents a pound?

$$\begin{array}{rrr} 7 \times & \$0.26 = & \$1.82 \\ 4 \times & 0.31 = & 1.24 \\ 10 \times & 0.34 = & 3.40 \\ \hline & & 21)\underline{\$6.46} \\ & & \$0.30\tfrac{16}{21}.\ \textit{Ans.} \end{array}$$

6. What is the cost of a gallon of the mixture in which 7 gals. cost 67 cents a gallon, 5 gals. cost 48 cents a gallon, and water, without cost, was added until there were 15 gals. of the mixture?

$$\begin{array}{rrr} 7 \times & \$0.67 = & \$4.69 \\ 5 \times & 0.48 = & 2.40 \\ \hline & & 15)\underline{\$7.09} \\ & & \$0.47.\ \textit{Ans.} \end{array}$$

7. If 7^1 of water be poured into a vessel containing 3^1 of sulphuric acid, specific gravity 1.840, and the mixture shrink to 9.972^1, what is the specific gravity of the mixture?

$$\begin{array}{r} 7 \times 1^1 = 7.00^1 \\ 3 \times 1.84 = 5.52 \\ \hline 9.972^1)\underline{12.52^1} \\ 1.256.\ \textit{Ans.} \end{array}$$

8. If 4^1 of water and 1^1 of sulphuric acid, specific gravity, 1.842, when mixed shrink $\frac{1}{4}$ of 1%, what is the specific gravity of the mixture?

$$\begin{array}{r} 4 \times 1^1 = 4.000^1 \\ 1 \times 1.842 = 1.842 \\ \hline 4.9875^1)\underline{5.842^1} \\ 1.171^1 \\ 4 + 1 = 5. \\ 5 - 0.01\tfrac{1}{4} = 4.9875.\ \textit{Ans.} \end{array}$$

9. In what proportions must tin of specific gravity 7.29 and lead of specific gravity 11.35 be mixed to make a solder of specific gravity 10.21, if no allowance be made for expansion or condensation? (Give the proportions in bulk.)

The specific gravity of tin lacks 2.92 of the required specific gravity; and the specific gravity of lead is 1.14 above the required specific gravity.

Therefore, the tin is to the lead in the inverse ratio of 292 to 114. That is,

Tin : lead = 114 : 292 = 57 : 146. *Ans.*

10. In what proportions must oils worth $1.25 and 80 cents a gallon be mixed to make a mixture worth $1 a gallon? (Test the answer.)

The cost of the better oil is $0.25 above the required cost; and the cost of the worse oil lacks $0.20 of the required cost.

Therefore, the better is to the worse in the inverse ratio of $0.25 to $0.20. That is,

The better : the worse = 20 : 25 = 4 : 5. *Ans.*

11. In what proportion may oils worth $1.20, 80 cents, and 60 cents a gallon be mixed so that the mixture shall be worth 70 cents a gallon?

When the 80-cent oil alone is taken, in what ratio to the 60-cent must it be used? When the $1.20 oil alone is taken, in what ratio to the 60-cent oil must it be used? When the $1.20 and 80-cent oils are mixed gallon for gallon, how much 60-cent oil must be added? When 1 gal. of the $1.20 oil and 3 of the 80-cent oil are taken, how much 60-cent oil must be added? If three-fourths of the mixture consist of the 60-cent oil, what per cent of each of the other two must be taken?

(1)

The cost of the $1.20 is $0.50 above the required cost; the cost of the $0.80 oil is $0.10 above the required cost; and the cost of the $0.60 oil lacks $0.10 of the required cost.

Therefore, the $1.20 oil is to the $0.80 oil is to the $0.60 oil in the inverse ratio of 50 to 10 to 10. That is,

The $1.20 oil : the $0.80 oil . the $0.60 oil = 10 : 10 : 50 = 1 : 1 : 5.

(2)

The cost of the \$0.80 oil is \$0.10 above the required cost; the cost of the \$0.60 oil lacks \$0.10 of the required cost.

Therefore, the \$0.80 oil is to the \$0.60 oil in the inverse ratio of 10 to 10. That is,

The \$0.80 oil : the \$0.60 oil = 10 : 10 = 1 : 1. *Ans.*

(3)

The cost of the \$1.20 oil is \$0.50 above the required cost; and the cost of the \$0.60 oil lacks \$0.10 of the required cost.

Therefore, the \$1.20 oil is to the \$0.60 oil in the inverse ratio of 50 to 10. That is,

The \$1.20 oil : the \$0.60 oil = 10 : 50 = 1 : 5. *Ans.*

(4)

When 1 gal. of the \$1.20 oil is mixed with 1 gal. of the \$0.80 oil, the mixture costs $\frac{\$1.20 + \$0.80}{2} = \$1.00$ per gallon.

The cost of the \$1.00 oil is \$0.30 above the required cost; the cost of the \$0.60 oil lacks \$0.10 of the required cost.

Therefore, the \$1.00 oil is to the \$0.60 oil in the inverse ratio of 30 : 10. That is,

The \$1.00 oil is to the \$0.60 oil = 10 : 30 = 1 : 3.

1 : 3 : : 2 gals. : what?

1 : 3 : : 2 gals. : 6 gals. *Ans.*

(5)

When 1 gal. of the \$1.20 oil is mixed with 3 gals. of the \$0.80 oil, the mixture costs $\frac{\$1.20 + 3 \times \$0.80}{4} = \$0.90$ per gal.

The cost of the \$0.90 oil is \$0.20 above the required cost; the cost of the \$0.60 oil lacks \$0.10 of the required cost.

Therefore, the \$0.90 oil is to the \$0.60 oil in the inverse ratio of 20 : 10. That is,

The \$0.90 oil : the \$0.60 oil = 10 : 20 = 1 : 2.

1 : 2 : : 4 gals. : what?

1 : 2 : : 4 gals. : 8 gals. *Ans.*

(6)

Each of the others will be $\frac{1 - \frac{3}{4}}{2} = \frac{1}{8}$ of the whole, or $12\frac{1}{2}\%$. *Ans.*

12. A solder composed of tin and lead, specific gravities 7.29 and 11.35, has a specific gravity of 10.44. What is the weight of each metal in a kilogram of solder?

The specific gravity of the tin lacks 3.15 of the required specific gravity; and the specific gravity of the lead is 0.91 above the required specific gravity.

Therefore, the tin is to the lead in the inverse ratio of 3.15 to 0.91. That is,

The tin : the lead = 91 : 315. $\quad 91 + 315 = 406.$

$\frac{91}{406}$ of $1000^g = 224\frac{4}{29}^g$ tin. $\quad \frac{315}{406}$ of $1000^g = 775\frac{25}{29}^g$ lead. *Ans.*

13. Find the equated time for the payment of \$250 due in 3 mos., \$400 due in 6 mos., \$700 due in 8 mos.

$$\begin{array}{ll} \$250 \times 0 = & \\ \$400 \times 3 = & \$1200 \\ \$700 \times 5 = & \$3500 \\ \hline \$1350 &)\$4700 \\ & 3\frac{13}{27} \end{array}$$

$3\frac{13}{27}$ mos. = 3 mos. 14 dys.

3 mos. 14 dys. after 3 mos. = 6 mos. 14 dys.

14. Find the equated time for the payment of \$300 due in 30 dys., \$500 due in 60 dys., and \$200 due in 90 dys.

$$\begin{array}{ll} \$300 \times 00 = & \\ \$500 \times 30 = & \$15,000 \\ \$200 \times 60 = & \$12,000 \\ \hline \$1000 &)\$27,000 \\ & 27 \end{array}$$

Hence, the equated time is 27 dys. after 30 dys. = 57 dys.

15. Find the equated time for the payment of \$325 due now, \$200 due in 30 dys., \$460 due in 60 dys., and \$150 due in 90 dys.

$$\begin{array}{ll} \$325 \times 00 = & \\ \$200 \times 30 = & \$\ 6,000 \\ \$460 \times 60 = & \$27,600 \\ \$150 \times 90 = & \$13,500 \\ \hline \$1135 &)\$47,100 \\ & 41 \end{array}$$

Hence, the equated time is 41 dys.

16. Find the equated time for the payment of \$240 due May 10, \$420 due July 2, \$310 due Sept. 14, and \$600 due Oct. 1.

$$\begin{array}{ll} \$240 \times \ 00 = & \\ \$420 \times \ 53 = & \$22,260 \\ \$310 \times 127 = & \$39,370 \\ \$600 \times 144 = & \$86,400 \\ \hline \$1570 &)\$148,030 \\ & 94 \end{array}$$

Hence, the equated time is 94 dys. after May 10 = Aug. 12.

17. Find the equated time for the payment of $275 due June 21, $175 due July 16, $200 due Aug. 6, and $150 due Sept. 3.

$$\begin{array}{lr}
\$275 \times 00 = & \\
\$175 \times 25 = & \$\ \ 4{,}375 \\
\$200 \times 46 = & \$\ \ 9{,}200 \\
\$150 \times 74 = & \$11{,}100 \\
\hline
\$800 &)\$24{,}675 \\
 & 31
\end{array}$$

Hence, the equated time is 31 dys. after June 21 = July 22.

18. Find the equated time for the payment of $112.30 due July 6, $115.25 due July 30, $232.15 due Sept. 4, and $102.36 due Oct. 1.

$$\begin{array}{lr}
\$112.30 \times 00 = & \\
\$115.25 \times 24 = & \$\ \ 2{,}766.00 \\
\$232.15 \times 60 = & \$13{,}929.00 \\
\$102.36 \times 87 = & \$\ \ 8{,}905.32 \\
\hline
\$562.06 &)\$25{,}600{,}32 \\
 & 46
\end{array}$$

Hence, the equated time is 46 dys. after July 6 = Aug. 21.

19. A owed B $2000 payable in 4 mos., but at the end of 1 mo. he paid him $500, at the end of 2 mos. $500, and at the end of 3 mos. $500. In how many months is the balance due?

$$\begin{array}{lr}
\$500 \times 3 = & \$1500 \\
\$500 \times 2 = & \$1000 \\
\$500 \times 1 = & \$\ \ 500 \\
\hline
\$1500 & \$3000
\end{array}$$

Therefore, he is entitled to keep the balance ($500) $\frac{3000}{500}$ mos. = 6 mos. after its maturity.

20. A merchant bought, Feb. 11, 1881, a bill of goods amounting to $1700, on 4 months' credit; but he paid March 22, $400, April 20, $220, May 10, $300. When is the balance due?

yrs.	mos.	dys.	
1881	2	11	
	4	0	
1881	6	11	= June 11, 1881.

$$\begin{array}{lr}
\$400 \times 81 = & \$32{,}400 \\
\$220 \times 52 = & \$11{,}440 \\
\$300 \times 32 = & \$\ \ 9{,}600 \\
\hline
\$920 & \$53{,}440
\end{array}$$

Therefore, he is entitled to keep the balance ($780) $\frac{53440}{780}$ dys = 69 dys. after its maturity, June 11, 1881, = Aug. 19, 1881.

Find the equated time of maturity of each of the following bills, and the amount due at settlement, including interest at 6%:

21. JAMES PRICE, *to* JOHN BATES, *Dr.*

1881.		
Apr. 5.	To mdse. on 4 mos. credit . . .	$120.50
Apr. 15.	" " 3 " " . . .	87.33
May 7.	" " 3 " " . . .	218.17
May 21.	" " 4 " " . . .	317.00
		$743.00
		Paid Oct. 18, 1881.

$\$\;87.33 \times 00 =$
$\$120.50 \times 21 = \$\;2{,}530.50$
$\$218.17 \times 23 = \$\;5{,}017.91$
$\$317.00 \times 68 = \$21{,}556.00$
$\$743.00\;)\$29{,}104.41$
39

Hence the equated time is 39 dys. after July 15, 1881 = Aug. 23, 1881.

From Aug. 23 to Oct. 18 is 56 dys.

56 dys. = $0.009\frac{1}{3}$

$743
$0.009\frac{1}{3}$
$6.93
743.00
$749.93

22. HALL & Co. *bought of* BOLES & Co.

1881.		
Feb. 11.	To mdse. on 30 dys.	$250.00
Apr. 20.	" " 2 mos.	500.00
May 31.	" " 3 mos.	150.00
July 6.	" " 60 dys.	1000.00
		Paid Nov. 10, 1881.

30 dys. after Feb. 11 = Mar. 13.
2 mos. after Apr. 20 = June 20.
3 mos. after May 31 = Aug. 31.
60 dys. after July 6 = Sept. 4.

$\$\;250 \times \;\;00 =$
$\$\;500 \times \;\;99 = \$\;49{,}500$
$\$\;150 \times 171 = \$\;25{,}650$
$\$1000 \times 175 = \$175{,}000$
$\$1900\;)\$250{,}150$
132

Hence, the equated time is 132 dys. after Mar. 13, 1881 = July 23, 1881.

From July 23 to Nov. 10 is 110 dys.

110 dys. = $0.018\frac{1}{3}$

$1900
$34.83
1900.00
$1934.83

23. Find the equated time of maturity of each side of the following account:

ADAMS & Co. *in account with* BACON & Co.

DR. CR.

1881.					
Jan. 3.	To mdse. 90 dys.	$250	Apr. 11.	By cash,	$200
Mar. 7.	" 60 "	150	Apr. 30.	"	100
May 3.	" 60 "	325	May 30.	"	125
June 7.	" 30 "	175	July 2.	"	400

90 dys. after Jan. 3 = April 3.
60 dys. after Mar. 7 = May 6.
60 dys. after May 3 = July 2.
30 dys. after June 7 = July 7.

DR.

$\$250 \times 00 =$
$\$150 \times 33 = \$\ 4,950$
$\$325 \times 90 = \$29,250$
$\$175 \times 95 = \$16,625$
$\$900 \quad)\$50,825$
56

Hence, the equated time is 56 days after April 3 = May 29.

CR.

$\$200 \times 00 =$
$\$100 \times 19 = \$\ 1,900$
$\$125 \times 49 = \$\ 6,125$
$\$400 \times 82 = \$32,800$
$\$825 \quad)\$40,825$
49

Hence, the equated time is 49 days after April 11 = May 30.

Find the time for paying the balance in the following equated bills:

	Average due.	DR.	*Average due.*	CR.
24.	May 17, 1881 . . .	$950	Apr. 12, 1881	$1000
25.	Apr. 12, 1881 . . .	$950	May 17, 1881	$1000
26.	May 30, 1881 . . .	$1000	June 23, 1881	$920
27.	July 6, 1881 . . .	$500	Apr. 14, 1881	$480

24.

Differences in equated time = 35 dys.
Balance of account = \$1000 − \$950 = \$50.
If the account were settled at the later date, May 17, the \$1000 on the Cr. side would have been on interest 35 dys., and this is equivalent to having the balance, \$50, on interest $\frac{1000}{50}$ of 35 dys. = 700 dys. Hence, the balance should begin to draw interest 700 dys. before May 17, 1881; that is, June 17, 1879.

25.

The difference in equated time = 35 dys.
Balance of account = \$1000 − \$950 = \$50.
If the account were settled at the later date, May 17, the \$950 would have been on interest 35 dys., which is equivalent to having the balance, \$50, on interest $\frac{950}{50}$ of 35 dys. = 665 dys.
Hence, to increase the Cr. side by an equal amount of interest, the balance should remain unpaid 665 dys.; that is, the balance is due March 13, 1883.

26.

The difference in equated time = 24 dys.
Balance of account = \$1000 − \$920 = \$80.
If the account were settled at the later date, June 23, the \$1000 on the Dr. side would have been on interest 24 dys., and this is equivalent to having the balance, \$80, on interest $\frac{1000}{80}$ of 24 dys. = 300 dys.
Hence, the balance should begin to draw interest 300 dys. before June 23, 1881; that is, Aug. 27, 1880.

27.

The difference in equated time is 83 dys.
Balance of account = \$20.
If the account were settled at the later date, July 6, the \$480 would have been on interest 83 dys., which is equivalent to having the balance, \$20, on interest $\frac{480}{20}$ of 83 dys. = 1992 dys.
Hence, to increase the Dr. side by an equal amount of interest, the balance should remain unpaid 1992 dys.; that is, the balance is due Dec. 19, 1886.

Find (by either method) the cash balance in the following bills, reckoning interest at 6%:

28.

1881.		Dr.	Int.	1881.		Cr.	Int.
Apr. 5.	To mdse.	$250	$3.17	Apr. 20.	By cash,	$200	$2.03
" 27.	"	610	5.49	" 30.	"	500	4.25
June 1.	"	200	0.63	June 4.	"	400	1.07
" 20.	To bal. acc.	40		" 20.	By bal. int.		1.94
		$1100	$9.29			$1100	$9.29

Hence, cash balance = $40 − $1.94 = $38.06. *Ans.*

29.

1881.		Dr.	Int.	1881.		Cr.	Int.
Apr. 15.	To mdse.	$250.00	$7.42	Apr. 26.	By cash,	$150.00	$4.18
May 25.	"	98.50	2.27	May 17.	"	150.00	3.65
June 7.	"	300.00	6.25	July 7.	"	200.00	3.17
				Oct. 10.	By bal. acc.	148.50	
				" 10.	" int.		4.94
		$648.50	$15.94			$648.50	$15.94

Hence, cash balance = $148.50 + $4.94 = $153.44. *Ans.*

30.

1881.		Dr.	Int.	1881.		Cr.	Int.
Feb. 2.	To mdse.	$100	$3.02	Feb. 25.	By cash,	$100	$2.63
Apr. 7.	"	200	3.90	Mar. 22.	"	150	3.33
June 2.	"	95	0.97	June 20.	"	200	1.43
" 9.		150	1.35	Aug. 2.	By bal. acc.	95	
				" 2.	" int.		1.85
		$545	$9.24			$545	$9.24

Hence, cash balance = $95 + $1.85 = $96.85. *Ans.*

31.

1881.		Dr.	Int.	1881.		Cr.	Int.
Apr. 5.	To mdse.	$250	$6.21	Apr. 20.	By cash,	$200	$4.47
" 27.	"	670	14.18	" 30.	"	500	10.33
June 4.	"	200	2.97	June 1.	"	400	6.13
				Sept. 1.	By bal. acc.	20	
				" 1.	" int.		2.43
		$1120	$23.36			$1120	$23.36

Hence, cash balance = $20 + $2.43 = $22.43. *Ans.*

32.

1881.		DR.	INT.	1881.		CR.	INT.
Mar. 10.	To mdse.	\$580	\$8.31	Mar. 15.	By cash,	\$500	\$6.75
Apr. 20.	"	200	1.50	Apr. 15.	"	300	2.50
May 5.	"	150	0.75	" 25.	"	120	0.80
" 17.	"	325	0.98	May 20.	"	225	0.56
				June 4.	By bal. acc.	110	
				" 4.	" int.		0.93
		\$1255	\$11.54			\$1255	\$11.54

Hence, cash balance = \$110 + \$0.93 = \$110.93. *Ans.*

EXERCISE LXXVII.

1. Find the square root of 2916.

```
  29'16(54
  25
104)416
    416
```

2. Find the square root of 7921.

```
  79'21(89
  64
169)1521
    1521
```

3. Find the square root of 494,209.

```
   49'42'09(703
   49
1403)4209
     4209
```

4. Find the square root of 20,164.

```
   2'01'64(142
   1
 24)101
     96
282)564
    564
```

5. Find the square root of 3,345,241.

```
    3'34'52'41(1829
    1
  28)234
     224
 362)1052
      724
3649)32841
     32841
```

6. Find the square root of 125,457.64.

```
       12′54′57.64(354.2
        9
       ---
    65) 354
        325
       ----
   704) 2957
        2816
        ----
  7082) 14164
        14164
        -----
```

7. Find the square root of 47,320,641.

```
        47′32′06′41(6879
        36
        --
   128) 1132
        1024
        ----
  1367) 10806
         9569
        -----
 13749) 123741
        123741
        ------
```

8. Find the square root of 21,609.

```
        2′16′09(147
        1
        -
    24) 116
         96
        ---
   287) 2009
        2009
        ----
```

9. Find the square root of 53.7289.

```
        53.72′89(7.33
        49
        --
   143) 472
        429
        ---
  1463) 4389
        4389
        ----
```

10. Find the square root of 883.2784.

```
        8′83.27′84(29.72
        4
        -
    49) 483
        441
        ---
   587) 4227
        4109
        ----
  5942) 11884
        11884
        -----
```

11. Find the square root of 1.97262025.

```
         1.97′26′20′25(1.4045
         1
         -
     24) 97
         96
         --
   2804) 12620
         11216
         -----
  28085) 140425
         140425
         ------
```

12. Find the square root of 0.0002090916.

```
0.00′02′09′09′16(0.01446
   1
24)109
    96
284)1309
    1136
2886)17316
     17316
```

13. Find the square root of 2.

```
2.00′00′00(1.414213
1
24)100
    96
281)400
    281
2824)11900
     11296
 2828)6040
      5656
       3840
       2828
      10120
       8484
```

14. Find the square root of 5.

```
5.00′00′00(2.236067
4
42)100
    84
443)1600
    1329
4466)27100
     26796
 4472)30400
      26832
       35680
       31304
```

15. Find the square root of 0.3.

```
0.30′00′00′00(0.547722
 25
104)500
    416
1087)8400
     7609
10947)79100
      76229
 10954)24710
       21908
        28020
        21908
```

16. Find the square root of $3\frac{1}{4}$.

```
3.25′00′00(1.802775
1
28)225
   224
3602)10000
      7204
 3604)27960
      25228
       27320
       25228
        20920
        18020
```

17. Find the square root of $8\frac{5}{6}$.

```
8.83′33′33(2.972092
4
49)483
   441
587)4233
    4109
5942)12433
     11884
      54933
      53496
       14373
       11888
```

18. Find the square root of 0.9.

```
       0.90′00′00′00(0.948683
        81
  184) 900
        736
 1888) 16400
        15104
18966) 129600
        113796
 18972) 158040
         151776
           62640
           56916
```

19. Find in yards the side of a square field containing 20 acres.

20 A. = 96,800 sq. yds.

```
     9′68′00.00′00(311.12
     9
   61) 68
       61
  621) 700
       621
 6221) 7900
       6221
62222) 167900
       124444
```

20. Find the side of a square the area of which is 150 sq. ft. 9 sq. in.

150 sq. ft. 9 sq. in. = 21,609 sq. in.

```
      2′16′09(147
      1
  24) 116
       96
 287) 2009
      2009
```

147 in. = 12 ft. 3 in.

21. Find the side of a square the area of which is 8 sq. yds. 7 sq. ft. 73 sq. in.

8 sq. yds. 7 sq. ft. 73 sq. in.
= 11,449 sq. in.

```
      1′14′49(107
      1
 207) 1449
      1449
```

107 in. = 8 ft. 11 in.

22. Find to six places of decimals the square roots of $\frac{4}{9}$; $\frac{5}{9}$; $\frac{1}{2}$; $\frac{3}{5}$; $\frac{5}{7}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{5}{6}$.

(1)

$\sqrt{\frac{4}{9}} = \frac{2}{3} = 0.666667.$

(2)

$\frac{5}{9} = 0.\dot{5}.$

```
   0.55′55′55′55′55′55(0.745355
    49
 144) 655
      576
 1485) 7955
       7425
 14903) 53055
        44709
         83465
         74515
          89505
          74515
```

(3)

$\frac{1}{2} = 0.5.$

0.50′00′00′00 (0.707106
49
1407) 10000
9849
14141) 15100
14141
14142) 95900
84852

(4)

$\frac{3}{5} = 0.6.$

0.60′00′00′00 (0.774596
49
147) 1100
1029
1544) 7100
6176
15485) 92400
77425
15490) 149750
139410
103400
92940

(5)

$\frac{5}{7} = 0.714285714285.$

0.71′42′85′71′42′85 (0.845154
64
164) 742
656
1685) 8685
8425
16901) 26071
16901
169025) 917042
845125
1690304) 7191785
6761216

(6)

$\frac{3}{4} = 0.75.$

0.75′00′00′00 (0.866025
64
166) 1100
996
1726) 10400
10356
17320) 44000
34640
93600
86600

(7)

$\frac{2}{3} = 0.66666666.$

0.66′66′66′66 (0.816496
64
161) 266
161
1626) 10566
9756
16324) 81066
65296
16328) 157706
146952
107546
97968

(8)

$\frac{5}{6} = 0.833333.$

0.83′33′33 (0.91287
81
181) 233
181
1822) 5233
3644
1824) 15893
14592
13013
12768
2453
1824

23. Find, by factoring, the square roots of 2025; 17.64; 2.0164; 533.61; 204.49.

$$2025 = 5^2 \times 9^2.$$

$$\therefore \sqrt{2025} = 5 \times 9 = 45. \text{ (1) } \textit{Ans.}$$

$$17.64 = 6^2 \times 0.7^2.$$

$$\therefore \sqrt{17.64} = 6 \times 0.7 = 4.2. \text{ (2) } \textit{Ans.}$$

$$2.0164 = 2^2 \times 0.71^2.$$

$$\therefore \sqrt{2.0164} = 2 \times 0.71 = 1.42. \text{ (3) } \textit{Ans.}$$

$$533.61 = 3^2 \times 0.7^2 \times 11^2.$$

$$\therefore \sqrt{533.61} = 3 \times 0.7 \times 11 = 23.1. \text{ (4) } \textit{Ans.}$$

$$204.49 = 11^2 \times 1.3^2.$$

$$\therefore \sqrt{204.49} = 11 \times 1.3 = 14.3. \text{ (5) } \textit{Ans.}$$

24. A ladder 13 ft. long standing on level ground reaches a window 12 ft. from the ground. How far from the wall is the foot of the ladder?

$$\sqrt{(13+12)(13-12)} = \sqrt{25} = 5 \text{ ft. } \textit{Ans.}$$

25. The two legs of a right triangle are 35 in. and 84 in. respectively. Find the hypotenuse.

$$\sqrt{35^2 + 84^2} = \sqrt{8281} = 91 \text{ in. } \textit{Ans.}$$

26. The hypotenuse of a right triangle is 61 in., and one leg 11 in. Find the other leg.

$$\sqrt{(61+11)(61-11)} = \sqrt{3600} = 60 \text{ in. } \textit{Ans.}$$

27. Find the longest straight line that can be drawn on the floor of a room 20 ft. by 15 ft.

$$\sqrt{20^2 + 15^2} = \sqrt{625} = 25 \text{ ft. } \textit{Ans.}$$

28. Find the longest line in a box that is 8 ft. long, 4 ft. wide, 1 ft. deep.

$\sqrt{8^2+4^2} = \sqrt{80}$. $\sqrt{\sqrt{80}^2 + 1^2} = \sqrt{81} = 9$ ft. *Ans.*

Exercise LXXVIII.

1. Find the cube root of 1331.

$$\begin{array}{r|l}
 & 1'331\,(11 \\
 & 1 \\
\hline
3 \times 10^2 = 300 & 331 \\
3\,(10 \times 1) = 30 & \\
1^2 = 1 & \\
\overline{331} & \underline{331}
\end{array}$$

2. Find the cube root of 1728.

$$\begin{array}{r|l}
 & 1'728\,(12 \\
 & 1 \\
\hline
3 \times 10^2 = 300 & 728 \\
3\,(10 \times 2) = 60 & \\
2^2 = 4 & \\
\overline{364} & \underline{728}
\end{array}$$

3. Find the cube root of 12.167.

$$\begin{array}{r|l}
 & 12.'167\,(2.3 \\
 & 8 \\
\hline
3 \times 20^2 = 1200 & 4167 \\
3(20 \times 3) = 180 & \\
3^2 = 9 & \\
\overline{1389} & \underline{4167}
\end{array}$$

4. Find the cube root of 300.763.

$$\begin{array}{r|l}
 & 300.'763\,(6.7 \\
 & 216 \\
\hline
3 \times 60^2 = 10800 & 84763 \\
3(60 \times 7) = 1260 & \\
7^2 = 49 & \\
\overline{12109} & \underline{84763}
\end{array}$$

5. Find the cube root of 148,877.

$$
\begin{array}{r|l}
 & 148'877\,(53 \\
 & 125 \\
\hline
3\times 50^2 = 7500 & 23877 \\
3(50\times 3) = 450 & \\
3^2 = 9 & \\
\hline
7959 & \underline{23877}
\end{array}
$$

6. Find the cube root of 2,048,383.

$$
\begin{array}{r|l}
 & 2'048'383\,(127 \\
 & 1 \\
\hline
3\times 10^2 = 300 & 1048 \\
3(10\times 2) = 60 & \\
2^2 = 4 & \\
\hline
364 & 728 \\
\hline
 & 320383 \\
3\times 120^2 = 43200 & \\
3(120\times 7) = 2520 & \\
7^2 = 49 & \\
\hline
45769 & \underline{320383}
\end{array}
$$

7. Find the cube root of 59.776471.

$$
\begin{array}{r|l}
 & 59.'776'471\,(3.91 \\
 & 27 \\
\hline
3\times 30^2 = 2700 & 32776 \\
3(30\times 9) = 810 & \\
9^2 = 81 & \\
\hline
3591 & 32319 \\
\hline
 & 457471 \\
3\times 390^2 = 456300 & \\
3(390\times 1) = 1170 & \\
1^2 = 1 & \\
\hline
457471 & \underline{457471}
\end{array}
$$

8. Find the cube root of 304957.115891.

$$
\begin{array}{rr|l}
 & & 304'957.115'891\,(67.31 \\
 & & 216 \\
3 \times 60^2 = & 10800 & \quad 88957 \\
3(60 \times 7) = & 1260 & \\
7^2 = & \underline{49} & \\
 & 12109 & \quad \underline{84763} \\
 & & \quad 4194115 \\
3 \times 670^2 = & 1346700 & \\
3(670 \times 3) = & 6030 & \\
3^2 = & \underline{9} & \\
 & 1352739 & \quad \underline{4058217} \\
 & & \quad 135898891 \\
3 \times 6730^2 = & 135878700 & \\
3(6730 \times 1) = & 20190 & \\
1^2 = & \underline{1} & \\
 & 135898891 & \quad \underline{135898891}
\end{array}
$$

9. Find the cube root of 0.007821346625.

$$
\begin{array}{rr|l}
 & & 0.007'821'346'625\,(0.1985 \\
 & & 1 \\
3 \times 10^2 = & 300 & \quad 6821 \\
3(10 \times 9) = & 270 & \\
9^2 = & \underline{81} & \\
 & 651 & \quad \underline{5859} \\
 & & \quad 962346 \\
3 \times 190^2 = & 108300 & \\
3(190 \times 8) = & 4560 & \\
8^2 = & \underline{64} & \\
 & 112924 & \quad \underline{903392} \\
 & & \quad 58954625 \\
3 \times 1980^2 = & 11761200 & \\
3(1980 \times 5) = & 29700 & \\
5^2 = & \underline{25} & \\
 & 11790925 & \quad \underline{58954625}
\end{array}
$$

10. Find the cube root of $104.\dot{6}00290750613$.

	104.′600′290′750′613 (4.7117
	64
$3 \times 40^2 = 4800$	40600
$3(40 \times 7) = 840$	
$7^2 = 49$	
5689	39823
	777290
$3 \times 470^2 = 662700$	
$3(470 \times 1) = 1410$	
$1^2 = 1$	
664111	664111
	113179750
$3 \times 4710^2 = 66552300$	
$3(4710 \times 1) = 14130$	
$1^2 = 1$	
66566431	66566431
	46613319613
$3 \times 47110^2 = 6658056300$	
$3(47110 \times 7) = 989310$	
$7^2 = 49$	
6659045659	46613319613

11. Find the cube root of 17,183,498,535,125.

	17′183′498′535′125 (25805
	8
$3 \times 20^2 = 1200$	9183
$3(20 \times 5) = 300$	
$5^2 = 25$	
1525	7625
	1558498
$3 \times 250^2 = 187500$	
$3(250 \times 8) = 6000$	
$8^2 = 64$	
193564	1548512
	9986535125
$3 \times 25800^2 = 1996920000$	
$3(25800 \times 5) = 387000$	
$5^2 = 25$	
1997307025	9986535125

12. Find the cube root of 122615.327232.

	122′615.327′232(49.68
	64
$3 \times 40^2 = 4800$	58615
$3(40 \times 9) = 1080$	
$9^2 = 81$	
5961	53649
	4966327
$3 \times 490^2 = 720300$	
$3(490 \times 6) = 8820$	
$6^2 = 36$	
729156	4374936
	591391232
$3 \times 4960^2 = 73804800$	
$3(4960 \times 8) = 119040$	
$8^2 = 64$	
73923904	591391232

13. Find the cube root of 10; $3\frac{5}{8}$; $8\frac{1}{3}$ to four places of decimals.

(1)

	10.000(2.1544
	8
$3 \times 20^2 = 1200$	2000
$3(20 \times 1) = 60$	
$1^2 = 1$	
1261	1261
61	739000
$3 \times 210^2 = 132300$	
$3(210 \times 5) = 3150$	
$5^2 = 25$	
135475	677375
3175	61625000
$3 \times 2150^2 = 13867500$	
$3(2150 \times 4) = 25800$	
$4^2 = 16$	
13893316	55573264
25816	60517360
$3 \times 2154^2 = 13919148$	53676592

(2)

$$\sqrt[3]{3\tfrac{5}{8}} = \sqrt[3]{\frac{29}{8}} = \frac{\sqrt[3]{29}}{2} = \frac{3.0722}{2} = 1.5361.$$

$$
\begin{array}{rrl}
 & & 29.000(3.0722 \\
 & & 27 \\
3 \times 300^2 = & 270000 & 2000000 \\
3(300 \times 7) = & 6300 & \\
7^2 = & 49 & \\
 & 276349 & 1934443 \\
 & 6349 & 65557000 \\
3 \times 3070^2 = & 28274700 & \\
3(3070 \times 2) = & 18420 & \\
2^2 = & 4 & \\
 & 28293124 & 56586248 \\
 & 18424 & 89707520 \\
3 \times 3072^2 = & 28311552 & 56623104
\end{array}
$$

(3)

$$
\begin{array}{rrl}
 & & 8.333'333(2.0274 \\
 & & 8 \\
3 \times 200^2 = & 120000 & 333333 \\
3(200 \times 2) = & 1200 & \\
2^2 = & 4 & \\
 & 121204 & 242408 \\
 & 1204 & 90925333 \\
3 \times 2020^2 = & 12241200 & \\
3(2020 \times 7) = & 42420 & \\
7^2 = & 49 & \\
 & 12283669 & 85985669 \\
 & 42469 & 49396643 \\
3 \times 2027^2 = & 12326187 & 44304748
\end{array}
$$

14. Find the cube root of 5; $\frac{5}{9}$; $7\frac{3}{5}$; $\frac{3}{4}$ to four places of decimals.

(1)

	5.000(1.7099
	1
$3 \times 10^2 = 300$	4000
$3(10 \times 7) = 210$	
$7^2 = 49$	
559	3913
259	87000000
$3 \times 1700^2 = 8670000$	
$3(1700 \times 9) = 45900$	
$9^2 = 81$	
8715981	78443829
45981	85561710
$3 \times 1709^2 = 8762043$	78858387

(2)

	0.555′555(0.8221
	512
$3 \times 80^2 = 19200$	43555
$3(80 \times 2) = 480$	
$2^2 = 4$	
19684	39368
484	41875
$3 \times 82^2 = 20172$	40344
	15315

(3)

	7.600(1.966
	1
$3 \times 10^2 = 300$	6600
$3(10 \times 9) = 270$	
$9^2 = 81$	5859
651	741000
351	
$3 \times 190^2 = 108300$	
$3(190 \times 6) = 3420$	
$6^2 = 36$	
111756	670536
3456	704640
$3 \times 196^2 = 115248$	691488

(4)

$$\begin{array}{rr|l}
 & & 0.750'000'000\,(0.9085 \\
 & & 729 \\
3\times 900^2 = & 2430000 & 21000000 \\
3(900\times 8) = & 21600 & \\
8^2 = & \underline{64} & \\
 & 2451664 & \underline{19613312} \\
 & \underline{21664} & 13866880 \\
 & 2473392 & \underline{12366960}
\end{array}$$

15. Find the entire surface of a cube the volume of which is 14 cu. ft. 705.088 cu. in.

$$\begin{array}{l}
14 \text{ cu. ft. } 705.088 \text{ cu. in.} \\
\underline{\times 1728} \\
24192 \text{ cu. in.} \\
\underline{705.088} \\
24897.088 \text{ cu. in}
\end{array}$$

$$\begin{array}{rr|l}
 & & 24'897.088\,(29.2 \text{ in.} \\
 & & 8 \\
3\times 20^2 = & 1200 & 16897 \\
3(20\times 9) = & 540 & \\
9^2 = & \underline{81} & \\
 & 1821 & \underline{16389} \\
 & \underline{621} & 508088 \\
3\times 290^2 = & 252300 & \\
3(290\times 2) = & 1740 & \\
2^2 = & \underline{4} & \\
 & 254044 & \underline{508088}
\end{array}$$

$$\begin{array}{l}
29.2 \\
\underline{\times 29.2} \\
852.64 \text{ sq. in. in each face.} \\
\underline{6} \\
144)\underline{5115.84} \text{ sq. in.} \\
35 \text{ sq. ft. } 75.84 \text{ sq. in.}
\end{array}$$

EXERCISE LXXIX.

1. If the diameter of the moon be reckoned at 2000 mi., and that of the earth at 8000 mi., find the ratio of the surface of the moon to that of the earth. Also, find the ratio of the volume of the moon to that of the earth.

$$2000^2 : 8000^2 = 1^2 : 4^2 = 1 : 16. \text{ (1) } \textit{Ans.}$$
$$2000^3 : 8000^3 = 1^3 : 4^3 = 1 : 64. \text{ (2) } \textit{Ans.}$$

2. If the diameter of the earth be reckoned at 8000 mi., that of Jupiter at 84,000 mi., and that of the sun at 880,000 mi., find the ratios of their volumes.

$$8000^3 : 84000^3 : 880000^3 = 2^3 : 21^3 : 220^3 = 8 : 9261 : 10648000. \textit{ Ans.}$$

3. If the diameters of two circles be 20 in. and 40 in. respectively, find the ratio of their circumferences and the ratio of their surfaces.

$$20 : 40 = 1 : 2.$$
$$20^2 : 40^2 = 1^2 : 2^2 = 1 : 4. \textit{ Ans.}$$

4. If the areas of two circles be 8000 sq. in. and 36,000 sq. in. respectively, find the ratio of their diameters to the nearest thousandth of an inch.

$$\sqrt{8000} : \sqrt{36000} = \sqrt{4} : \sqrt{18} = 2 : 4.242 = 1 : 2.121. \textit{ Ans.}$$

5. If the volumes of two spheres be 100 cu. in. and 1000 cu. in. respectively, find the ratio of their diameters to the nearest thousandth of an inch.

$$\sqrt[3]{100} : \sqrt[3]{1000} = \sqrt[3]{1} : \sqrt[3]{10} = 1 : 2.154. \textit{ Ans.}$$

6. If two stacks of hay of the same shape contain 4 t. 6 cwt. and 1 t. 8 cwt. respectively, find the ratio of their heights.

$$\sqrt[3]{86} : \sqrt[3]{28} = 4.414 : 3.037 = 1 : 0.688. \textit{ Ans.}$$

7. If an ox 7 ft. in girth weigh 1500 lbs., what will be the girth of a similar ox weighing 2500 lbs.?

$\sqrt[3]{1500} : \sqrt[3]{2500} :: 7$ ft. : what?
$\sqrt[3]{1} : \sqrt[3]{\frac{2500}{1500}} :: 7$ ft. : what?
$1 : 1.186 :: 7$ ft. : what?
$1 : 1.186 :: 7$ ft. : 8.3 ft. *Ans.*

8. The surface of a pyramid is 560 sq. in. What is the surface of a similar pyramid whose volume is 27 times as great?

$\sqrt[3]{1} : \sqrt[3]{27} = 1 : 3.$
$1^2 : 3^2 :: 560$ sq. in. : what?
$1 : 9 :: 560$ sq. in. : 5040 sq. in. *Ans.*

9. The volume of a pyramid is 1331 cu. in. What is the volume of a similar pyramid whose surface is 4 times as great?

$\sqrt{1} : \sqrt{4} = 1 : 2.$
$1^3 : 2^3 :: 1331$ cu. in. : what?
$1 : 8 :: 1331$ cu. in. : 10648 cu. in. *Ans.*

10. If a well-proportioned man 5 ft. 10 in. high weigh 160 lbs., what should a man 6 ft. high weigh, to the nearest tenth of a pound? What should be the height, to the nearest tenth of an inch, of a man weighing 210 lbs.?

5 ft. 10 in. = 70 in. 6 ft. = 72 in.
$70^3 : 72^3 :: 160$ lbs. : what?
$343000 : 373248 :: 160$ lbs. : 174.1 lbs. (1) *Ans.*
$\sqrt[3]{160} : \sqrt[3]{210} :: 70$ in. : what?
$5.43 : 5.95 :: 70$ in. : 76.6 in.
76.6 in. = 6 ft. 4.6 in. (2) *Ans.*

11. A three-gallon jug and a one-gallon jug are of the same shape What, to the nearest thousandth, is the ratio of their diameters?

$\sqrt[3]{3} : \sqrt[3]{1} = 1.443 : 1 = 1 : 0.693.$ *Ans.*

12. Two hills have exactly the same shape; one is 900 ft. high, the other 1200 ft. Find the ratio of their surfaces, and also the ratio of their volumes.

$$900^2 : 1200^2 = 3^2 : 4^2 = 9 : 16. \text{ (1) } \textit{Ans.}$$
$$900^3 : 1200^3 = 3^3 : 4^3 = 27 : 64. \text{ (2) } \textit{Ans.}$$

13. A ball 3 in. in diameter weighs 4 lbs.; another ball of the same metal weighs 9 lbs. Find the diameter of the second ball to the nearest thousandth of an inch.

$$\sqrt[3]{4} : \sqrt[3]{9} : : 3 \text{ in.} : \text{what?}$$
$$1.587 : 2.080 : : 3 \text{ in.} : 3.931 \text{ in. } \textit{Ans.}$$

14. If Apollo's altar were a perfect cube 10 ft. on a side, what, to the nearest hundredth of an inch, would be the dimensions of a new cubical altar containing twice as much stone?

$$10 \times 10 \times 10 = 1000 \text{ cu. ft.}$$
$$2 \times 1000 = 2000 \text{ cu. ft.}$$
$$\sqrt[3]{1000} : \sqrt[3]{2000} : : 10 : \text{what?}$$
$$10 : 12.599 : : 10 : 12.599 \text{ ft.}$$
$$12.599 \text{ ft.} = 12 \text{ ft. } 7.19 \text{ in. } \textit{Ans.}$$

15. A man standing 40 ft. from a building 24 ft. wide observed that, when he closed one eye, the width of the building hid from view 90 rods of fence which was parallel to the width of the building. Find the distance from the eye of the observer to the fence.

$$24 : 40 : : 90 \text{ rds.} : \text{what?}$$
$$24 : 40 : : 90 \text{ rds.} : 150 \text{ rds. } \textit{Ans.}$$

16. A bushel measure and a peck measure are of the same shape. Find the ratio of their heights.

$$1 \text{ bushel} = 4 \text{ pecks.}$$
$$\sqrt[3]{4} : \sqrt[3]{1} = 1.587 : 1 = 1 : 0.63. \textit{ Ans.}$$

Exercise LXXX.

Given: $\log 2 = 0.3010$; $\log 3 = 0.4771$; $\log 5 = 0.6990$; $\log 7 = 0.8451$.

Find the logarithms of the following numbers by resolving the numbers into factors, and taking the sum of the logarithms of the factors.

1.

$$\log 6 = \log(2 \times 3) = \log 2 + \log 3.$$

$\log 2 = 0.3010$
$\log 3 = 0.4771$
0.7781. *Ans.*

2.

$$\log 15 = \log(3 \times 5) = \log 3 + \log 5.$$

$\log 3 = 0.4771$
$\log 5 = 0.6990$
1.1761. *Ans.*

3.

$$\log 21 = \log(3 \times 7) = \log 3 + \log 7.$$

$\log 3 = 0.4771$
$\log 7 = 0.8451$
1.3222. *Ans.*

4.

$$\log 14 = \log(2 \times 7) = \log 2 + \log 7.$$

$\log 2 = 0.3010$
$\log 7 = 0.8451$
1.1461. *Ans.*

5.

$$\log 35 = \log(5 \times 7) = \log 5 + \log 7.$$

$\log 5 = 0.6990$
$\log 7 = 0.8451$
1.5441. *Ans.*

6.

$$\log 9 = \log(3 \times 3) = \log 3 + \log 3.$$

$\log 3 = 0.4771$
$\log 3 = 0.4771$
0.9542. *Ans.*

7.

$$\log 8 = \log(2 \times 2 \times 2) = \log 2 + \log 2 + \log 2.$$

$\log 2 = 0.3010$
$\log 2 = 0.3010$
$\log 2 = 0.3010$
0.9030. *Ans.*

8.

$$\log 49 = \log(7 \times 7) = \log 7 + \log 7.$$

$\log 7 = 0.8451$
$\log 7 = 0.8451$
1.6902. *Ans.*

9.

$$\begin{aligned}\log 25 &= \log(5 \times 5)\\ &= \log 5 + \log 5.\\ \log 5 &= 0.6990\\ \log 5 &= \underline{0.6990}\\ & 1.3980. \textit{ Ans.}\end{aligned}$$

10.

$$\begin{aligned}\log 30 &= \log(2 \times 3 \times 5)\\ &= \log 2 + \log 3 + \log 5.\\ \log 2 &= 0.3010\\ \log 3 &= 0.4771\\ \log 5 &= \underline{0.6990}\\ & 1.4771. \textit{ Ans.}\end{aligned}$$

11.

$$\begin{aligned}\log 42 &= \log(2 \times 3 \times 7)\\ &= \log 2 + \log 3 + \log 7.\\ \log 2 &= 0.3010\\ \log 3 &= 0.4771\\ \log 7 &= \underline{0.8451}\\ & 1.6232. \textit{ Ans.}\end{aligned}$$

12.

$$\begin{aligned}\log 420 &= \log(2\times2\times3\times5\times7)\\ &= \log 2 + \log 2 + \log 3\\ &\qquad + \log 5 + \log 7.\\ \log 2 &= 0.3010\\ \log 2 &= 0.3010\\ \log 3 &= 0.4771\\ \log 5 &= 0.6990\\ \log 7 &= \underline{0.8451}\\ & 2.6232. \textit{ Ans.}\end{aligned}$$

13.

$$\begin{aligned}\log 12 &= \log(2 \times 2 \times 3)\\ &= \log 2 + \log 2 + \log 3.\\ \log 2 &= 0.3010\\ \log 2 &= 0.3010\\ \log 3 &= \underline{0.4771}\\ & 1.0791. \textit{ Ans.}\end{aligned}$$

14.

$$\begin{aligned}\log 60 &= \log(2 \times 2 \times 3 \times 5)\\ &= \log 2 + \log 2 + \log 3\\ &\qquad + \log 5.\\ \log 2 &= 0.3010\\ \log 2 &= 0.3010\\ \log 3 &= 0.4771\\ \log 5 &= \underline{0.6990}\\ & 1.7781. \textit{ Ans.}\end{aligned}$$

15.

$$\begin{aligned}\log 75 &= \log(3 \times 5 \times 5)\\ &= \log 3 + \log 5 + \log 5.\\ \log 3 &= 0.4771\\ \log 5 &= 0.6990\\ \log 5 &= \underline{0.6990}\\ & 1.8751. \textit{ Ans.}\end{aligned}$$

16.

$$\begin{aligned}\log 7.5 &= \log(5 \times 5 \times 3 \times 0.1)\\ &= \log 5 + \log 5 + \log 3\\ &\qquad + \log 0.1.\\ \log 5 &= 0.6990\\ \log 5 &= 0.6990\\ \log 3 &= 0.4771\\ \log 0.1 &= \underline{9.0000 - 10}\\ & 0.8751. \textit{ Ans.}\end{aligned}$$

17.

$$\begin{aligned}\log\ 0.021 &= \log(3\times 7\times 0.001)\\ &= \log 3+\log 7\\ &\quad +\log 0.001.\\ \log\ 3 &= 0.4771\\ \log\ 7 &= 0.8451\\ \log\ 0.001 &= \underline{7.0000-10}\\ & \ 8.3222-10.\ \textit{Ans.}\end{aligned}$$

18.

$$\begin{aligned}\log\ 0.35 &= \log(5\times 7\times 0.01)\\ &= \log 5+\log 7\\ &\quad +\log 0.01.\\ \log\ 5 &= 0.6990\\ \log\ 7 &= 0.8451\\ \log\ 0.01 &= \underline{8.0000-10}\\ & \ 9.5441-10.\ \textit{Ans.}\end{aligned}$$

19.

$$\begin{aligned}\log 0.0035 &= \log(5\times 7\times 0.0001)\\ &= \log 5+\log 7\\ &\quad +\log 0.0001.\\ \log\ 5 &= 0.6990\\ \log\ 7 &= 0.8451\\ \log 0.0001 &= \underline{6.0000-10}\\ & \ 7.5441-10.\ \textit{Ans.}\end{aligned}$$

20

$$\begin{aligned}\log\ 0.004 &= \log(2\times 2\times 0.001)\\ &= \log 2+\log 2\\ &\quad +\log 0.001.\\ \log\ 2 &= 0.3010\\ \log\ 2 &= 0.3010\\ \log\ 0.001 &= \underline{7.0000-10}\\ & \ 7.6020-10.\ \textit{Ans.}\end{aligned}$$

21.

$$\begin{aligned}\log 0.05 &= \log(5\times 0.01)\\ &= \log 5+\log 0.01.\\ \log\ 5 &= 0.6990\\ \log 0.01 &= \underline{8.0000-10}\\ & \ 8.6990-10.\ \textit{Ans.}\end{aligned}$$

22.

$$\begin{aligned}\log 12.5 &= \log(5\times 5\times 5\times 0.1)\\ &= \log 5+\log 5+\log 5\\ &\quad +\log 0.1.\\ \log\ 5 &= 0.6990\\ \log\ 5 &= 0.6990\\ \log\ 5 &= 0.6990\\ \log\ 0.1 &= \underline{9.0000-10}\\ & \ 1.0970.\ \textit{Ans.}\end{aligned}$$

23.

$$\begin{aligned}\log 1.25 &= \log(5\times 5\times 5\times 0.01)\\ &= \log 5+\log 5+\log 5\\ &\quad +\log 0.01.\\ \log\ 5 &= 0.6990\\ \log\ 5 &= 0.6990\\ \log\ 5 &= 0.6990\\ \log 0.01 &= \underline{8.0000-10}\\ & \ 0.0970.\ \textit{Ans.}\end{aligned}$$

24.

$$\begin{aligned}\log 37.5 &= \log(3\times 5\times 5\times 5\times 0.1)\\ &= \log 3+\log 5+\log 5\\ &\quad +\log 5+\log 0.1.\\ \log\ 3 &= 0.4771\\ \log\ 5 &= 0.6990\\ \log\ 5 &= 0.6990\\ \log\ 5 &= 0.6990\\ \log\ 0.1 &= \underline{9.0000-10}\\ & \ 1.5741.\ \textit{Ans.}\end{aligned}$$

25.

$$\log\ 2.1 = \log(3 \times 7 \times 0.1) = \log 3 + \log 7 + \log 0.1.$$

$$\begin{aligned} \log 3 &= 0.4771 \\ \log 7 &= 0.8451 \\ \log 0.1 &= \underline{9.0000 - 10} \\ &\ 0.3222. \end{aligned}$$ *Ans.*

26.

$$\log 16 = \log(2 \times 2 \times 2 \times 2) = \log 2 + \log 2 + \log 2 + \log 2$$

$$\begin{aligned} \log 2 &= 0.3010 \\ \log 2 &= 0.3010 \\ \log 2 &= 0.3010 \\ \log 2 &= \underline{0.3010} \\ &\ 1.2040. \end{aligned}$$ *Ans.*

27.

$$\log 0.056 = \log(2\times2\times2\times7\times0.001) = \log 2 + \log 2 + \log 2 + \log 7 + \log 0.001.$$

$$\begin{aligned} \log 2 &= 0.3010 \\ \log 2 &= 0.3010 \\ \log 2 &= 0.3010 \\ \log 7 &= 0.8451 \\ \log 0.001 &= \underline{7.0000 - 10} \\ &\ 8.7481 - 10. \end{aligned}$$ *Ans.*

28.

$$\log 0.63 = \log(3 \times 3 \times 7 \times 0.01) = \log 3 + \log 3 + \log 7 + \log 0.01.$$

$$\begin{aligned} \log 3 &= 0.4771 \\ \log 3 &= 0.4771 \\ \log 7 &= 0.8451 \\ \log 0.01 &= \underline{8.0000 - 10} \\ &\ 9.7993 - 10. \end{aligned}$$ *Ans.*

29.

$$\log 1.75 = \log(5 \times 5 \times 7 \times 0.01) = \log 5 + \log 5 + \log 7 + \log 0.01.$$

$$\begin{aligned} \log 5 &= 0.6990 \\ \log 5 &= 0.6990 \\ \log 7 &= 0.8451 \\ \log 0.01 &= \underline{8.0000 - 10} \\ &\ 0.2431. \end{aligned}$$ *Ans.*

30.

$$\log 105 = \log(3 \times 5 \times 7) = \log 3 + \log 5 + \log 7.$$

$$\begin{aligned} \log 3 &= 0.4771 \\ \log 5 &= 0.6990 \\ \log 7 &= \underline{0.8451} \\ &\ 2.0212. \end{aligned}$$ *Ans.*

31.

$$\log 0.0105 = \log(3\times5\times7\times0.0001) = \log 3 + \log 5 + \log 7 + \log 0.0001.$$

$$\begin{aligned} \log 3 &= 0.4771 \\ \log 5 &= 0.6990 \\ \log 7 &= 0.8451 \\ \log 0.0001 &= \underline{6.0000 - 10} \\ &\ 8.0212 - 10. \end{aligned}$$ *Ans.*

32.

$$\log 1.05 = \log(3 \times 5 \times 7 \times 0.01) = \log 3 + \log 5 + \log 7 + \log 0.01.$$

$$\begin{aligned} \log 3 &= 0.4771 \\ \log 5 &= 0.6990 \\ \log 7 &= 0.8451 \\ \log 0.01 &= \underline{8.0000 - 10} \\ &\ 0.0212. \end{aligned}$$ *Ans.*

EXERCISE LXXXI.

Given: $\log 2 = 0.3010$; $\log 3 = 0.4771$; $\log 5 = 0.6990$; $\log 7 = 0.8451$.

Find logarithms of the following:

1. $\log 2^3 = 3 \times \log 2 = 3 \times 0.3010 = 0.9030$. *Ans.*

2. $\log 5^2 = 2 \times \log 5 = 2 \times 0.6990 = 1.3980$. *Ans.*

3. $\log 7^4 = 4 \times \log 7 = 4 \times 0.8451 = 3.3804$. *Ans.*

4. $\log 3^8 = 8 \times \log 3 = 8 \times 0.4771 = 3.8168$. *Ans.*

5. $\log 7^3 = 3 \times \log 7 = 3 \times 0.8451 = 2.5353$. *Ans.*

6. $\log 5^5 = 5 \times \log 5 = 5 \times 0.6990 = 3.4950$. *Ans.*

7. $\log 2^{\frac{1}{3}} = \frac{1}{3}$ of $\log 2 = \frac{1}{3}$ of $0.3010 = 0.1003$. *Ans.*

8. $\log 5^{\frac{1}{2}} = \frac{1}{2}$ of $\log 5 = \frac{1}{2}$ of $0.6990 = 0.3495$. *Ans.*

9. $\log 3^{\frac{1}{8}} = \frac{1}{8}$ of $\log 3 = \frac{1}{8}$ of $0.4771 = 0.0596$. *Ans.*

10. $\log 7^{\frac{1}{5}} = \frac{1}{5}$ of $\log 7 = \frac{1}{5}$ of $0.8451 = 0.1690$. *Ans.*

11. $\log 5^{\frac{1}{5}} = \frac{1}{5}$ of $\log 5 = \frac{1}{5}$ of $0.6990 = 0.1398$. *Ans.*

12. $\log 7^{\frac{1}{11}} = \frac{1}{11}$ of $\log 7 = \frac{1}{11}$ of $0.8451 = 0.0768$. *Ans.*

13. $\log 2^{\frac{3}{4}} = \frac{3}{4}$ of $\log 2 = \frac{3}{4}$ of $0.3010 = 0.2258$. *Ans.*

14. $\log 5^{\frac{2}{3}} = \frac{2}{3}$ of $\log 5 = \frac{2}{3}$ of $0.6990 = 0.4660$. *Ans.*

15. $\log 3^{\frac{3}{7}} = \frac{3}{7}$ of $\log 3 = \frac{3}{7}$ of $0.4771 = 0.2045$. *Ans.*

16. $\log 7^{\frac{2}{7}} = \frac{2}{7}$ of $\log 7 = \frac{2}{7}$ of $0.8451 = 0.2415$. *Ans.*

17. $\log 5^{\frac{5}{3}} = \frac{5}{3}$ of $\log 5 = \frac{5}{3}$ of $0.6990 = 1.1650$. *Ans.*

18. $\log 3^{\frac{9}{11}} = \frac{9}{11}$ of $\log 3 = \frac{9}{11}$ of $0.4771 = 0.3904$. *Ans.*

19. $\log 7^{\frac{7}{2}} = \frac{7}{2}$ of $\log 7 = \frac{7}{2}$ of $0.8451 = 2.9579$. *Ans.*

20. $\log 3^{\frac{4}{3}} = \frac{4}{3}$ of $\log 3 = \frac{4}{3}$ of $0.4771 = 0.6361$. *Ans.*

21. $\log 5^{\frac{7}{2}} = \frac{7}{2}$ of $\log 5 = \frac{7}{2}$ of $0.6990 = 2.4465$. *Ans.*

22. $\log 2^{\frac{11}{7}} = \frac{11}{7}$ of $\log 2 = \frac{11}{7}$ of $0.3010 = 0.4730$. *Ans.*

23. $\log 5^{\frac{3}{4}} = \frac{3}{4}$ of $\log 5 = \frac{3}{4}$ of $0.6990 = 0.5243$. *Ans.*

24. $\log 7^{\frac{11}{7}} = \frac{11}{7}$ of $\log 7 = \frac{11}{7}$ of $0.8451 = 1.3280$. *Ans.*

25. $\log 21^{\frac{7}{8}} = \frac{7}{8}$ of $\log 21 = \frac{7}{8}$ of $\log (3 \times 7)$
$= \frac{7}{8}$ of $(0.4771 + 0.8451)$
$= \frac{7}{8}$ of $1.3222 = 1.1569$. *Ans.*

Exercise LXXXII.

Given: $\log 2 = 0.3010$; $\log 3 = 0.4771$; $\log 5 = 0.6990$; $\log 7 = 0.8451$.

Find logarithms for the following quotients:

1.

$$\begin{array}{ll} \log \frac{2}{5} & = \log 2 + \text{colog}\, 5 - 10. \\ \log 2 & = 0.3010 \\ \text{colog}\, 5 - 10 & = \underline{9.3010 - 10} \\ & \ 9.6020 - 10. \ \textit{Ans.} \end{array}$$

2.

$$\begin{array}{ll} \log \frac{2}{7} & = \log 2 + \text{colog}\, 7 - 10. \\ \log 2 & = 0.3010 \\ \text{colog}\, 7 - 10 & = \underline{9.1549 - 10} \\ & \ 9.4559 - 10. \ \textit{Ans.} \end{array}$$

3.

$$\begin{array}{ll} \log \frac{3}{5} & = \log 3 + \text{colog}\, 5 - 10. \\ \log 3 & = 0.4771 \\ \text{colog}\, 5 - 10 & = \underline{9.3010 - 10} \\ & \ 9.7781 - 10. \ \textit{Ans.} \end{array}$$

4.

$$\begin{array}{ll} \log \frac{3}{7} & = \log 3 + \text{colog}\, 7 - 10. \\ \log 3 & = 0.4771 \\ \text{colog}\, 7 - 10 & = \underline{9.1549 - 10} \\ & \ 9.6320 - 10. \ \textit{Ans.} \end{array}$$

5.

$$\begin{array}{ll} \log \frac{5}{7} & = \log 5 + \text{colog}\, 7 - 10. \\ \log 5 & = 0.6990 \\ \text{colog}\, 7 - 10 & = \underline{9.1549 - 10} \\ & \ 9.8539 - 10. \ \textit{Ans.} \end{array}$$

6.

$$\begin{array}{ll} \log \frac{7}{5} & = \log 7 + \text{colog}\, 5 - 10. \\ \log 7 & = 0.8451 \\ \text{colog}\, 5 - 10 & = \underline{9.3010 - 10} \\ & \ 0.1461. \ \textit{Ans.} \end{array}$$

7.

$$\log \tfrac{5}{3} = \log 5 + \text{colog}\, 3 - 10.$$
$$\log 5 = 0.6990$$
$$\text{colog}\, 3 - 10 = \underline{9.5229 - 10}$$
$$0.2219.\ \textit{Ans.}$$

8.

$$\log \tfrac{5}{2} = \log 5 + \text{colog}\, 2 - 10.$$
$$\log 5 = 0.6990$$
$$\text{colog}\, 2 - 10 = \underline{9.6990 - 10}$$
$$0.3980.\ \textit{Ans.}$$

9.

$$\log \tfrac{7}{3} = \log 7 + \text{colog}\, 3 - 10.$$
$$\log 7 = 0.8451$$
$$\text{colog}\, 3 - 10 = \underline{9.5229 - 10}$$
$$0.3680.\ \textit{Ans.}$$

10.

$$\log \tfrac{7}{2} = \log 7 + \text{colog}\, 2 - 10.$$
$$\log 7 = 0.8451$$
$$\text{colog}\, 2 - 10 = \underline{9.6990 - 10}$$
$$0.5441.\ \textit{Ans.}$$

11.

$$\log \tfrac{3}{2} = \log 3 + \text{colog}\, 2 - 10.$$
$$\log 3 = 0.4771$$
$$\text{colog}\, 2 - 10 = \underline{9.6990 - 10}$$
$$0.1761.\ \textit{Ans.}$$

12.

$$\log \frac{7}{0.5} = \log 7 + \text{colog}\, 0.5 - 10.$$
$$\log 7 = 0.8451$$
$$\text{colog}\, 0.5 - 10 = \underline{10.3010 - 10}$$
$$1.1461.\ \textit{Ans.}$$

13.

$$\log \frac{0.05}{3} = \log 0.05 + \text{colog}\, 3 - 10.$$
$$\log 0.05 = 8.6990 - 10$$
$$\text{colog}\, 3 - 10 = \underline{9.5229 - 10}$$
$$8.2219 - 10.\ \textit{Ans.}$$

14.

$$\log \frac{0.005}{2} = \log 0.005 + \text{colog}\, 2 - 10.$$
$$\log 0.005 = 7.6990 - 10$$
$$\text{colog}\, 2 - 10 = \underline{9.6990 - 10}$$
$$7.3980 - 10.\ \textit{Ans.}$$

15.

$$\log \frac{0.07}{5} = \log 0.07 + \text{colog}\, 5 - 10.$$
$$\log 0.07 = 8.8451 - 10$$
$$\text{colog}\, 5 - 10 = \underline{9.3010 - 10}$$
$$8.1461 - 10.\ \textit{Ans.}$$

16.

$$\log \frac{5}{0.07} = \log 5 + \text{colog}\, 0.07 - 10.$$

$$\begin{array}{lrl} \log & 5 = & 0.6990 \\ \text{colog} & 0.07 - 10 = & \underline{11.1549 - 10} \\ & & 1.8539. \textit{ Ans.} \end{array}$$

17.

$$\log \frac{3}{0.007} = \log 3 + \text{colog}\, 0\,007 - 10.$$

$$\begin{array}{lrl} \log & 3 = & 0.4771 \\ \text{colog} & 0.007 - 10 = & \underline{12.1549 - 10} \\ & & 2.6320. \textit{ Ans.} \end{array}$$

18.

$$\log \frac{0.003}{7} = \log 0.003 + \text{colog}\, 7 - 10.$$

$$\begin{array}{lrl} \log & 0.003 = & 7.4771 - 10 \\ \text{colog} & 7 - 10 = & \underline{9.1549 - 10} \\ & & 6.6320 - 10. \textit{ Ans.} \end{array}$$

19.

$$\log \frac{0.05}{0.003} = \log 0.05 + \text{colog}\, 0.003 - 10.$$

$$\begin{array}{lrl} \log & 0.05 = & 8.6990 - 10 \\ \text{colog} & 0.003 - 10 = & \underline{12.5229 - 10} \\ & & 1.2219. \textit{ Ans.} \end{array}$$

20.

$$\log \frac{0.007}{0.02} = \log 0.007 + \text{colog}\, 0.02 - 10.$$

$$\begin{array}{lrl} \log & 0.007 = & 7.8451 - 10 \\ \text{colog} & 0.02 - 10 = & \underline{11.6990 - 10} \\ & & 9.5441 - 10. \textit{ Ans.} \end{array}$$

21.

$$\log \frac{0.02}{0.007} = \log 0.02 + \operatorname{colog} 0.007 - 10.$$

$$\begin{aligned} \log\ 0.02 &= 8.3010 - 10 \\ \operatorname{colog} 0.007 - 10 &= \underline{12.1549 - 10} \\ &\ 0.4559.\ \textit{Ans.} \end{aligned}$$

22.

$$\log \frac{0.005}{0.07} = \log 0.005 + \operatorname{colog} 0.07 - 10.$$

$$\begin{aligned} \log\ 0.005 &= 7.6990 - 10 \\ \operatorname{colog}\ 0.07 - 10 &= \underline{11.1549 - 10} \\ &\ 8.8539 - 10.\ \textit{Ans.} \end{aligned}$$

23.

$$\log \frac{0.03}{7} = \log 0.03 + \operatorname{colog} 7 - 10,$$

$$\begin{aligned} \log\ 0.03 &= 8.4771 - 10 \\ \operatorname{colog}\ 7 - 10 &= \underline{9.1549 - 10} \\ &\ 7.6320 - 10.\ \textit{Ans.} \end{aligned}$$

24.

$$\log \frac{0.0007}{0.2} = \log 0.0007 + \operatorname{colog} 0.2 - 10.$$

$$\begin{aligned} \log\ 0.0007 &= 6.8451 - 10 \\ \operatorname{colog}\ 0.2 - 10 &= \underline{10.6990 - 10} \\ &\ 7.5441 - 10.\ \textit{Ans.} \end{aligned}$$

25.

$$\log \frac{0.02^2}{3^3} = \log 0.02^2 + \operatorname{colog} 3^3 - 10.$$

$$\begin{aligned} \log\ 0.02^2 &= 6.6020 - 10 \\ \operatorname{colog}\ 3^3 - 10 &= \underline{8.5687 - 10} \\ &\ 5.1707 - 10.\ \textit{Ans.} \end{aligned}$$

26.

$$\begin{array}{lrl}
\log & \dfrac{3^3}{0.02^2} = & \log 3^3 + \operatorname{colog} 0.02^2 - 10. \\
\log & 3^3 = & 1.4313 \\
\operatorname{colog} & 0.02^2 - 10 = & \underline{13.3980 - 10} \\
 & & 4.8293. \ \textit{Ans.}
\end{array}$$

27.

$$\begin{array}{lrl}
\log & \dfrac{7^3}{0.02^2} = & \log 7^3 + \operatorname{colog} 0.02^2 - 10. \\
\log & 7^3 = & 2.5353 \\
\operatorname{colog} & 0.02^2 - 10 = & \underline{13.3980 - 10} \\
 & & 5.9333. \ \textit{Ans.}
\end{array}$$

28.

$$\begin{array}{lrl}
\log & \dfrac{0.07^3}{0.003^3} = & \log 0.07^3 + \operatorname{colog} 0.003^3 - 10. \\
\log & 0.07^3 = & 6.5353 - 10 \\
\operatorname{colog} & 0.003^3 - 10 = & \underline{17.5687 - 10} \\
 & & 4.1040. \ \textit{Ans.}
\end{array}$$

29.

$$\begin{array}{lrl}
\log & \dfrac{0.005^2}{7^3} = & \log 0.005^2 + \operatorname{colog} 7^3 - 10. \\
\log & 0.005^2 = & 5.3980 - 10 \\
\operatorname{colog} & 7^3 - 10 = & \underline{7.4647 - 10} \\
 & & 2.8627 - 10. \ \textit{Ans.}
\end{array}$$

30.

$$\begin{array}{lrl}
\log & \dfrac{7^3}{0.005^2} = & \log 7^3 + \operatorname{colog} 0.005^2 - 10. \\
\log & 7^3 = & 2.5353 \\
\operatorname{colog} & 0.005^2 - 10 = & \underline{14.6020 - 10} \\
 & & 7.1373. \ \textit{Ans.}
\end{array}$$

Exercise LXXXIII.

Find logarithms of the following numbers.

1.

$\log 70 = 1.8451$. *Ans.*

2.

$\log 101 = 2.0043$. *Ans.*

3.

$\log 333 = 2.5224$. *Ans.*

4.

$\log 3491 = 3.5428 + (\frac{1}{10} \text{ of } 13) = 3.5429$. *Ans.*

5.

$\log 1866 = 3.2695 + (\frac{6}{10} \text{ of } 23) = 3.2709$. *Ans.*

6.

$\log 6897 = 3.8382 + (\frac{7}{10} \text{ of } 6) = 3.8386$. *Ans.*

7.

$\log 9901 = 3.9956 + (\frac{1}{10} \text{ of } 5) = 3.9957$. *Ans.*

8.

$\log 4389 = 3.6415 + (\frac{9}{10} \text{ of } 10) = 3.6424$. *Ans.*

9.

$\log 1111 = 3.0453 + (\frac{1}{10} \text{ of } 39) = 3.0457$. *Ans.*

10.

$\log 58343 = 4.7657 + (\frac{43}{100} \text{ of } 7) = 4.7660$. *Ans.*

11.

$\log 77860 = 4.8910 + (\frac{60}{100} \text{ of } 5) = 4.8913$. *Ans.*

12.

$\log 30127 = 4.4786 + (\frac{27}{100} \text{ of } 14) = 4.4790$. *Ans.*

13.

$\log 730.84 = 2.8633 + (\frac{84}{100} \text{ of } 6) = 2.8638$. *Ans.*

14.

$\log 0.008765 = 7.9425 + (\frac{5}{10} \text{ of } 5) - 10 = 7.9428 - 10.$ *Ans.*

15.

$\log 8.0808 = 0.9074 + (\frac{8}{100} \text{ of } 5) = 0.9074.$ *Ans.*

16.

$\log 5.0009 = 0.6990 + (\frac{9}{100} \text{ of } 8) = 0.6991.$ *Ans.*

17.

$\log 0.3769 = 9.5752 + (\frac{9}{10} \text{ of } 11) - 10 = 9.5762 - 10.$ *Ans.*

18.

$\log 0.070707 = 8.8494 + (\frac{7}{100} \text{ of } 6) - 10 = 8.8494 - 10.$ *Ans.*

19.

$\log 0.03723 = 8.5705 + (\frac{5}{10} \text{ of } 12) - 10 = 8.5709 - 10.$ *Ans.*

20.

$\log 98.871 = 1.9948 + (\frac{71}{100} \text{ of } 4) = 1.9951.$ *Ans.*

21. Find antilog 3.9017.

The number corresponding to the mantissa 9015 is 7970.
The number corresponding to the mantissa 9020 is 7980.
The difference between these numbers is 10,
and $7970 + \frac{2}{5} \text{ of } 10 = 7974.$ *Ans.*

22. Find antilog 1.2076.

The number corresponding to the mantissa 2068 is 1610.
The number corresponding to the mantissa 2095 is 1620.
The difference between these numbers is 10,
and $1610 + \frac{8}{27} \text{ of } 10 = 1613.$
Therefore, the number required is 16.13. *Ans.*

23. Find antilog 0.4442.

The number corresponding to the mantissa 4440 is 2780.
The number corresponding to the mantissa 4456 is 2790.
The difference between these numbers is 10,
and $2780 + \frac{2}{16} \text{ of } 10 = 2781.$
Therefore, the number required is 2.781. *Ans.*

24. Find antilog 1.0090.

The number corresponding to the mantissa 0086 is 1020.
The number corresponding to the mantissa 0128 is 1030.
The difference between these numbers is 10,

and $1020 + \frac{4}{42}$ of $10 = 1021$.

Therefore, the number required is 10.21. *Ans.*

25. Find antilog 2.9850.

The number corresponding to the mantissa 9850 is 9660.
Therefore, the number required is 966. *Ans.*

26. Find antilog 4.5388.

The number corresponding to the mantissa 5378 is 3450.
The number corresponding to the mantissa 5391 is 3460.
The difference between the numbers is 10,

and $3450 + \frac{10}{13}$ of $10 = 3458$.

Therefore, the number required is 34,580. *Ans.*

27. Find antilog 0.8550.

The number corresponding to the mantissa 8549 is 7160.
The number corresponding to the mantissa 8555 is 7170.
The difference between these number is 10,

and $7160 + \frac{1}{6}$ of $10 = 7162$.

Therefore, the number required is 7.162. *Ans.*

28. Find antilog 9.9992 – 10.

The number corresponding to the mantissa 9991 is 9980.
The number corresponding to the mantissa 9996 is 9990.
The difference between these numbers is 10,

and $9980 + \frac{1}{5}$ of $10 = 9982$.

Therefore, the number required is 0.9982. *Ans.*

29. Find antilog 8.7324 – 10.

The number corresponding to the mantissa 7324 is 5400.
Therefore, the number required is 0.0540. *Ans.*

30. Find antilog 9.5555 − 10.

The number corresponding to the mantissa 5551 is 3590.
The number corresponding to the mantissa 5563 is 3600.
The difference between these numbers is 10,

and $3590 + \frac{4}{12}$ of $10 = 3593$.

Therefore, the number required is 0.3593. *Ans.*

31. Find antilog 6.0216 − 10.

The number corresponding to the mantissa 0212 is 1050.
The number corresponding to the mantissa 0253 is 1060.
The difference between these numbers is 10,

and $1050 + \frac{4}{41}$ of $10 = 1051$.

Therefore, the number required is 0.0001051. *Ans.*

32. Find antilog 7.0080 − 10.

The number corresponding to the mantissa 0043 is 1010.
The number corresponding to the mantissa 0086 is 1020.
The difference between these numbers is 10,

and $1010 + \frac{37}{43}$ of $10 = 1019$.

Therefore, the number required is 0.001019. *Ans.*

33. Find by logarithms the product 948.22×0.4387.

$$\begin{aligned} \log\ 948.22 &= 2.9769 \\ \log\ 0.4387 &= \underline{9.6422 - 10} \\ & \ 2.6191 \qquad = \log 416. \ \textit{Ans.} \end{aligned}$$

34. Find by logarithms the product 1.9704×0.0786.

$$\begin{aligned} \log\ 1.9704 &= 0.2946 \\ \log\ 0.0786 &= \underline{8.8954 - 10} \\ & \ 9.1900 - 10 = \log 0.1549. \ \textit{Ans.} \end{aligned}$$

35. Find by logarithms the product 380.25×0.00673.

$$\begin{aligned} \log\ 380.25 &= 2.5801 \\ \log 0.00673 &= \underline{7.8280 - 10} \\ & \ 0.4081 \qquad = \log 2.559. \ \textit{Ans.} \end{aligned}$$

36. Find by logarithms the product 216.21×0.76312.

$$\begin{aligned} \log\ 216.21 &= 2.3349 \\ \log 0.76312 &= \underline{9.8826 - 10} \\ &\ 2.2175 \qquad = \log 165. \ \textit{Ans.} \end{aligned}$$

37. Find by logarithms the product 0.56127×1.2312.

$$\begin{aligned} \log 0.56127 &= 9.7492 - 10 \\ \log\ 1.2312 &= \underline{0.0903} \\ &\ 9.8395 - 10 = \log 0.691. \ \textit{Ans.} \end{aligned}$$

38. Find by logarithms the product 0.86311×56.371.

$$\begin{aligned} \log 0.86311 &= 9.9361 - 10 \\ \log\ 56.371 &= \underline{1.7511} \\ &\ 1.6872 \qquad = \log 48.67. \ \textit{Ans.} \end{aligned}$$

39. Find by logarithms the product 59.795×0.7955.

$$\begin{aligned} \log\ 59.795 &= 1.7767 \\ \log\ 0.7955 &= \underline{9.9007 - 10} \\ &\ 1.6774 \qquad = 47.58. \ \textit{Ans.} \end{aligned}$$

40. Find by logarithms the product 270.05×0.0087.

$$\begin{aligned} \log\ 270.05 &= 2.4315 \\ \log\ 0.0087 &= \underline{7.9395 - 10} \\ &\ 0.3710 \qquad = \log 2.349. \ \textit{Ans.} \end{aligned}$$

41. Find by logarithms the product 11.163×0.3333.

$$\begin{aligned} \log\ 11.163 &= 1.0478 \\ \log\ 0.3333 &= \underline{9.5228 - 10} \\ &\ 0.5706 \qquad = \log 3.721. \ \textit{Ans.} \end{aligned}$$

42. Find by logarithms the product 777.78×0.0787.

$$\begin{aligned} \log\ 777.78 &= 2.8909 \\ \log\ 0.0787 &= \underline{8.8960 - 10} \\ &\ 1.7869 \qquad = \log 61.21. \ \textit{Ans.} \end{aligned}$$

43. Find by logarithms the product 2.6537×0.2313.

$$\begin{aligned} \log\ 2.6537 &= 0.4238 \\ \log\ 0.2313 &= \underline{9.3642 - 10} \\ &\ 9.7880 - 10 = \log 0.6137. \quad \textit{Ans.} \end{aligned}$$

44. Find by logarithms the product 37.587×12.371.

$$\begin{aligned} \log\ 37.587 &= 1.5750 \\ \log\ 12.371 &= \underline{1.0924} \\ &\ 2.6674 \qquad = \log 464.9. \quad \textit{Ans.} \end{aligned}$$

45. Find by logarithms the product 89.313×2.3781.

$$\begin{aligned} \log\ 89.313 &= 1.9510 \\ \log\ 2.3781 &= \underline{0.3762} \\ &\ 2.3272 \qquad = \log 212.4. \quad \textit{Ans.} \end{aligned}$$

46. Find by logarithms the product 9.1765×0.00089.

$$\begin{aligned} \log\ 9.1765 &= 0.9627 \\ \log 0.00089 &= \underline{6.9494 - 10} \\ &\ 7.9121 - 10 = \log 0.008168. \quad \textit{Ans.} \end{aligned}$$

47. Find the quotient of $\frac{56.407}{13.045}$.

$$\begin{aligned} \log\ 56.407 &= 1.7513 \\ \text{colog}\ 13.045 &= \underline{8.8846 - 10} \\ &\ 0.6359 \qquad = \log 4.324. \quad \textit{Ans.} \end{aligned}$$

48. Find the quotient of $\frac{857.06}{3079.8}$.

$$\begin{aligned} \log\ 857.06 &= 2.9330 \\ \text{colog}\ 3079.8 &= \underline{6.5114 - 10} \\ &\ 9.4444 - 10 = \log 0.2783. \quad \textit{Ans.} \end{aligned}$$

49. Find the quotient of $\frac{0.9387}{598.6}$.

$$\begin{aligned} \log\ 0.9387 &= 9.9726 - 10 \\ \text{colog}\ 598.6 &= \underline{7.2229 - 10} \\ &\ 7.1955 - 10 = \log 0.001569. \quad \textit{Ans.} \end{aligned}$$

50. Find the quotient of $\frac{3069}{0.7891}$.

$$\begin{aligned}
\log\ 3069 &= 3.4870\\
\text{colog}\ 0.7891 &= \underline{10.1028 - 10}\\
&\ 3.5898 \qquad = \log 3889. \textit{ Ans.}
\end{aligned}$$

51. Find the quotient of $\frac{75.46 \times 0.0765}{93.08 \times 98.071}$.

$$\begin{aligned}
\log\ 75.46 &= 1.8777\\
\log\ 0.0765 &= 8.8837 - 10\\
\text{colog}\ 93.08 &= 8.0312 - 10\\
\text{colog}\ 98.071 &= \underline{8.0084 - 10}\\
&\ 6.8010 - 10 \quad = \log 0.0006324. \textit{ Ans.}
\end{aligned}$$

52. Find the quotient of $\frac{98 \times 537 \times 0.0079}{67309 \times 0.0947}$.

$$\begin{aligned}
\log\ 98 &= 1.9912\\
\log\ 537 &= 2.7300\\
\log\ 0.0079 &= 7.8976 - 10\\
\text{colog}\ 67309 &= 5.1719 - 10\\
\text{colog}\ 0.0947 &= \underline{11.0237 - 10}\\
&\ 8.8144 - 10 = \log 0.06523. \textit{ Ans.}
\end{aligned}$$

53. Find the quotient of $\frac{314 \times 7.18 \times 8132}{519 \times 827 \times 3.215}$.

$$\begin{aligned}
\log\ 314 &= 2.4969\\
\log\ 7.18 &= 0.8561\\
\log\ 8132 &= 3.9102\\
\text{colog}\ 519 &= 7.2848 - 10\\
\text{colog}\ 827 &= 7.0825 - 10\\
\text{colog}\ 3.215 &= \underline{9.4928 - 10}\\
&\ 1.1233 \qquad = \log 13.28. \textit{ Ans.}
\end{aligned}$$

54. Find the quotient of $\frac{212 \times 2.16 \times 8002}{536 \times 351 \times 7.256}$.

$$\begin{array}{lrl} \log & 212 = & 2.3263 \\ \log & 2.16 = & 0.3345 \\ \log & 8002 = & 3.9032 \\ \text{colog} & 536 = & 7.2708 - 10 \\ \text{colog} & 351 = & 7.4547 - 10 \\ \text{colog} & 7.256 = & \underline{9.1393 - 10} \\ & & 0.4288 \qquad = \log 2.684. \ \textit{Ans.} \end{array}$$

55. Find by logarithms 5.06^3.

$$3 \log 5.06 = 3 \times 0.7042 = 2.1126$$
$$2.1126 = \log 129.6. \ \textit{Ans.}$$

56. Find by logarithms 2.501^5.

$$5 \log 2.501 = 5 \times 0.3981 = 1.9905$$
$$1.9905 = \log 97.84. \ \textit{Ans.}$$

57. Find by logarithms 1.716^7.

$$7 \log 1.716 = 7 \times 0.2345 = 1.6415$$
$$1.6415 = \log 43.8. \ \textit{Ans.}$$

58. Find by logarithms 1.178^{10}.

$$10 \log 1.178 = 10 \times 0.0712 = 0.7120$$
$$0.7120 = \log 5.153. \ \textit{Ans.}$$

59. Find by logarithms 0.7685^6.

$$6 \log 0.7685 = 6 \times (9.8857 - 10) = 9.3142 - 10$$
$$9.3142 - 10 = \log 0.2061. \ \textit{Ans.}$$

60. Find by logarithms 0.9611^8.

$$8 \log 0.9611 = 8 \times (9.9828 - 10) = 9.8624 - 10$$
$$9.8624 - 10 = \log 0.7285. \ \textit{Ans.}$$

61. Find by logarithms 0.0231^2.

$$2 \log 0.0231 = 2 \times (8.3636 - 10) = 6.7272 - 10$$
$$6.7272 - 10 = \log 0.0005336. \quad \textit{Ans.}$$

62. Find by logarithms 0.8567^3.

$$3 \log 0.8567 = 3 \times (9.9329 - 10) = 9.7987 - 10$$
$$9.7987 - 10 = \log 0.629. \quad \textit{Ans.}$$

63. Find by logarithms $(\frac{61}{73})^4$.

$$\begin{aligned} 4 \log 61 &= 4 \times 1.7853 &&= 7.1412 \\ 4 \operatorname{colog} 73 &= 4 \times (8.1367 - 10) &&= \underline{2.5468 - 10} \\ & && \ 9.6880 - 10 = \log 0.4876. \quad \textit{Ans.} \end{aligned}$$

64. Find by logarithms $(\frac{13}{71})^3$.

$$\begin{aligned} 3 \log 13 &= 3 \times 1.1139 &&= 3.3417 \\ 3 \operatorname{colog} 71 &= 3 \times (8.1487 - 10) &&= \underline{4.4461 - 10} \\ & && \ 7.7878 - 10 = \log 0.006134. \quad \textit{Ans.} \end{aligned}$$

65. Find by logarithms $(\frac{16}{9})^5$.

$$\begin{aligned} 5 \log 16 &= 5 \times 1.2041 &&= 5.0205 \\ 5 \operatorname{colog} 9 &= 5 \times (9.0458 - 10) &&= \underline{5.2290 - 10} \\ & && \ 0.2495 \qquad = \log 17.76. \quad \textit{Ans.} \end{aligned}$$

66. Find by logarithms $(\frac{35}{4})^3$.

$$\begin{aligned} 3 \log 35 &= 3 \times 1.5441 &&= 4.6323 \\ 3 \operatorname{colog} 4 &= 3 \times (9.3979 - 10) &&= \underline{8.1937 - 10} \\ & && \ 2.8260 \qquad = \log 699.9. \quad \textit{Ans.} \end{aligned}$$

67. Find by logarithms $(5\frac{5}{11})^2$.

$$\begin{aligned} 2 \log 60 &= 2 \times 1.7782 &&= 3.5564 \\ 2 \operatorname{colog} 11 &= 2 \times (8.9586 - 10) &&= \underline{7.9172 - 10} \\ & && \ 1.4736 \qquad = \log 29.76. \quad \textit{Ans.} \end{aligned}$$

68. Find by logarithms $(4\frac{4}{31})^3$.

$$\begin{aligned} 3\log\ 128 &= 3 \times 2.1072 &&= 6.3216 \\ 3\,\text{colog}\ 31 &= 3 \times (8.5086 - 10) &&= \underline{5.5258\ - 10} \\ & && \ 1.8474 \quad = \log 70.37. \ \textit{Ans.} \end{aligned}$$

69. Find by logarithms $(2\frac{29}{37})^5$.

$$\begin{aligned} 5\log\ 103 &= 5 \times 2.0128 &&= 10.0640 \\ 5\,\text{colog}\ 37 &= 5 \times (8.4318 - 10) &&= \underline{\ 2.1590 - 10} \\ & && \ \ 2.2230 \quad = \log 167.1. \ \textit{Ans.} \end{aligned}$$

70. Find by logarithms $(\frac{871}{711})^3$.

$$\begin{aligned} 3\log\ 871 &= 3 \times 2.9400 &&= 8.8200 \\ 3\,\text{colog}\ 711 &= 3 \times (7.1481 - 10) &&= \underline{1.4443 - 10} \\ & && \ 0.2643 \quad = \log 1.838. \ \textit{Ans.} \end{aligned}$$

71. Find by logarithms $13^{\frac{1}{3}}$.

$\frac{1}{3}\log\ 13 = \frac{1}{3}$ of $1.1139 = 0.3713 = \log 2.351$. *Ans.*

72. Find by logarithms $29^{\frac{1}{5}}$.

$\frac{1}{5}\log\ 29 = \frac{1}{5}$ of $1.4624 = 0.2925 = \log 1.961$. *Ans.*

73. Find by logarithms $879^{\frac{1}{10}}$.

$\frac{1}{10}\log 879 = \frac{1}{10}$ of $2.9440 = 0.2944 = \log 1.97$. *Ans.*

74. Find by logarithms $0.609^{\frac{1}{4}}$.

$$\begin{aligned} \log 0.609 &= 9.7846 - 10 \\ & \ \underline{30. \qquad - 30} \\ & 4\overline{)39.7846 - 40} \\ & \ 9.9462 - 10 = \log 0.8834. \ \textit{Ans.} \end{aligned}$$

75. Find by logarithms $93.73^{\frac{1}{2}}$.

$\frac{1}{2}\log 93.73 = \frac{1}{2}$ of $1.9719 = 0.9860 = \log 9.683$. *Ans.*

76. Find by logarithms $21.97^{\frac{5}{6}}$.

$\frac{5}{6}\log 21.97 = \frac{5}{6}$ of $1.3418 = 1.1182 = \log 13.13$. *Ans.*

77. Find by logarithms $7.935^{\frac{5}{7}}$.

$\frac{5}{7}\log 7.935 = \frac{5}{7}$ of $0.8996 = 0.6426 = \log 4.391$. *Ans.*

78. Find by logarithms $0.815^{\frac{3}{4}}$.

$$
\begin{array}{rl}
\log 0.815 = & 9.9112 - 10 \\
 & 3 \\
\hline
 & 29.7336 - 30 \\
 & 10 - 10 \\
\hline
4) & 39.7336 - 40 \\
\hline
 & 9.9334 - 10 = \log 0.8578. \quad \textit{Ans.}
\end{array}
$$

79. What weight of sulphuric acid, specific gravity 1.841, will fill a silver sphere 138^{mm} in diameter?

$$
\begin{array}{rl}
\log \quad 138^3 = & 6.4197 \\
\log 0.5236 = & 9.7190 - 10 \\
\log \ 1.841 = & 0.2650 \\
\hline
 & 6.4037 = \log 2534000.
\end{array}
$$

That is, $2534000^{cmm} = 2.534^{kg}$. *Ans.*

80. What is the area of a circle 13.75 in. in diameter?

$$
\begin{array}{rl}
\log 6.875^2 = & 1.6746 \\
\log 3.1416 = & 0.4971 \\
\hline
 & 2.1717 = \log 148.5.
\end{array}
$$

That is, 148.5 sq. in. *Ans.*

81. Find the depth of a cubical bin that holds 75 bu.

$$
\begin{array}{rl}
\log \qquad 75 = & 1.8751 \\
\log 2150.42 = & 3.3325 \\
\hline
3) & 5.2076 \\
\hline
 & 1.7359 = \log 54.44.
\end{array}
$$

That is, 54.44 in. *Ans.*

82. Find the diameter of a 24-lb. shot, specific gravity 7.6.

$$\begin{array}{lrl}
\log & 24 = & 1.3802 \\
\log & 1728 = & 3.2375 \\
\text{colog} & 7.6 = & 9.1192 - 10 \\
\text{colog} & 62.5 = & 8.2041 - 10 \\
\text{colog} & 0.5236 = & \underline{10.2810 - 10} \\
 & & 3\overline{)2.2220} \\
 & & 0.7407 = \log 5.504.
\end{array}$$

That is, 5.504 in. *Ans.*

Exercise LXXXIV.

1. What number is 3 less than its square?

Assume 2.3 and 2.4.

$2.3^2 - 2.3 = 5.29 - 2.3 = 2.99$, an error of -0.01.

$2.4^2 - 2.4 = 5.76 - 2.4 = 3.36$, an error of $+0.36$.

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.37.

Hence, error of $2.3 : 0.1 :: 0.01 : 0.37$,

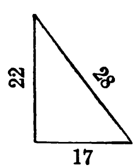
or, error of $2.3 : 0.1 :: 1 : 37$.

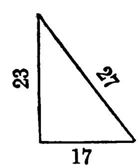
Therefore, the error of $2.3 = \frac{1 \times 0.1}{37} = 0.0028$.

$2.3 + 0.0028 = 2.3028$. *Ans.*

2. A flag-staff 50 ft. high broke, and the top falling over rested one end on the stump and the other 17 ft. from its base. How high was the stump?

Assume 22 ft. and 23 ft.





$22^2 + 17^2 = 484 + 289 = 773$, an error of -11.
$23^2 + 17^2 = 529 + 289 = 818$, an error of $+89$.
The difference of the assumed numbers is 1, and the difference of the errors is 100.
Hence, the error of 22 : 1 : : 11 : 100.

Therefore, the error of $22 = \frac{1 \times 11}{100} = 0.11$.

$22 + 0.111 = 22.11$ ft. *Ans.*

3. What number added to eight times its reciprocal is equal to 8?

Assume 1.1 and 1.2.

$1.1 + 8 \times \frac{1}{1.1} = 1.1 + 7.273 = 8.373$, an error of $+0.373$.

$1.2 + 8 \times \frac{1}{1.2} = 1.2 + 6.667 = 7.867$, an error of -0.133.

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.506.
Hence, the error of 1.1 : 0.1 : : 0.373 : 0.506,
or, the error of 1.1 : 0.1 : : 373 : 506.

Therefore, the error of $1.1 = \frac{1 \times 373}{506} = 0.0737$.

$1.1 + 0.0737 = 1.1737$. *Ans.*

Assume 6.8 and 6.9.

$6.8 + 8 \times \frac{1}{6.8} = 6.8 + 1.177 = 7.977$, an error of -0.023.

$6.9 + 8 \times \frac{1}{6.9} = 6.9 + 1.159 = 8.059$, an error of $+0.059$.

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.082.
Hence, the error of 6.8 : 0.1 : : 0.023 : 0.082,
or, the error of 6.8 : 0.1 : : 23 : 82.

Therefore, the error of $6.8 = \frac{1 \times 23}{82} = 0.028$.

$6.8 + 0.028 = 6.828$. *Ans.*

4. Find a number whose reciprocal is equal to 4 minus the number.

Assume 3 and 4.
$\frac{1}{3} = 4 - 3$, or $\frac{1}{3} = 1$, an error of $+\frac{2}{3}$.
$\frac{1}{4} = 4 - 4$, or $\frac{1}{4} = 0$, an error of $-\frac{1}{4}$.
The difference of the assumed numbers is 1, and the difference of the errors is $\frac{11}{12}$.
Hence, the error of $3 : 1 :: \frac{2}{3} : \frac{11}{12}$,
or, the error of $3 : 1 :: 8 : 11$.

Therefore, the error of $3 = \frac{1 \times 8}{11} = 0.73$.

$3 + 0.73 = 3.73$. *Ans.*

Assume 0.26 and 0.27.

$\frac{1}{0.26} = 4 - 0.26$, or $3.846 = 3.74$, an error of -0.106.

$\frac{1}{0.27} = 4 - 0.27$, or $3.704 = 3.73$, an error of $+0.026$.

The difference of the assumed numbers is 0.01, and the difference of the errors is 0.132.
Hence, the error of $0.26 : 0.01 :: 0.106 : 0.132$,
or, the error of $0.26 : 0.01 :: 53 : 66$.

Therefore, the error of $0.26 = \frac{0.01 \times 53}{66} = 0.008$.

$0.26 + 0.008 = 0.268$. *Ans.*

5. What number is ten times its own logarithm?

Assume 1.3 and 1.5.
$1.3 = 10 \times 0.1139$, or $1.3 = 1.139$, an error of -0.161.
$1.5 = 10 \times 0.1761$, or $1.5 = 1.761$, an error of $+0.261$.
The difference of the assumed numbers is 0.2, and the difference of the errors is 0.422.
Hence, the error of $1.3 : 0.2 :: 0.161 : 0.422$,
or, the error of $1.3 : 0.2 :: 161 : 422$.

Therefore, the error of $1.3 = \frac{161 \times 0.2}{422} = 0.07$

Assume 1.37 and 1.38.

$1.37 = 10 \times 0.1367$, or $1.37 = 1.367$, an error of $- 0.003$.

$1.38 = 10 \times 0.1399$, or $1.38 = 1.399$, an error of $+ 0.019$.

The difference of the assumed numbers is 0.01, and the difference of the errors is 0.022.

Hence, the error of $1.37 : 0.01 : : 0.003 : 0.022$.

or, the error of $1.37 : 0.01 : : 3 : 22$

Therefore, the error of $1.37 = \dfrac{3 \times 0.01}{22} = 0.0013$.

Therefore the number is $1.37 + 0.0013 = 1.3713$. *Ans.*

The number 10 also satisfies the conditions.

6. What number is double its own cube root?

Assume 2.8 and 2.9.

$2\sqrt[3]{2.8} = 2.82$, an error of $+ 0.02$.

$2\sqrt[3]{2.9} = 2.85$, an error of $- 0.05$.

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.07.

Hence, the error of $2.8 : 0.1 : : 0.02 : 0.07$,

or, the error of $2.8 : 0.1 : : 2 : 7$.

Therefore, the error of $2.8 = \dfrac{0.1 \times 2}{7} = 0.0284$.

$2.8 + 0.0284 = 2.8284$. *Ans.*

7. What number exceeds its cube root by $6\frac{1}{4}$?

Assume 8.2 and 8.3.

$8.2 - \sqrt[3]{8.2} = 6.184$, an error of $- 0.066$.

$8.3 - \sqrt[3]{8.3} = 6.275$, an error of $+ 0.025$.

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.091.

Hence, the error of $8.2 : 0.1 : : 0.066 : 0.091$,

or, the error of $8.2 : 0.1 : : 66 : 91$.

Therefore, the error of $8.2 = \dfrac{0.1 \times 66}{91} = 0.072$.

$8.2 + 0.072 = 8.272$. *Ans.*

8. What is the number which added to its own square makes 11?

Assume 2.8 and 2.9.

$2.8 + 2.8^2 = 10.64$, an error of -0.36.

$2.9 + 2.9^2 = 11.31$, an error of $+0.31$.

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.67.

Hence, the error of $2.8 : 0.1 :: 0.36 : 0.67$,

or, the error of $2.8 : 0.1 :: 36 : 67$.

Therefore, the error of $2.8 = \dfrac{0.1 \times 36}{67} = 0.054$.

$2.8 + 0.054 = 2.854$. *Ans.*

9. What is the number which multiplied by 10 makes 8 more than the square of the number?

Assume 9.1 and 9.2.

$10 \times 9.1 = 9.1^2 + 8$, or $91 = 90.81$, an error of -0.19.

$10 \times 9.2 = 9.2^2 + 8$, or $92 = 92.64$, an error of $+0.64$.

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.83.

Hence, the error of $9.1 : 0.1 :: 0.19 : 0.83$,

or, the error of $9.1 : 0.1 :: 19 : 83$.

Therefore, the error of $9.1 = \dfrac{0.1 \times 19}{83} = 0.023$.

$9.1 + 0.023 = 9.123$. *Ans.*

Assume 0.87 and 0.88.

$10 \times 0.87 = 0.87^2 + 8$, or $8.7 = 8.7569$, an error of $+0.0569$.

$10 \times 0.88 = 0.88^2 + 8$, or $8.8 = 8.7744$, an error of -0.0256.

The difference of the assumed numbers is 0.01, and the difference of the errors is 0.0825

Hence, the error of $0.87 : 0.01 :: 0.0569 : 0.0825$,

or, the error of $0.87 : 0.01 :: 569 : 825$.

Therefore, the error of $0.87 = \dfrac{0.01 \times 569}{825} = 0.0069$.

$0.87 + 0.0069 = 0.8769$. *Ans.*

10. A certain number is equal to the sum of $\frac{1}{6}$ its own cube plus $\frac{1}{4}$ its own square. What is the number?

Assume 1.81 and 1.82.

$\frac{1.81^3}{6}+\frac{1.81^2}{4}=1.8073$, an error of -0.0027.

$\frac{1.82^3}{6}+\frac{1.82^2}{4}=1.8329$, an error of $+0.0128$.

The difference of the assumed numbers is 0.01, and the difference of the errors is 0.0156.

Hence, the error of $1.81 : 0.01 :: 0.0027 : 0.0156$,

or, the error of $1.81 : 0.01 :: 9 : 52$.

Therefore, the error of $1.81=\frac{0.01\times 9}{52}=0.0017$.

$1.81+0.0017=1.8117$. *Ans.*

11. What number is equal to its square minus three times its logarithm?

Assume 1.1 and 1.2.

$1.1^2-3\times 0.0414=1.0858$, an error of -0.0142.

$1.2^2-3\times 0.0792=1.2024$, an error of $+0.0024$.

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.0166.

Hence, the error of $1.1 : 0.1 :: 0.0142 : 0.0166$,

or, the error of $1.1 : 0.1 :: 71 : 83$.

Therefore, the error of $1.1=\frac{0.1\times 71}{83}=0.086$.

$1.1+0.086=1.186$. *Ans.*

12. The sum of the square and the square root of a number, being divided by 1 plus the number, gives a quotient of $2\frac{1}{2}$. What is the number?

Assume 2.7 and 2.8.

$\frac{2.7^2+\sqrt{2.7}}{1+2.7}=\frac{8.933}{3.7}=2.415$, an error of -0.085.

$$\frac{2.8^2 + \sqrt{2.8}}{1 + 2.8} = \frac{9.513}{3.8} = 2.503, \text{ an error of } + 0.003.$$

The difference of the assumed numbers is 0.1, and the difference of the errors is 0.088.

Hence, the error of 2.7 : 0.1 : : 0.085 : 0.088,

or, the error of 2.7 : 0.1 : : 85 : 88.

Therefore, the error of $2.7 = \dfrac{0.1 \times 85}{88} = 0.096.$

$2.7 + 0.096 = 2.796$. *Ans.*

Exercise LXXXV.

1. Convert $\frac{3}{11}$, $\frac{13}{75}$, $\frac{29}{127}$, $\frac{135}{64}$ into continued fractions.

(1)

```
3)11(3
   9
   2)3(1
     2
     1)2(2
       2
```

$$\therefore \frac{3}{11} = \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2}}}. \quad \textit{Ans.}$$

(2)

```
13)75(5
   65
   10)13(1
      10
       3)10(3
          9
          1)3(3
            3
```

$$\therefore \frac{13}{75} = \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{3}}}}. \quad \textit{Ans.}$$

(3)

```
29)127(4
   116
   ---
    11)29(2
       22
       --
        7)11(1
           7
          --
          4)7(1
            4
            -
            3)4(1
              3
              -
              1)3(3
                3
                -
```

$$\therefore \frac{29}{127} = \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{3}}}}}}. \ \textit{Ans.}$$

(4)

$$\frac{135}{64} = 2\tfrac{7}{64}.$$

```
7)64(9
  63
  --
   1)7(7
     7
     -
```

$$\therefore \frac{135}{64} = 2 + \cfrac{1}{9 + \frac{1}{7}}. \ \textit{Ans.}$$

2. Find the approximate values of $\frac{20}{27}$; $\frac{53}{47}$; $\frac{734}{851}$.

(1)

```
20)27(1
   20
   --
    7)20(2
      14
      --
       6)7(1
         6
         -
         1)6(6
           6
           -
```

$$\therefore \frac{20}{27} = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{1}{6}}}}$$

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1 + \frac{1}{2}} = \frac{2}{3}.$$

$$\cfrac{1}{1 + \cfrac{1}{2 + \frac{1}{1}}} = \frac{3}{4}.$$

$1, \frac{2}{3}, \frac{3}{4}$. *Ans.*

(2)

$$\frac{53}{47} = 1\tfrac{6}{47}.$$

```
6)47(7
  42
  --
   5)6(1
     5
     -
     1)5(5
       5
       -
```

$$\therefore 1\tfrac{6}{47} = 1 + \cfrac{1}{7 + \cfrac{1}{1 + \frac{1}{5}}}$$

$$1 = 1.$$

$$1 + \frac{1}{7} = \frac{8}{7}.$$

$$1 + \cfrac{1}{7 + \frac{1}{1}} = \frac{9}{8}.$$

$1, \frac{8}{7}, \frac{9}{8}$. *Ans.*

(3)

```
734)851(1
    734
    117)734(6
        702
         32)117(3
             96
             21)32(1
                21
                11)21(1
                   11
                   10)11(1
                      10
                       1)10(10
                         10
```

$$\therefore \tfrac{734}{851} = \cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{10}}}}}}}$$

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1+\cfrac{1}{6}} = \frac{6}{7}.$$

$$\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{3}}} = \frac{19}{22}.$$

$$\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{3+\cfrac{1}{1}}}} = \frac{25}{29}.$$

$$\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1}}}}} = \frac{44}{51}.$$

$$\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}} = \frac{69}{80}.$$

1, $\frac{6}{7}$, $\frac{19}{22}$, $\frac{25}{29}$, $\frac{44}{51}$, $\frac{69}{80}$. *Ans.*

3. Find common fractions approximating to 0.236; 0.2361; 1.609.

(1)

$$0.236 = \tfrac{236}{1000} = \tfrac{59}{250}.$$

```
59)250(4
   236
    14)59(4
       56
        3)14(4
          12
           2)3(1
             2
             1)2(2
               2
```

$$\therefore \tfrac{59}{250} = \cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{2}}}}}$$

$$\frac{1}{4}=\frac{1}{4}$$

$$\cfrac{1}{4+\cfrac{1}{4}}=\frac{4}{17}$$

$$\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4}}}=\frac{17}{72}$$

$$\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{1}}}}=\frac{21}{89}$$

$\frac{1}{4}, \frac{4}{17}, \frac{17}{72}, \frac{21}{89}$. *Ans.*

(2)

$$.2361=\frac{2361}{10000}.$$

```
2361)10000(4
      9444
      ----
       556)2361(4
           2224
           ----
            137)556(4
                548
                ---
                  8)137(17
                    136
                    ---
                      1)8(8
                        8
```

$$\therefore \frac{2361}{10000}=\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{17+\cfrac{1}{8}}}}}$$

$$\frac{1}{4}=\frac{1}{4}$$

$$\cfrac{1}{4+\cfrac{1}{4}}=\frac{4}{17}$$

$$\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4}}}=\frac{17}{72}$$

$$\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{17}}}}=\frac{293}{1241}$$

$\frac{1}{4}, \frac{4}{17}, \frac{17}{72}, \frac{293}{1241}$. *Ans.*

(3)

$$1.609 = 1 + \tfrac{609}{1000}.$$

```
609)1000(1
    609
    391)609(1
        391
        218)391(1
            218
            173)218(1
                173
                 45)173(3
                    135
                     38)45(1
                        38
                         7)38(5
                           35
                            3)7(2
                              6
                              1)3(3
                                3
```

$$\therefore\ 1 + \tfrac{609}{1000} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{3}}}}}}}}}$$

$$1 + \frac{1}{1} = 2.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1}} = \frac{3}{2}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}} = \frac{5}{3}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}} = \frac{8}{5}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3}}}}} = \frac{29}{18}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1}}}}}} = \frac{37}{23}.$$

$2,\ \frac{3}{2},\ \frac{5}{3},\ \frac{8}{5},\ \frac{29}{18},\ \frac{37}{23}.$ *Ans.*

4. Find common fractions approximating to 0.382; 1.732; 0.6253.

(1)

$$0.382 = \tfrac{382}{1000} = \tfrac{191}{500}.$$

```
191)500(2
    382
    118)191(1
        118
         73)118(1
             73
             45)73(1
                45
                28)45(1
                   28
                   17)28(1
                      17
                      11)17(1
                         11
                          6)11(1
                             6
                             5)6(1
                               5
                               1)5(5
                                 5
```

$$\therefore \frac{191}{500} = \cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{5}}}}}}}}}}$$

$$\frac{1}{2} = \frac{1}{2}.$$

$$\cfrac{1}{2+\frac{1}{1}} = \frac{1}{3}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\frac{1}{1}}} = \frac{2}{5}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}} = \frac{3}{8}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}}} = \frac{5}{13}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}}}} = \frac{8}{21}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}}} = \frac{13}{34}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}}}} = \frac{21}{55}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}}}}} = \frac{34}{89}.$$

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{13}$, $\frac{8}{21}$, $\frac{13}{34}$, $\frac{21}{55}$, $\frac{34}{89}$. *Ans.*

(2)

$$1.732 = 1\tfrac{183}{250}.$$

```
183)250(1
    183
     67)183(2
        134
         49)67(1
            49
            18)49(2
               36
               13)18(1
                  13
                   5)13(2
                     10
                      3)5(1
                        3
                        2)3(1
                          2
                          1)2(2
                            2
```

$$\therefore\ 1\tfrac{183}{250} = 1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}}}}}}$$

[illegible]

$$\cdots = \cfrac{1}{\cdots - \cfrac{1}{\cdots - \cfrac{1}{1 - \cfrac{1}{2 - \cfrac{1}{51 - \cfrac{1}{1 - \cfrac{1}{2 - \cfrac{1}{3}}}}}}}}$$

[illegible]

24,124[illegible]51
1224
17,2[illegible]1
17
7

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1+\cfrac{1}{1}} = \frac{1}{2}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}} = \frac{2}{3}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}} = \frac{5}{8}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51}}}}} = \frac{257}{411}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\cfrac{1}{1}}}}}} = \frac{262}{419}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\cfrac{1}{1+\cfrac{1}{2}}}}}}} = \frac{781}{1249}.$$

$1, \frac{1}{2}, \frac{2}{3}, \frac{5}{8}, \frac{257}{411}, \frac{262}{419}, \frac{781}{1249}$. *Ans.*

5. Find approximate values of $\frac{171}{457}$; $\frac{613}{757}$; $\frac{711}{818}$; $\frac{937}{115}$.

(1)

171)457(2
342
115)171(1
115
56)115(2
112
3)56(18
54
2)3(1
2
1)2(2
2

$$\therefore \frac{171}{457} = \cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{18+\cfrac{1}{1+\cfrac{1}{2}}}}}}$$

$$\frac{1}{2} = \frac{1}{2}. \qquad \cfrac{1}{2+\cfrac{1}{1}} = \frac{1}{3}.$$

$$1+\frac{1}{1}=2.$$

$$1+\cfrac{1}{1+\cfrac{1}{2}}=\frac{5}{3}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1}}}=\frac{7}{4}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}}=\frac{19}{11}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1}}}}}=\frac{26}{15}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}}}}=\frac{71}{41}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1}}}}}}}=\frac{97}{56}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1}}}}}}}}=\frac{168}{97}.$$

$2, \frac{5}{3}, \frac{7}{4}, \frac{19}{11}, \frac{26}{15}, \frac{71}{41}, \frac{97}{56}, \frac{168}{97}$. *Ans.*

(3)

$$0.6253=\frac{6253}{10000}.$$

```
6253)10000(1
     6253
     3747)6253(1
          3747
          2506)3747(1
               2506
               1241)2506(2
                    2482
                      24)1241(51
                         1224
                           17)24(1
                              17
                               7
 7)17(2
   14
    3)7(2
      6
      1)3(3
        3
```

$$\therefore \frac{6253}{10000}=\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3}}}}}}}}$$

$$\frac{1}{1} = 1.$$

$$\frac{1}{1+\frac{1}{1}} = \frac{1}{2}.$$

$$\frac{1}{1+\cfrac{1}{1+\frac{1}{1}}} = \frac{2}{3}.$$

$$\frac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}} = \frac{5}{8}.$$

$$\frac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\frac{1}{51}}}}} = \frac{257}{411}.$$

$$\frac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\frac{1}{1}}}}}} = \frac{262}{419}.$$

$$\frac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\cfrac{1}{1+\frac{1}{2}}}}}}} = \frac{781}{1249}.$$

$1, \frac{1}{2}, \frac{2}{3}, \frac{5}{8}, \frac{257}{411}, \frac{262}{419}, \frac{781}{1249}$. *Ans.*

5. Find approximate values of $\frac{171}{457}$; $\frac{613}{757}$; $\frac{711}{813}$; $\frac{937}{113}$.

(1)

```
171)457(2
    342
    115)171(1
        115
         56)115(2
            112
              3)56(18
                54
                 2)3(1
                   2
                   1)2(2
                     2
```

$$\therefore \frac{171}{457} = \frac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{18+\cfrac{1}{1+\frac{1}{2}}}}}}$$

$$\frac{1}{2} = \frac{1}{2}. \qquad \frac{1}{2+\frac{1}{1}} = \frac{1}{3}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}=\frac{3}{8}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{18}}}}=\frac{55}{147}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{18+\cfrac{1}{1}}}}}=\frac{58}{155}.$$

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{55}{147}$, $\frac{58}{155}$. *Ans.*

(2)

```
613)757(1
    613
    144)613(4
        576
         37)144(3
            111
             33)37(1
                33
                 4)33(8
                   32
                    1)4(4
                      4
```

$$\therefore \frac{613}{757}=\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{8+\cfrac{1}{4}}}}}}$$

$$\frac{1}{1}=1. \qquad \cfrac{1}{1+\cfrac{1}{4}}=\frac{4}{5}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3}}}=\frac{13}{16}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1}}}}=\frac{17}{21}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{8}}}}}=\frac{149}{184}.$$

1, $\frac{4}{5}$, $\frac{13}{16}$, $\frac{17}{21}$, $\frac{149}{184}$. *Ans.*

(3)

$\frac{711}{813} = \frac{237}{271}$.

```
237)271(1
    237
     34)237(6
        204
         33)34(1
            33
             1)33(33
               33
```

$$\therefore \frac{237}{271} = \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{33}}}}$$

$$\frac{1}{1} = 1. \qquad \cfrac{1}{1 + \cfrac{1}{6}} = \frac{6}{7}. \qquad \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{1}}} = \frac{7}{8}.$$

$1, \frac{6}{7}, \frac{7}{8}$. *Ans.*

(4)

$\frac{937}{113} = 8\frac{33}{113}$.

```
33)113(3
    99
    14)33(2
       28
        5)14(2
          10
           4)5(1
             4
             1)4(4
               4
```

$$\therefore 8\tfrac{33}{113} = 8 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{4}}}}}$$

$$8 = 8.$$

$$8 + \tfrac{1}{3} = 8\tfrac{1}{3}.$$

$$8 + \cfrac{1}{3 + \cfrac{1}{2}} = 8\tfrac{2}{7}.$$

$$8 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{2}}} = 8\tfrac{5}{17}. \qquad 8 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1}}}} = 8\tfrac{7}{24}.$$

$8, 8\frac{1}{3}, 8\frac{2}{7}, 8\frac{5}{17}, 8\frac{7}{24}$. *Ans.*

6. Find the proper fraction that, when reduced to a continued fraction, will have 2, 3, 5, 6, 7 as quotients.

$$\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{6+\cfrac{1}{7}}}}}=\frac{709}{1640}.\quad \textit{Ans.}$$

$$\frac{1}{6\frac{1}{7}}=\frac{7}{43};\qquad \frac{1}{5\frac{7}{43}}=\frac{43}{222};$$

$$\frac{1}{3\frac{43}{222}}=\frac{222}{709};\qquad \frac{1}{2\frac{222}{709}}=\frac{709}{1640}.$$

7. Find a series of fractions approximating to the ratio of the pound troy (5760 grs.) to the pound avoirdupois (7000 grs.).

$$\frac{5760}{7000}=\frac{144}{175}.$$

```
144)175(1
    144
     31)144(4
        124
         20)31(1
            20
            11)20(1
               11
                9)11(1
                   9
                   2)9(4
                     8
                     1)2(2
                       2
```

$$\therefore \frac{144}{175}=\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{2}}}}}}}$$

$$\frac{1}{1}=1.\qquad \cfrac{1}{1+\cfrac{1}{4}}=\frac{4}{5}.\qquad \cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1}}}=\frac{5}{6}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1}}}}=\frac{9}{11}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}=\frac{14}{17}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4}}}}}}=\frac{65}{79}.$$

$1, \frac{4}{5}, \frac{5}{6}, \frac{9}{11}, \frac{14}{17}, \frac{65}{79}$. *Ans.*

8. Find a series of fractions approximating to the ratio of the side of a square to its diagonal; that ratio being 1 : 1.414214 nearly.

$$\frac{1}{1.414214}=\frac{1000000}{1414214}=\frac{7071}{10000}.$$

$$\begin{array}{l}
7071)10000(1\\
\quad 7071\\
\quad 2929)7071(2\\
\qquad 5858\\
\qquad 1213)2929(2\\
\qquad\quad 2426\\
\qquad\quad 503)1213(2\\
\qquad\qquad 1006\\
\qquad\qquad 207)503(2\\
\qquad\qquad\quad 414\\
\qquad\qquad\quad 89)207(2\\
\qquad\qquad\qquad 178\\
\qquad\qquad\qquad 29)89(3\\
\qquad\qquad\qquad\quad 87\\
\qquad\qquad\qquad\quad 2)29(14\\
\qquad\qquad\qquad\qquad 28\\
\qquad\qquad\qquad\qquad 1)2(2\\
\qquad\qquad\qquad\qquad\quad 2
\end{array}$$

$$\therefore \tfrac{7071}{10000}=\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{14+\cfrac{1}{2}}}}}}}}}$$

$$\frac{1}{1}=1.$$

$$\cfrac{1}{1+\cfrac{1}{2}}=\frac{2}{3}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2}}}=\frac{5}{7}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}=\frac{12}{17}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}}=\frac{29}{41}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}}}=\frac{70}{99}.$$

$1, \frac{2}{3}, \frac{5}{7}, \frac{12}{17}, \frac{29}{41}, \frac{70}{99}$. *Ans.*

9. Find a series of fractions approximating to the ratio of the ar to the square chain, from the equality 1 ar = 0.2471 of a square chain.

$$0.2471 = \tfrac{2471}{10000}.$$

```
2471)10000(4
      9884
      116)2471(21
          2436
            35)116(3
               105
                11)35(3
                   33
                    2)11(5
                      10
                       1)2(2
                         2
```

$$\therefore \tfrac{2471}{10000} = \cfrac{1}{4 + \cfrac{1}{21 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{2}}}}}}$$

$$\frac{1}{4} = \frac{1}{4}. \qquad \frac{1}{4 + \frac{1}{21}} = \frac{21}{85}. \qquad \cfrac{1}{4 + \cfrac{1}{21 + \cfrac{1}{3}}} = \frac{64}{259}.$$

$\frac{1}{4}$, $\frac{21}{85}$, $\frac{64}{259}$. *Ans.*

10. Find a series of fractions approximating to the ratio of the 48-pound shot to the weight of the French shot of 24^{kg}.

$$48 \text{ lbs.} = 48 \times 0.45359 = 21.77232^{kg}.$$

$$\frac{21.77232}{24} = \frac{907}{1000}.$$

```
907)1000(1
    907
     93)907(9
        837
         70)93(1
            70
            23)70(3
               69
                1)23(23
                  23
```

$$\therefore \tfrac{907}{1000} = \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{23}}}}}$$

$$\frac{1}{1} = 1. \qquad \frac{1}{1 + \frac{1}{9}} = \frac{9}{10}. \qquad \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{1}}} = \frac{10}{11}. \qquad \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{3}}}} = \frac{39}{43}.$$

1, $\frac{9}{10}$, $\frac{10}{11}$, $\frac{39}{43}$. *Ans.*

11. If the mean diameter of the Earth is reckoned at 7912 mi., and that of Mars 4189 mi., find a series of fractions approximating to the ratio of the mean diameters of these two planets.

```
4189)7912(1
     4189
     3723)4189(1
          3723
           466)3723(7
               3262
                461)466(1
                    461
                      5
```

$$\therefore \frac{4189}{7912} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{1}}}}$$

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1 + \cfrac{1}{1}} = \frac{1}{2}.$$

$$\cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7}}} = \frac{8}{15}.$$

$$\cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{1}}}} = \frac{9}{17}.$$

$1, \frac{1}{2}, \frac{8}{15}, \frac{9}{17}$. *Ans.*

12. Find a series of fractions approximating to the ratio of a cubic yard to a cubic meter from the equality

1 cu. yd. = 0.76453 of a cubic meter.

$$0.76453 = \frac{76453}{100000}.$$

```
76453)100000(1
       76453
       23547)76453(3
             70641
              5812)23547(4
                   23248
                     299)5812(19
                         5681
                          131)299(2
                              262
                               37
```

$$\therefore \frac{76453}{100000} = \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{19 + \cfrac{1}{2}}}}}$$

$$\frac{1}{1} = 1. \qquad \frac{1}{1+\frac{1}{3}} = \frac{3}{4}. \qquad \frac{1}{1+\cfrac{1}{3+\frac{1}{4}}} = \frac{13}{17}.$$

$$\frac{1}{1+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{19}}}} = \frac{250}{327}.$$

$1, \frac{3}{4}, \frac{13}{17}, \frac{250}{327}$. *Ans.*

13. Find a series of fractions approximating to the ratio of the kilometer to the mile, from the equality $1^{m} = 1.09362$ yds.

$1^{m} = 1.09362$ yds. $\quad 1^{km} = 1093.62$ yds. $\quad 1^{km} = 0.621$ mi.

$0.621 = \frac{621}{1000}$.

621)1000(1
621
379)621(1
379
242)379(1
242
137)242(1
137
105)137(1
105
32)105(3
96
9)32(3
27
5

$$\therefore \frac{621}{1000} = \frac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{3}}}}}}$$

$$\frac{1}{1} = 1. \qquad \frac{1}{1+\frac{1}{1}} = \frac{1}{2}. \qquad \frac{1}{1+\cfrac{1}{1+\frac{1}{1}}} = \frac{2}{3}.$$

$$\frac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}} = \frac{3}{5}. \qquad \frac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}}} = \frac{5}{8}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3}}}}}}=\frac{18}{29}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{3}}}}}}}=\frac{59}{95}.$$

$1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{18}{29}, \frac{59}{95}$. *Ans.*

Exercise LXXXVI.

1. Find the seventh term of the series 3, 5, 7.....

$3 + (6 \times 2) = 3 + 12 = 15$. *Ans.*

2. Find the fifteenth term of the series 2, 7, 12.....

$2 + (14 \times 5) = 2 + 70 = 72$. *Ans.*

3. Find the sixth term of the series 2, $2\frac{5}{7}$, $3\frac{3}{7}$.....

$2 + (5 \times \frac{5}{7}) = 2 + 3\frac{4}{7} = 5\frac{4}{7}$. *Ans*

4. Find the twentieth term of the series 2, $3\frac{1}{4}$, $4\frac{1}{2}$.....

$2 + (1\frac{1}{4} \times 19) = 2 + 23\frac{3}{4} = 25\frac{3}{4}$.

5. Find the seventh term of the series 21, 19, 17.....

$21 - (6 \times 2) = 21 - 12 = 9$. *Ans.*

6. Find the twelfth term of the series 18, $17\frac{1}{3}$, $16\frac{2}{3}$.....

$18 - (11 \times \frac{2}{3}) = 18 - 7\frac{1}{3} = 10\frac{2}{3}$.

7. When the first term of a series is 5, and the common difference $2\frac{1}{4}$, find the thirteenth and eighteenth terms.

$5 + (12 \times 2\frac{1}{4}) = 5 + 27 = 32$. (1)

$5 + (17 \times 2\frac{1}{4}) = 5 + 38\frac{1}{4} = 43\frac{1}{4}$. (2)

8. Find the common difference in a series whose fourth term is 12 and seventh term 27.

$$\frac{27 - 12}{3} = 5. \text{ Ans.}$$

9. Find the common difference in a series whose first term is 20 and fourth term 40.

$$\frac{40 - 20}{3} = 6\frac{2}{3}. \text{ Ans.}$$

10. Find the common difference in a series whose first term is 2 and eleventh term 20.

$$\frac{20 - 2}{10} = 1\frac{4}{5}. \text{ Ans.}$$

11. Find the common difference in a series whose third term is 7 and eighth term $12\frac{1}{2}$.

$$\frac{12\frac{1}{2}-7}{5}=1.1. \textit{ Ans.}$$

12. Find the common difference in a series whose first term is 1 and fourth term 19.

$$\frac{19-1}{3}=6. \textit{ Ans.}$$

Exercise LXXXVII.

1. Find the sum of $1+5+9+\dots$ to twenty terms.

$$l=1+(19\times 4)=77.$$

$$s=20\times\frac{1+77}{2}=780. \textit{ Ans.}$$

2. Find the sum of $4+5\frac{1}{2}+7+\dots$ to eight terms.

$$l=4+(7\times 1\frac{1}{2})=14\frac{1}{2}.$$

$$s=8\times\frac{4+14\frac{1}{2}}{2}=74. \textit{ Ans.}$$

3. Find the sum of $8+7\frac{2}{3}+7\frac{1}{3}+\dots$ to sixteen terms.

$$l=8-(15\times\tfrac{1}{3})=3.$$

$$s=16\times\frac{8+3}{2}=88. \textit{ Ans.}$$

4. Find the sum of $20+18\frac{1}{4}+16\frac{1}{2}+\dots$ to seven terms.

$$l=20-(6\times 1\frac{3}{4})=9\frac{1}{2}.$$

$$s=7\times\frac{20+9\frac{1}{2}}{2}=103\frac{1}{4}. \textit{ Ans.}$$

5. Find the sum of the first twenty natural numbers.

$$s=20\times\frac{1+20}{2}=210. \textit{ Ans.}$$

6. Find the sum of the natural numbers from 37 to 53 inclusive.

$$s=17\times\frac{37+53}{2}=765. \textit{ Ans.}$$

7. Find the sum of a series of thirty terms, of which the first is 21 and the last 59.

$$s=30\times\frac{21+59}{2}=1200. \textit{ Ans.}$$

8. Find the sum of the series whose first two terms are 3 and 9 and last 75.

$$l=a+(n-1)d.$$

$$75=3+6n-6.$$

$$6n=78.$$

$$n=13.$$

$$s=13\times\frac{3+75}{2}=507. \textit{ Ans.}$$

9. Find the sum of a series of twenty terms whose third and fifth terms are 10 and 15.

$$d=\frac{10+15}{2}=2\frac{1}{2}.$$

$$l=5+(19\times 2\frac{1}{2})=52\frac{1}{2}.$$

$$s=20\times\frac{5+52\frac{1}{2}}{2}=575. \textit{ Ans.}$$

10. A stone, when dropped from a height, falls through 16.1 ft. in the first second, 48.3 in the next, 80.5 in the third, and so on, in arithmetical progression. How far will it fall in the seventh second? and how far in 7 sec.?

$$7\text{th term is } 16.1 + (6 \times 32.2) = 209.3 \text{ ft. (1) } \textit{Ans.}$$

$$s = 7 \times \frac{16.1 + 209.3}{2} = 788.9 \text{ ft. (2) } \textit{Ans.}$$

11. A, who travels 8 mi. the first day, 11 the second, 14 the third, and so on, overtakes in 17 dys. B, who started at the same time, and travelled uniformly. What is B's rate per day?

$$l = 8 + (16 \times 3) = 56.$$

$$s = 17 \times \frac{8 + 56}{2} = 544.$$

$$\tfrac{544}{17} = 32 \text{ mi. } \textit{Ans.}$$

12. One hundred stones lie in a straight line, 1 yd. apart. A boy starts at the first stone, brings each of the others in separately, and piles them with the first stone. How far does he travel?

$$l = 2 + (98 \times 2) = 198 \text{ yds.}$$

$$s = 99 \times \tfrac{1}{2}(2 + 198) = 9900 \text{ yds.}$$

$$9900 \text{ yds} = 5\tfrac{5}{8} \text{ mi. } \textit{Ans.}$$

Exercise LXXXVIII.

1. Find the eighth term of the series 2, 6, 18.....

$2 \times 3^7 = 2 \times 2187 = 4374$. *Ans.*

2. Find the fifth term of the series 8, 4, 2.....

$8 \times (\frac{1}{2})^4 = 8 \times \frac{1}{16} = \frac{1}{2}$. *Ans.*

3. Find the seventh term of the series 2, 3, $4\frac{1}{2}$.....

$2 \times (\frac{3}{2})^6 = 2 \times \frac{729}{64} = 22\frac{25}{32}$. *Ans.*

4. Find the sixth term of the series 4, $2\frac{2}{3}$, $1\frac{7}{9}$.....

$4 \times (\frac{2}{3})^5 = 4 \times \frac{32}{243} = \frac{128}{243}$. *Ans.*

5. Find the eighth term of the series 4, 10, 25.....

$4 \times (\frac{5}{2})^7 = 4 \times \frac{78125}{128} = 2441\frac{13}{32}$.

6. Write the first three terms of the series whose fifth and sixth terms are 112 and 224, respectively.

$r = 2$.

$\frac{112}{2^4} = 7$. (1) *Ans.*

$2 \times 7 = 14$. (2) *Ans.*

$2^2 \times 7 = 28$. (3) *Ans.*

7. The seventh and ninth terms of a series are 100 and 144, respectively. Find the twelfth term.

The 9th term = 7th term $\times r^2$.

$\therefore r^2 = \frac{144}{100}$.

$\therefore r = \frac{12}{10} = \frac{6}{5}$.

12th term $= 144 \times (\frac{6}{5})^3$

$= 144 \times \frac{216}{125}$

$= 248.832$. *Ans*

8. A capital of \$1000 is increased by $\frac{1}{10}$ of itself each year. What will it be at the beginning of the fifth year?

$\$1000 \times (\frac{11}{10})^4 = \$1000 \times \frac{14641}{10000}$

$= \$1464.10$. *Ans.*

9. A capital of \$1000 is increased by $\frac{6}{100}$ of itself each year. What will it be at the beginning of the sixth year?

$\$1000 \times (\frac{106}{100})^5$

$= \$1000 \times \frac{13382255776}{10000000000}$

$= \$1338.23$. *Ans.*

Exercise LXXXIX.

1. Find the sum of 2 + 6 + 18 + ····· to six terms.

$$s = 2 \times \frac{3^6 - 1}{3 - 1} = 2 \times \frac{728}{2} = 728. \textit{ Ans.}$$

2. Find the sum of 1 + 2 + 4 + ····· to nine terms.

$$s = 1 \times \frac{2^9 - 1}{2 - 1} = 1 \times \frac{511}{1} = 511. \textit{ Ans.}$$

3. Find the sum of 3 + 9 + 27 + ····· to five terms.

$$s = 3 \times \frac{3^5 - 1}{3 - 1} = 3 \times \frac{242}{2} = 363. \textit{ Ans.}$$

4. Find the sum of $2 + 3 + 4\frac{1}{2} +$ ····· to eight terms.

$$s = 2 \times \frac{(\frac{3}{2})^8 - 1}{\frac{3}{2} - 1} = 2 \times \frac{24\frac{161}{256}}{\frac{1}{2}} = 98\frac{33}{64}. \textit{ Ans.}$$

5. Find the sum of $1 + \frac{1}{3} + \frac{1}{9} + \cdots\cdots$ to eight terms.

$$s = 1 \times \frac{1-(\frac{1}{3})^8}{1-\frac{1}{3}} = \frac{1-\frac{1}{6561}}{\frac{2}{3}} = \frac{6560}{6561} \times \frac{3}{2} = 1\tfrac{1093}{2187}. \textit{ Ans.}$$

6. Find the sum of $1 + \frac{1}{2} + \frac{1}{4} + \cdots\cdots$ to ten terms.

$$s = 1 \times \frac{1-(\frac{1}{2})^{10}}{1-\frac{1}{2}} = 1 \times \frac{1-\frac{1}{1024}}{\frac{1}{2}} = 1 \times \frac{1023}{1024} \times 2 = 1\tfrac{511}{512}. \textit{ Ans.}$$

7. Find the sum of $\frac{1}{2} + \frac{1}{3} + \frac{2}{9} + \cdots\cdots$ to eight terms.

$$s = \frac{1}{2} \times \frac{1-(\frac{2}{3})^8}{1-\frac{2}{3}} = \frac{1}{2} \times \frac{1-\frac{256}{6561}}{\frac{1}{3}} = \frac{1}{2} \times \frac{6305}{6561} \times 3 = 1\tfrac{1931}{4374}. \textit{ Ans.}$$

8. Find the sum of the first six terms of the series whose first term is 3 and ratio 5.

$$s = 3 \times \frac{5^6-1}{5-1} = 3 \times \tfrac{15624}{4} = 11718. \textit{ Ans.}$$

9. Find the sum of the first eight terms of the series whose first term is 3 and ratio $\frac{1}{3}$.

$$s = 3 \times \frac{1-(\frac{1}{3})^8}{1-\frac{1}{3}} = 3 \times \frac{1-\frac{1}{6561}}{\frac{2}{3}} = 3 \times \frac{6560}{6561} \times \frac{3}{2} = 4\tfrac{364}{729}. \textit{ Ans.}$$

10. A person saved in one year \$64, and in each succeeding year, for 9 years more, $1\frac{1}{2}$ times as much as in the preceding year. Find the whole amount saved.

$$s = \$64 \times \frac{(\frac{3}{2})^{10}-1}{\frac{3}{2}-1} = \$64 \times \frac{56\frac{681}{1024}}{\frac{1}{2}} = \$7253.13. \textit{ Ans.}$$

Exercise XC.

1. Find the sum of the infinite series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\cdots$

$$s = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1. \textit{ Ans.}$$

2. Find the sum of the infinite series $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots\cdots$

$$s = \frac{\frac{2}{3}}{1-\frac{2}{3}} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2. \textit{ Ans.}$$

3. Find the sum of the infinite series $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots\cdots$

$$s = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}. \textit{ Ans.}$$

4. Find the sum of the infinite series $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \cdots\cdots$

$$s = \frac{\frac{1}{5}}{1 - \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}. \textit{ Ans.}$$

5. Find the sum of the infinite series 0.171717.

$$s = \frac{0.17}{1 - 0.01} = \frac{0.17}{0.99} = \frac{17}{99}. \textit{ Ans.}$$

6. Find the sum of the infinite series 0.212121.....

$$s = \frac{0.21}{1 - 0.01} = \frac{0.21}{0.99} = \frac{21}{99}. \textit{ Ans.}$$

7. Find the sum of the infinite series 0.9999.....

$$s = \frac{0.9}{1 - 0.1} = \frac{0.9}{0.9} = 1. \textit{ Ans.}$$

8. Find the sum of the infinite series 0.232323.....

$$s = \frac{0.23}{1 - 0.01} = \frac{0.23}{0.99} = \frac{23}{99}. \textit{ Ans.}$$

9. Find the sum of the infinite series 0.36848484.....

$$s = \frac{0.0084}{1 - 0.01} = \frac{0.0084}{0.99} = \frac{84}{9900}. \qquad \frac{36}{100} + \frac{84}{9900} = \frac{3648}{9900}. \textit{ Ans.}$$

10. Find the sum of the infinite series 0.15272727.

$$s = \frac{0.0027}{1 - 0.01} = \frac{0.0027}{0.99} = \frac{27}{9900}. \qquad \frac{15}{100} + \frac{27}{9900} = \frac{1512}{9900}. \textit{ Ans.}$$

Exercise XCI.

1. A deposits \$60 in a savings bank, and draws it out at the end of 8 yrs., with 4% compound interest. What does he receive?

$$\log A = \log P + n \times \log (1 + r),$$
$$\log A = \log 60 + 8 \times \log 1.04.$$
$$\log 60 = 1.7782$$
$$8 \times \log 1.04 = 0.1360$$
$$1.9142 = \log 82.08$$

That is, \$82.08. *Ans.*

2. What will \$100 amount to in 7 years, interest at 8% per annum, compounded semi-annually?

$$\log A = \log P + n \times \log (1 + r),$$
$$\log A = \log 100 + 14 \times \log 1.04.$$

$$\begin{aligned} \log 100 &= 2.0000 \\ 14 \times \log 1.04 &= \underline{0.2380} = \log 173. \\ & 2.2380 \end{aligned}$$

That is, \$173. *Ans.*

3. In how many years will a sum of money double itself at 6%, compounded annually?

$$\log A = \log P + n \times \log(1 + r),$$
$$\log 2 = \log 1 + n \times \log 1.06,$$
$$\log 2 - \log 1 = n \times \log 1.06.$$
$$\frac{\log 2 - \log 1}{\log 1.06} = n.$$
$$\therefore \frac{0.3010}{0.0253} = 11.8 = n.$$

That is, 11.8 yrs. *Ans.*

4. In how many years will a sum of money treble itself at 6%, compounded annually?

$$\log A = \log P + n \times \log(1 + r),$$
$$\log 3 = \log 1 + n \times \log 1.06,$$
$$\log 3 - \log 1 = n \times \log 1.06,$$
$$\frac{\log 3 - \log 1}{\log 1.06} = n.$$
$$\therefore \frac{0.4771}{0.0253} = 18.9 = n.$$

That is, 18.9 yrs. *Ans.*

5. In how many years will \$87 amount to \$99 at 3%, compounded annually?

$$\log A = \log P + n \times \log(1 + r),$$
$$\log 99 = \log 87 + n \times \log 1.03,$$
$$\log 99 - \log 87 = n \times \log 1.03,$$
$$\frac{\log 99 - \log 87}{\log 1.03} = n.$$
$$\therefore \frac{0.0561}{0.0128} = 4.38 = n.$$

That is, 4.38 yrs. *Ans.*

6. In how many years will \$100 amount to \$175 at 4%, compounded annually?

$$\log A = \log P + n \times \log(1+r),$$
$$\log 175 = \log 100 + n \times \log 1.04,$$
$$\log 175 - \log 100 = n \times \log 1.04,$$
$$\frac{\log 175 - \log 100}{\log 1.04} = n.$$
$$\therefore \frac{0.2430}{0.00170} = 14.29 = n.$$

That is, 14.29 yrs. *Ans.*

7. At what rate per cent will a sum of money double itself in 12 years, compound interest?

$$\log A = \log P + n \times \log(1+r),$$
$$\log 2 = \log 1 + 12 \times \log(1+r),$$
$$\log 2 - \log 1 = 12 \times \log(1+r),$$
$$\frac{\log 2 - \log 1}{12} = \log(1+r).$$
$$\therefore \frac{0.3010}{12} = 0.0251 = \log(1+r).$$
$$\therefore 1.0595 = 1 + r, \text{ or } r = 0.0595.$$

That is, 5.95%. *Ans.*

8. At what rate will a sum of money treble itself in 15 years, at compound interest?

$$\log A = \log P + n \times \log(1+r),$$
$$\log 3 = \log 1 + 15 \times \log(1+r),$$
$$\log 3 - \log 1 = 15 \times \log(1+r),$$
$$\frac{\log 3 - \log 1}{15} = \log(1+r).$$
$$\therefore \frac{0.4771}{15} = 0.0318 = \log(1+r).$$
$$\therefore 1 + r = 1.076, \text{ or } r = 0.076.$$

That is, 7.6%. *Ans.*

9. At what rate will \$80 at compound interest amount to \$110 in 8 yrs.?

$$\log A = \log P + n \times \log(1+r),$$
$$\log 110 = \log 80 + 8 \times \log(1+r),$$
$$\log 110 - \log 80 = 8 \times \log(1+r),$$

$$\frac{\log 110 - \log 80}{8} = \log(1+r).$$

$$\therefore \frac{0.1383}{8} = 0.0173 = \log(1+r).$$

$$\therefore \quad 1 + r = 1.041, \text{ or } r = 0.041.$$

That is, 4.1%. *Ans.*

10. What sum must be invested at 5%, compound interest, to amount to \$1200 in 7 yrs.

$$\log \quad A = \log P + n \times \log(1+r),$$
$$\log 1200 = \log P + 7 \times \log 1.05,$$
$$\log \quad P = \log 1200 - 7 \times \log 1.05.$$

$$\begin{aligned} \log 1200 &= 3.0792 \\ 7 \times \log 1.05 &= \underline{0.1484} \\ & 2.9308 \\ &= \log 852.8. \end{aligned}$$

That is, \$852.80. *Ans.*

11. What sum must be invested at 4%, compound interest, to amount to \$2000 in 10 yrs.? To amount to \$5000 in 8 yrs.?

$$\log \quad A = \log P + n \times \log(1+r),$$
$$\log 2000 = \log P + 10 \times \log 1.04,$$
$$\log \quad P = \log 2000 - 10 \times \log 1.04.$$

$$\begin{aligned} \log 2000 &= 3.3010 \\ 10 \times \log 1.04 &= \underline{0.1700} \\ & 3.1310 \\ &= \log 1352. \end{aligned}$$

That is, \$1352. (1) *Ans.*

$$\log \quad A = \log P + n \times \log(1+r),$$
$$\log 5000 = \log P + 8 \times \log 1.04,$$
$$\log \quad P = \log 5000 - 8 \times \log 1.04.$$

$$\begin{aligned} \log 5000 &= 3.6990 \\ 8 \times \log 1.04 &= \underline{0.1360} \\ & 3.5630 \\ &= \log 3656. \end{aligned}$$

That is, \$3656. (2) *Ans.*

12. If A puts \$100 a year into a savings bank that pays 4% per annum, compound interest, what will he have in the bank at the end of 10 years?

From § 438,

$$s = \frac{a(r^n - 1)}{r - 1}.$$

$$s = \frac{\$100(1.04^{10} - 1)}{1.04 - 1}.$$

$$s = \frac{\$100(1.479 - 1)}{0.04}.$$

$$s = \frac{\$100 \times 0.479}{0.04}.$$

$$s = \frac{\$47.9}{0.04}.$$

$s = \$1197.50$. *Ans.*

13. What will be the amount in the last problem if the bank pays $4\frac{1}{2}$% per annum?

$$s = \frac{a(r^n - 1)}{r - 1} = \frac{\$100(1.045^{10} - 1)}{1.045 - 1} = \frac{\$100(1.5525 - 1)}{0.045}$$

$$= \frac{\$100 \times 0.5525}{0.045} = \frac{\$55.25}{0.045} = \$1227.78. \textit{ Ans.}$$

14. What should be paid to-day for an annuity of \$500 a year, for 12 years, if money is worth $3\frac{1}{2}$%, compound interest?

$$s = \frac{a(r^n - 1)}{r - 1} = \frac{500(1.035^{12} - 1)}{0.035}.$$

$12 \log 1.035 = 0.1788$
$= \log 1.509.$

Present worth of s is

$$\frac{500 \times 0.509}{0.035 \times 1.035^{12}}.$$

$$\begin{aligned} \log\ 500 &= 2.6990 \\ \log 0.509 &= 9.7067 - 10 \\ \text{colog}\, 0.035 &= 1.4559 \\ \text{colog}\, 1.035 &= 9.8212 - 10 \\ \hline & 3.6828 \\ &= \log 4818. \end{aligned}$$

That is, \$4818. *Ans.*

15. What should be paid to-day for an annuity of \$300 a year, for 10 years, if money is worth 4%, compound interest?

$$s = \frac{300(1.04^{10} - 1)}{0.04}.$$

Present worth of s

$$\frac{300 \times 0.479}{0.04 \times 1.04^{10}}.$$

$$\begin{aligned}
\log\ 300 &= 2.4771 \\
\log 0.479 &= 9.6803 - 10 \\
\text{colog}\, 0.04 &= 1.3979 \\
\text{colog}\, 1.04^{10} &= 9.8300 - 10 \\
\hline
& \quad 3.3853 \\
& = \log 2428.
\end{aligned}$$

That is, \$2428. *Ans.*

$$\begin{aligned}
\log 1.04 &= 0.0170 \\
& \quad\ \ 10 \\
\hline
& \quad 0.1700 \\
& = \log 1.479.
\end{aligned}$$

16. What should be paid to-day for the assurance that 5 years hence I shall begin to receive \$500 a year, for 8 years, if money is worth $4\frac{1}{2}$, compound interest?

$$s = \frac{500(1.045^8 - 1)}{0.045}.$$

Present worth of s is

$$\frac{500 \times 0.422}{0.045 \times 1.045^{13}}.$$

$$\begin{aligned}
\log\ 500 &= 2.6990 \\
\log\ 0.422 &= 9.6253 - 10 \\
\text{colog}\ 0.045 &= 1.3468 \\
\text{colog}\, 1.045^{13} &= 9.7517 - 10 \\
\hline
& \quad 3.4228 \\
& = \log 2647.50.
\end{aligned}$$

That is, \$2647.50. *Ans.*

$$\begin{aligned}
\log 1.045 &= 0.0191 \\
& \quad\ \ 8 \\
\hline
& \quad 0.1528 \\
& = \log 1.422.
\end{aligned}$$

17. If interest is reckoned at 6%, what sum of money must be paid annually, beginning a year hence, to clear off a debt of \$10,000 in 5 equal payments?

$s = \dfrac{(1.06^5 - 1)}{0.06}$, the amount of \$1 annually deposited at 6% for 5 yrs.

Each payment must be $\dfrac{10000 \times 1.06^5}{\dfrac{(1.06^5 - 1)}{0.06}} = \dfrac{10000 \times 1.06^5 \times 0.06}{0.338}.$

$$\begin{aligned} \log 10000 &= 4.0000 \\ \log\ 1.06^5 &= 0.1265 \\ \log\ 0.06 &= 8.7782-10 \\ \text{colog}\ 0.338 &= 0.4711 \\ \hline & 3.3758 = \log 2376. \end{aligned} \qquad \begin{aligned} \log 1.06 &= 0.0253 \\ &\quad\ 5 \\ \hline 0.1265 &= \log 1.338. \end{aligned}$$

That is, \$2376. *Ans.*

18. If interest is reckoned at 6%, what is the amount of each of 12 equal semi-annual payments, the first to be paid 6 months hence, required to clear off a debt of \$24,000?

$s = \dfrac{(1.03^{12}-1)}{0.03}$, the amount of \$1 annually deposited at 3% for 12 yrs.

Each payment must be $\dfrac{24000 \times 1.03^{12}}{\dfrac{(1.03^{12}-1)}{0.03}} = \dfrac{24000 \times 1.03^{12} \times 0.03}{0.424}$

$$\begin{aligned} \log 24000 &= 4.3802 \\ \log\ 1.03^{12} &= 0.1536 \\ \log\ 0.03 &= 8.4771-10 \\ \text{colog}\ 0.424 &= 0.3726 \\ \hline & 3.3835 = \log 2418. \end{aligned} \qquad \begin{aligned} \log 1.03 &= 0.0128 \\ &\quad\ 12 \\ \hline 0.1536 &= \log 1.424. \end{aligned}$$

That is, \$2418. *Ans.*

Miscellaneous Problems.

1. Make six different numbers with the digits 1, 2, 3, and find their sum.

$$\begin{array}{r} 123 \\ 132 \\ 213 \\ 231 \\ 312 \\ 321 \\ \hline 1332. \end{array}$$ *Ans.*

2. Make six different numbers with the digits 2, 3, 5, and find, by logarithms, their continued product.

$235 \times 253 \times 325 \times 352 \times 523 \times 532.$

$$\begin{aligned} \log 235 &= 2.3711 \\ \log 253 &= 2.0431 \\ \log 325 &= 2.5119 \\ \log 352 &= 2.5465 \\ \log 523 &= 2.7185 \\ \log 532 &= 2.7259 \\ \hline & 15.2770 \end{aligned}$$

$= \log 1{,}892{,}000{,}000{,}000{,}000.$

3. Make six different numbers with the digits 8, 7, 3, and find, by logarithms, their continued product.

$873 \times 837 \times 783 \times 738 \times 387 \times 378.$

$$\begin{aligned}\log 873 &= 2.9410\\ \log 837 &= 2.9227\\ \log 783 &= 2.8938\\ \log 738 &= 2.8681\\ \log 387 &= 2.5877\\ \log 378 &= 2.5775\\ &\overline{16.7908}\end{aligned}$$

$= \log 61{,}770{,}000{,}000{,}000{,}000.$

4. Find, by logarithms, the missing term in each of the following proportions:

(1)

$7.13 : 3.57 :: 4.18 : ?$

$$\frac{3.57 \times 4.18}{7.13} = ?$$

$$\begin{aligned}\log 3.57 &= 0.5527\\ \log 4.18 &= 0.6212\\ \text{colog } 7.13 &= 9.1469 - 10\\ &\overline{0.3208}\\ &= \log 2.093. \quad \textit{Ans.}\end{aligned}$$

(2)

$5.89 : 76.3 :: ? : 38.7.$

$$\frac{5.89 \times 38.7}{76.3} = ?$$

$$\begin{aligned}\log 5.89 &= 0.7701\\ \log 38.7 &= 1.5877\\ \text{colog } 76.3 &= 8.1175 - 10\\ &\overline{0.4753}\\ &= \log 2.987. \quad \textit{Ans.}\end{aligned}$$

(3)

$7.37 : ? :: 86.1 : 43.7.$

$$\frac{7.37 \times 43.7}{86.1} = ?$$

$$\begin{aligned}\log 7.37 &= 0.8675\\ \log 43.7 &= 1.6405\\ \text{colog } 86.1 &= 8.0650 - 10\\ &\overline{0.5730}\\ &= \log 3.741. \quad \textit{Ans.}\end{aligned}$$

(4)

$? : 69.7 :: 3.79 : 29.4.$

$$\frac{69.7 \times 3.79}{29.4} = ?$$

$$\begin{aligned}\log 69.7 &= 1.8432\\ \log 3.79 &= 0.5786\\ \text{colog } 29.4 &= 8.5317 - 10\\ &\overline{0.9535}\\ &= \log 8.984. \quad \textit{Ans.}\end{aligned}$$

5. Find, by logarithms, the values of $0.08^{\frac{1}{3}}$; $2734^{\frac{1}{3}}$; $21.97^{\frac{1}{3}}$; $7^{3.6}$.

$$\begin{aligned}\tfrac{1}{3}\log 0.08 &= \tfrac{1}{3} \text{ of } (8.9031 - 10) = 9.6344 - 10 = \log 0.4309.\\ \tfrac{1}{3}\log 2734 &= \tfrac{1}{3} \text{ of } 3.4368 = 1.1456 = \log 13.98.\\ \tfrac{1}{3}\log 21.97 &= \tfrac{1}{3} \text{ of } 1.3418 = 0.4473 = \log 2.801.\\ 3.6 \log 7 &= 3.6 \times 0.8451 = 3.0424 = \log 1103.\end{aligned}$$

6. Find, by logarithms, the values of $9.71^{\frac{7}{3}}$; $7.935^{\frac{5}{7}}$.

$$\tfrac{7}{3}\log 9.71 = \tfrac{7}{3} \times 0.9872 = 2.3035 = \log 201.1. \text{ (1) } \textit{Ans.}$$

$$\tfrac{5}{7}\log 7.935 = \tfrac{5}{7} \times 0.8996 = 0.6426 = \log 4.391. \text{ (2) } \textit{Ans.}$$

7. What is the horizontal distance between two points, when the air-line distance is 1534 ft., and the difference of level 34 ft.?

$$\sqrt{1534^2 - 34^2} = \sqrt{2352000} = 1533.623 \text{ ft. } \textit{Ans.}$$

8. Find the horizontal distance when the road distance is 1 mile, and the rise 347 ft.

$$\sqrt{(5280 + 347)(5280 - 347)} = \sqrt{27757991} = 5268.585 \text{ ft. } \textit{Ans.}$$

9. If the road distance is half a mile, and the horizontal distance 2513 ft., find the difference of level.

$$\sqrt{(2640 + 2513)(2640 - 2513)} = \sqrt{654431} = 808.97 \text{ ft. } \textit{Ans.}$$

10. The diagonal of a rectangular floor is 34.6 ft., and the width is 17.8 ft. Find the length of the floor.

$$\sqrt{(34.6 + 17.8)(34.6 - 17.8)} = \sqrt{880.32} = 29.67 \text{ ft. } \textit{Ans.}$$

11. The height of a tower on a river's bank is 55 ft., the length of a line from the top to the opposite bank is 78 ft. Find the breadth of the river.

$$\sqrt{(78 + 55)(78 - 55)} = \sqrt{3059} = 55.31 \text{ ft. } \textit{Ans.}$$

12. The number of seamen at Portsmouth is 800, at Charlestown 404, and at Brooklyn 756. A ship is commissioned whose complement is 490 seamen. Determine the number to be drafted from each place in order to obtain a proportionate number from each.

$$800 + 404 + 756 = 1960. \qquad \tfrac{404}{1960} \times \tfrac{490}{1} = 101, \text{ C.}$$

$$\tfrac{800}{1960} \times \tfrac{490}{1} = 200, \text{ P.} \qquad \tfrac{756}{1960} \times \tfrac{490}{1} = 189, \text{ B.}$$

13. Show, without division, that 36,432 contains 8, 9, 11 as factors

$$432 = 54 \times 8.$$

$$3 + 6 + 4 + 3 + 2 = 18.$$

$$3 + 4 + 2 = 6 + 3.$$

(See § 222.)

14. Find the smallest multiplier that will make 47,250 a perfect cube.

$$47{,}250 = 2 \times 3^3 \times 5^3 \times 7.$$

$$2^2 \times 7^2 = 4 \times 49 = 196. \quad \textit{Ans.}$$

15. Find the proper fraction which, when reduced to a continued fraction, has for quotients, 1, 3, 5, 7, 2, 4.

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{2+\cfrac{1}{4}}}}}} = \frac{1099}{1443}. \quad \textit{Ans.}$$

16. If the meter is equal to 1.09362 yds., find a series of four fractions that will express more and more nearly the true ratio of the meter to the yard.

$$1.09362 = 1\tfrac{9362}{100000} = 1\tfrac{4681}{50000}.$$

```
4681)50000(10
     46810
      3190)4681(1
           3190
           1491)3190(2
                2982
                 208)1491(7
                     1456
                       35
```

$$\therefore\ 1\tfrac{4681}{50000} = 1 + \cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{7}}}}.$$

$$1 + \frac{1}{10} = \frac{11}{10}.$$

$$1 + \cfrac{1}{10+\cfrac{1}{1}} = \frac{12}{11}.$$

$$1 + \cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{2}}} = \frac{35}{32}.$$

$$1 + \cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{7}}}} = \frac{257}{235}.$$

$$\frac{11}{10},\ \frac{12}{11},\ \frac{35}{32},\ \frac{257}{235}. \quad \textit{Ans.}$$

17. Find the square factors contained in 33,075.

$33075 = 3^3 \times 5^2 \times 7^2.$ $\qquad 3^2 \times 5^2 = 225,$

$3^2 = 9,$ $\qquad 3^2 \times 7^2 = 441,$

$5^2 = 25,$ $\qquad 5^2 \times 7^2 = 1225,$

$7^2 = 49.$ $\qquad 3^2 \times 5^2 \times 7^2 = 11025.$

9, 25, 49, 225, 441, 1225, 11,025.

18. The top of St. Peter's, Rome, is $\frac{9}{110}$ of a mile above the ground, and that of St. Paul's, London, is $\frac{17}{264}$ of a mile. By how many feet does the height of St. Peter's exceed that of St. Paul's?

$$\frac{9}{\cancel{110}} \text{ of } \frac{\overset{48}{\cancel{5280}}}{1} = 432 \text{ ft.} \qquad \frac{17}{\cancel{264}} \text{ of } \frac{\overset{20}{\cancel{5280}}}{1} = 340 \text{ ft.}$$

432 ft. – 340 ft. = 92 ft. *Ans.*

19. How many days elapsed between the annular eclipse of May 15, 1836, and that of March 15, 1858?

$$\begin{array}{r} 1858 - 3 - 15 \\ 1836 - 5 - 15 \\ \hline 21 - 10 \end{array}$$

During the interval there were five leap years, and in the ten months from May 15 to March 15 there are 304 days.

$365 \times 21 = 7665.$ $\qquad 7665 + 304 + 5 = 7974$ days. *Ans.*

20. In a gale, a flag-staff 60 ft. high snaps 28.8 ft. from the bottom; and, not being wholly broken off, the top touches the ground. If the ground is level, how far is the top from the bottom?

$$\text{The distance} = \sqrt{(31.2 + 28.8)(31.2 - 28.8)} = \sqrt{60 \times 2.4}$$
$$= \sqrt{144} = 12 \text{ ft. } \textit{Ans.}$$

21. Seventeen trees are standing in a line, 20 yds. apart from each other; a person walks from the first to the second and back, then to the third and back, and so on to the end. How far does he walk?

The total distance is the sum of an arithmetical series in which

$$n = 16,\ d = 40,\ a = 40.$$

$l = a + (n - 1)d = 40 + 15 \times 40 = 640.$

$s = \frac{n}{2}(a + l) = \frac{16}{2} \times (40 + 640) = 5440$ yds. $= 3$ mi. 160 yds. *Ans.*

22. A level reach in a canal is $14\frac{3}{4}$ mi. long and 48 ft. broad. At one end is a lock 80 ft. long, 12 ft. broad, and with a fall of 8 ft. 6 in. How many barges can pass through the lock before the water in the canal is lowered 1 in.?

The amount of water that can be drained off in lowering the level 1 in. is $14\frac{3}{4} \times 5280 \times 48 \times \frac{1}{12} = 311{,}520$ cu. ft. The amount of water wasted each time a barge goes through the lock is $80 \times 12 \times 8\frac{1}{2} = 8160$ cu. ft. Hence,

$$311{,}520 \div 8160 = 38 \text{ barges. } \textit{Ans.}$$

23. Find the capacity, in liters and in bushels, of a box 1.7^{m} long, 87^{cm} wide, and 31^{cm} deep.

$$1.7^{\text{m}} = 170^{\text{cm}}.$$
$$1^{\text{l}} = 0.908 \text{ qt.}$$
$$170 \times 87 \times 31 = 458{,}490^{\text{ccm}} = 458.49^{\text{l}}. \text{ (1) } \textit{Ans.}$$
$$458.49 \times 0.908 = 416.309 \text{ qts.}$$
$$1 \text{ bu.} = 32 \text{ qts.}$$
$$416.309 \div 32 = 13 \text{ bu. (2) } \textit{Ans.}$$

24. Find the number of kilograms of olive oil, specific gravity 0.915, to fill a vessel 2.3^{m} long, 1.8^{m} wide, and 74^{cm} deep.

$$74^{\text{cm}} = 0.74^{\text{m}}.$$
$$2.3 \times 1.8 \times 0.74 = 3.0636^{\text{cbm}} = 3063.6^{\text{kg}}.$$
$$3063.6^{\text{kg}} \times 0.915 = 2803.194^{\text{kg}}. \textit{ Ans.}$$

25. How many tons in a block of marble 4 ft. long, 34 in. wide, 17.3 in. thick, if its specific gravity is 2.73?

$$4 \text{ ft.} = 48 \text{ in.}$$
$$48 \times 34 \times 17.3 = 28233.6 \text{ cu. in.}$$
$$28233.6 \div 1728 = 16.34 \text{ cu. ft.}$$
$$16.34 \times 62\tfrac{1}{2} \text{ lbs.} = 1021.25 \text{ lbs.}$$
$$1021.25 \text{ lbs.} \times 2.73 = 2788.0125 \text{ lbs.} = 1.394 \text{ t. } \textit{Ans.}$$

26. Find the surface of a sphere 18.3 in. in diameter.

$$\text{The radius} = 9.15 \text{ in.}$$
$$9.15 \times 9.15 \times 3.1416 = 263.0226 \text{ sq. in.}$$
$$263.0226 \times 4 = 1052.09 \text{ sq. in. } \textit{Ans.}$$

27. Find the number of acres in a circular field 213 yds. 2 ft. across.

213 yds. 2 ft. = 641 ft. in diameter.

The radius is 320.5 ft.

$$1 \text{ A.} = 43560 \text{ sq. ft.}$$

$$\text{Area} = \frac{3.1416 \times 320.5^2}{43560}$$

$$\begin{aligned} \log 3.1416 &= 0.4971 \\ \log\ 320.5^2 &= 5.0116 \\ \text{colog}\ 43560 &= 5.3609 - 10 \\ \hline & 0.8696 = \log 7.407. \end{aligned}$$

7.407 A. *Ans.*

28. How many cubic inches in a 10-inch globe? in a 20-inch globe? What is the ratio of their volumes?

The ratio of their volumes is $10^3 : 20^3 = 1^3 . 2^3 = 1 : 8$. *Ans.*
$10^3 \times 0.5236 = 523.6$ cu in. (1). *Ans*
$523.6 \times 8 = 4188.8$ cu. in. (2). *Ans.*

29. How many balls 3 in. in diameter can be cast from a pig of iron 7 ft. long, 6.7 in. wide, 3.8 in. thick, if the waste in melting and casting is reckoned at $3\frac{1}{4}\%$?

$$7 \text{ ft.} = 84 \text{ in.}$$

$$\text{No. of balls} = \frac{84 \times 6.7 \times 3.8 \times 0.9675}{3^3 \times 5236}.$$

$$\begin{aligned} \log\quad 84 &= 1.9243 \\ \log\quad 6.7 &= 0.8261 \\ \log\quad 3.8 &= 0.5798 \\ \log 0.9675 &= 9.9857 - 10 \\ \text{colog}\quad 3^3 &= 8.5686 - 10 \\ \text{colog}\, 0.5236 &= 0.2810 \\ \hline & 2.1655 = \log 146.4. \end{aligned}$$

Hence, number of balls is 146. *Ans.*

30. Find the difference in length, at 80° F., of a glass and a steel rod, each 3 ft. long at freezing point, if the expansion at 100° C. is 0.00085 for glass and 0.0012 for steel.

$$80^\circ \text{ F} = \tfrac{5}{9}(80^\circ - 32^\circ) \text{ C.} = 26\tfrac{2}{3}^\circ \text{ C.}$$

Difference in length at 100° C. is $0.0012 - 0.00085 = 0.00035$.

$$3 \text{ ft.} = 36 \text{ in.}$$

$$0.00035 \times 36 \times 0.26\tfrac{2}{3} = 0.00366 \text{ in. } \textit{Ans.}$$

31. A grain of gold is beaten out in leaf to cover 56 sq. in. What weight will be required for gilding the faces of a cube whose edge is $3\frac{1}{2}$ ft.?

$$6 \times 3\tfrac{1}{2} \times 3\tfrac{1}{2} = 73\tfrac{1}{2} \text{ sq. ft.}$$

$$144 \times 73\tfrac{1}{2} \text{ sq. ft.} = 10584 \text{ sq. in.}$$

$$10584 \div 56 = 189 \text{ grs.} = 7 \text{ dwt. } 21 \text{ grs. } \textit{Ans.}$$

32. What premium must be paid, at the rate of $1\frac{7}{8}\%$, for insuring a vessel worth \$117,750, in order that in the event of loss the owner may receive both the value of the ship and the premium?

$$1 - 0.01875 = 0.98125.$$

$$\$117,750 \div 0.98125 = \$120,000.$$

$$\$120,000 - 117,750 = \$2250. \textit{ Ans.}$$

33. By selling goods at 60 cts. a pound, 8% on the cost is lost; what advance must be made in the price in order to gain 15% on the cost?

If the selling price is 92, and we wish it to be 115, we must add 23, or $\frac{1}{4}$ of 92. That is, we must add $\frac{1}{4}$ of 60 cts. = 15 cts. a pound. *Ans.*

34. Divide \$27.12$\frac{1}{2}$ among three persons, giving the second \$5 less than the first, and twice as much as the third.

Let a represent the amount given to the first.

$a - 5$ will represent the amount given to the second.

$\frac{a}{2} - \frac{5}{2}$ will represent the amount given to the third.

Then, $2\frac{1}{2}a - 7\frac{1}{2}$ will represent the amount given to all three.

But $27.12½ is the amount given to all three.

Hence, $2\frac{1}{2}a - 7\frac{1}{2} = 27.12\frac{1}{2}$.

Add to each side 7½, $2\frac{1}{2}a = 34.625$

Divide each side by 2½, $a = 13.85$

$$a - 5 = 8.85$$

$$\tfrac{1}{2}(a-5) = 4.42\tfrac{1}{2}$$

Hence, 1st gets $13.85; 2d, $8.85; 3d, $4.42½. *Ans.*

35. The population of a city in 1880 was 12,298, showing a decrease of 8⅓% on its population in 1870; in 1870 there was an increase of 7½% on the census of 1860. What was its population in 1860?

$12{,}298 \div 0.91\frac{2}{3} = 13{,}416$, population in 1870.

$13{,}416 \div 1.075 = 12{,}480$, population in 1860. *Ans.*

36. Find the increase of income obtained by transferring $2500 from 3% stocks at 94½ to 4% stocks at 105.

0.03 of $2500 = $75, income from the 3% stock.

0.94½ of $2500 = $2362.50, amount from the 3% stock.

$1.05 is paid for $1 worth of 4% stock.

Hence, $2362.50 is paid for $2362.50 \div 1.05 =$ $2250 stock.

0.04 of $2250 = $90, income from 4% stock.

$90 − $75 = $15, increase of income. *Ans.*

37. Each person breathing in a closed room spoils the air at the rate of about 8 cu. ft. a minute. A congregation of 400 persons enter a closed room 70 ft. by 40 ft. and 20 ft. high. How long will it take them to spoil the air?

The air in the room will last one person $\frac{70 \times 40 \times 20}{8}$ min., or 400 persons,

$$\frac{\overset{7}{\cancel{70}} \times \cancel{40} \times \overset{5}{\cancel{20}}}{\underset{2}{\cancel{8}} \times \underset{10}{\cancel{400}}} = \frac{35}{2} = 17\tfrac{1}{2} \text{ min. } \textit{Ans.}$$

38. How long can the windows and doors of a school-room be safely kept closed when occupied by 50 children, if the room is 25 ft. by 20 ft. and 10 ft. high?

The air in the room will last one person $\frac{25 \times 20 \times 10}{8}$ min., or

50 persons,

$$\frac{\overset{5}{\cancel{25}} \times \overset{5}{\cancel{20}} \times \cancel{10}}{\underset{2}{\cancel{8}} \times \underset{\cancel{2}}{\cancel{50}}} = \frac{25}{2} = 12\tfrac{1}{2} \text{ min. } \textit{Ans.}$$

39. Find the square root to four decimal places, of the reciprocal of 0.0043.

$$\tfrac{1}{2} \operatorname{colog} 0.0043 = 1.1833 = \log 15.25. \ \textit{Ans.}$$

40. A pays B \$230 as the present value of \$300 due in 5 yrs. Which gains by the payment, and how much, if interest is reckoned at 5%?

The present worth of \$300 due in 5 yrs. is $\frac{100}{127.63}$ of \$300 $= \$235.05$. Hence, A gains \$5.05 by the transaction.

41. Find the quantity of coal required by a steamer for a voyage of 4043 mi., if her rate per hour is 14.04 knots, and her consumption of coal 87 t. per day.

$$14.04 \times 6086 \times 24 = 2{,}050{,}738.56 \text{ ft. per day.}$$
$$4043 \times 5280 = 21{,}347{,}040 \text{ ft.}$$
$$21{,}347{,}040 \div 2{,}050{,}738.56 = 10.40944 \text{ days.}$$
$$10.40944 \times 87 \text{ t.} = 905 \text{ t. } 1392 \text{ lbs. } \textit{Ans.}$$

42. Find the area of a circular ring of which the inner and outer diameters are 7.36 and 10.64 in.

$$\begin{aligned} \text{Area} &= 0.7854(10.64^2 - 7.36^2) \\ &= 0.7854\,(113.2096 - 54.1696) \\ &= 0.7854 \times 59.04 \\ &= 46.37 \text{ sq. in. } \textit{Ans.} \end{aligned}$$

43. A and B can do a piece of work in $13\frac{1}{3}$ dys., A and C in $10\frac{2}{3}$ dys., A, B, and C in $7\frac{1}{2}$ dys. In how many days can A do it alone?

In 1 day A and B can do $\frac{3}{40}$ of the work.
In 1 day A and C can do $\frac{3}{32}$ of the work.

In 1 day A, B, and C can do $\frac{2}{15}$ of the work.

$\therefore$ in 1 day B can do $\frac{2}{15} - \frac{3}{32}$ of the work.

And in 1 day C can do $\frac{2}{15} - \frac{3}{40}$ of the work.

And in 1 day A can do $\frac{3}{40} - (\frac{2}{15} - \frac{3}{32})$ of the work.

$$\frac{3}{40} - \left(\frac{2}{15} - \frac{3}{32}\right) = \frac{3}{40} - \frac{2}{15} + \frac{3}{32} = \frac{36 - 64 + 45}{480} = \frac{17}{480}.$$

$\therefore$ it will take A $\frac{480}{17} = 28\frac{4}{17}$ dys. *Ans.*

44. If 3 men working 11 hrs. a day can reap 20 A. in 11 dys., how many men working 12 hrs. a day can reap a field 360 yds. long and 320 yds. broad in 4 dys.?

$$360 \text{ yds.} \times 320 \text{ yds.} = 115{,}200 \text{ sq. yds.}$$
$$115{,}200 \div 30\tfrac{1}{4} \div 160 = \tfrac{2880}{121} \text{ A.}$$

12	11	3 : what?
4	11	
20	$\frac{2880}{121}$	

$$\frac{\overset{\overset{\overset{3}{\cancel{12}}}{\cancel{144}}}{\cancel{11} \times \cancel{11} \times \cancel{2880}} \times 3}{\underset{\cancel{11}}{\cancel{12} \times \cancel{4} \times \cancel{121} \times \cancel{20}}} = 9 \text{ men. } \textit{Ans.}$$

45. Find the area of a triangle whose sides are 12, 5, and 13 in.

Observe that $13^2 = 12^2 + 5^2$. Hence the triangle is a right triangle. Area $= \frac{1}{2}$ of $12 \times 5 = 30$ sq. in. *Ans.*

46. Find the area of a triangle whose sides are 73, 57, and 48 ft.

$$\frac{53 + 57 + 48}{2} = 89.$$

$$\text{Area} = \sqrt{89 \times 16 \times 32 \times 41}.$$

$$\begin{aligned} \log 89 &= 1.9494 \\ \log 16 &= 1.2041 \\ \log 32 &= 1.5051 \\ \log 41 &= 1.6128 \\ \hline 2)\,&6.2714 \\ &3.1357 = \log 1367. \end{aligned}$$

1367 sq. ft. *Ans.*

47. Find the number of hektars in a triangular field whose sides are 37.5^m, 91.7^m, and 78.9^m.

$$\frac{37.5 + 91.7 + 78.9}{2} = 104.05.$$

$$\text{Area} = \sqrt{104.05 \times 66.55 \times 12.35 \times 25.15}.$$

$$\begin{array}{l}
\log 104.05 = 2.0172 \\
\log\ \ 66.55 = 1.8232 \\
\log\ \ 12.35 = 1.0917 \\
\log\ \ 25.15 = \underline{1.4006} \\
\qquad\qquad 2)\underline{6.3327} \\
\qquad\qquad\ \ 3.1663 = \log 1467. \\
\qquad\qquad\ \ 1467^{qm} = 0.1467^{ha}.\ \textit{Ans.}
\end{array}$$

48. Find the number of hektars in a triangular field whose sides are 67.5^m, 81.2^m, and 102.7^m.

$$\frac{67.5 + 81.2 + 102.7}{2} = 125.7.$$

$$\text{Area} = \sqrt{125.7 \times 58.2 \times 44.5 \times 23}$$

$$\begin{array}{l}
\log 125.7 = 2.0994 \\
\log\ \ 58.2 = 1.7649 \\
\log\ \ 44.5 = 1.6484 \\
\log\ \ 23\ \ \ = \underline{1.3617} \\
\qquad\qquad 2)\underline{6.8744} \\
\qquad\qquad\ \ 3.4372 = \log 2736. \\
\qquad\quad 2736^{qm} = 0.2736^{ha}.\ \textit{Ans.}
\end{array}$$

49. Find the number of acres in a triangular field whose sides are 227, 342, and 416 ft.

$$1\ \text{A.} = 43{,}560\ \text{sq. ft.}$$

$$\frac{227 + 342 + 416}{2} = 492.5.$$

$$\text{Area} = \frac{\sqrt{492.5 \times 265.5 \times 150.5 \times 76.5}}{43560}\ \text{A.}$$

$$
\begin{aligned}
\log 492.5 &= 2.6924\\
\log 265.5 &= 2.4241\\
\log 150.5 &= 2.1776\\
\log \ \ 76.5 &= 1.8837\\
&\ 2)\underline{9.1778}\\
&\ \ \ \ 4.5889\\
\text{colog } 43{,}560 &= \underline{5.3609 - 10}\\
&\ \ \ \ 9.9498 - 10 = \log 0.8908.
\end{aligned}
$$

0.8908 A. *Ans.*

50. Find the number of acres in a triangular field whose sides are 79 chains 8 links, 57 chains 3 links, and 102 chains 19 links.

$$\frac{79.08 + 57.03 + 102.19}{2} = 119.15.$$

$$\text{Area} = \frac{\sqrt{119.15 \times 40.07 \times 62.12 \times 16.96}}{10}$$

$$
\begin{aligned}
\log 119.15 &= 2.0761\\
\log \ \ 40.07 &= 1.6028\\
\log \ \ 62.12 &= 1.7932\\
\log \ \ 16.96 &= 1.2294\\
&\ 2)\underline{6.7015}\\
&\ \ \ \ 3.3508\\
\text{colog } 10 &= \underline{9.0000 - 10}\\
&\ \ \ \ 2.3508 = \log 224.3.
\end{aligned}
$$

224.3 A. *Ans.*

51. Find the number of square rods in a triangle whose sides are 7 rds. 2 yds., 6 rds. 5 yds., and 9 rds. $4\frac{1}{2}$ ft.

7 rds. 2 yds. = 121.5 ft.
6 rds. 5 yds. = 114 ft.
9 rds. $4\frac{1}{2}$ ft. = 153 ft.
1 rd. = 272.25 sq. ft.

$$\frac{121.5 + 114 + 153}{2} = 194.25.$$

$$\text{Area} = \frac{\sqrt{194.25 \times 72.75 \times 80.25 \times 41.25}}{272.25}$$

$$\begin{aligned}
\log 194.25 &= 2.2884 \\
\log\ \ 72.75 &= 1.8618 \\
\log\ \ 80.25 &= 1.9043 \\
\log\ \ 41.25 &= \underline{1.6155} \\
2)&\underline{7.6700} \\
&3.8350 \\
\text{colog}\, 272.25 &= \underline{7.5650 - 10} \\
&1.4000 = \log 25.12. \qquad 25.12 \text{ sq. rds. } \textit{Ans.}
\end{aligned}$$

52. Find the number of acres in a four-sided field, the sides of which are in order 361, 561, 443, and 357 ft.; and the distance from the beginning of the first side to the end of the second side is 682 ft.

$$\frac{361 + 561 + 682}{2} = 802.$$

$$\text{Area} = \frac{\sqrt{802 \times 441 \times 241 \times 120}}{43560} \text{ A.}$$

$$\begin{aligned}
\log 802 &= \ 2.9042 \\
\log 441 &= \ 2.6444 \\
\log 241 &= \ 2.3820 \\
\log 120 &= \ \underline{2.0792} \\
2)&\underline{10.0098} \\
&5.0049 \\
\text{colog}\, 43560 &= \ \underline{5.3609} - 10 \\
&0.3658 = \log 2.322.
\end{aligned}$$

$$\frac{357 + 443 + 682}{2} = 741.$$

$$\text{Area} = \frac{\sqrt{741 \times 384 \times 298 \times 59}}{43560}$$

$$\begin{aligned}
\log 741 &= 2.8698 \\
\log 384 &= 2.5843 \\
\log 298 &= 2.4742 \\
\log\ \ 59 &= \underline{1.7709} \\
2)&\underline{9.6992} \\
&4.8496 \\
\text{colog}\, 43560 &= \underline{5.3609} - 10 \\
&0.2105 = 1.624
\end{aligned}$$

$1.624 + 2.322 = 3.946$ A. *Ans.*

53. Find the number of hektars in a field of three sides, one of which is 82.1^{m}, and the distance from this side to the opposite corner is 47.3^{m}.

$$\tfrac{1}{2}(82.1 \times 47.3) = 1941.67^{qm} = 0.194167^{ha}. \textit{ Ans.}$$

54. Find the number of acres in a triangular lot, one side of which is 343.6 ft., and the distance from this side to the opposite corner is 163.2 ft.

$$\tfrac{1}{2}(343.6 \times 163.2) = 28037.76 \text{ sq. ft.}$$
$$28037.76 \div 43560 = 0.6436 \text{ A. } \textit{Ans.}$$

55. Find the altitude of a triangle, if each side is 1000 ft.

$$\frac{1000 + 1000 + 1000}{2} = 1500.$$
$$\text{Area} = \sqrt{1500 \times 500 \times 500 \times 500}$$
$$= 250000\sqrt{3} = 433012.5 \text{ sq. ft.}$$
$$433012.5 \div 500 = 866.025 \text{ ft. } \textit{Ans.}$$

56. Find the distances of the vertices from the opposite sides of a triangle, when these sides are 17.8^{mm}, 23.6^{mm}, and 31.5^{mm}.

$$\frac{17.8 + 23.6 + 31.5}{2} = 36.45.$$
$$\text{Area} = \sqrt{36.45 \times 18.65 \times 12.85 \times 4.95}.$$

$$\begin{aligned}
\log 36.45 &= 1.5617\\
\log 18.65 &= 1.2707\\
\log 12.85 &= 1.1089\\
\log 4.95 &= \underline{0.6946}\\
&\;\; 2)\underline{4.6359}\\
\log \text{area} &= 2.3179\\
\text{colog } 8.9 &= \underline{9.0506 - 10}\\
&\;\; 1.3685\\
&= \log 23.36.
\end{aligned}$$

23.36^{mm}. (1) *Ans.*

$$\begin{aligned}
\log \text{area} &= 2.3179\\
\text{colog } 11.8 &= \underline{8.9281 - 10}\\
&\;\; 1.2460\\
&= \log 17.62.
\end{aligned}$$

17.62^{mm}. (2) *Ans.*

$$\begin{aligned}
\log \text{area} &= 2.3179\\
\text{colog } 15.75 &= \underline{8.8027 - 10}\\
&\;\; 1.1206\\
&= \log 13.2.
\end{aligned}$$

13.2^{mm}. (3) *Ans.*

57. If the four sides of a field measured in succession are 237, 253, 244, and 261 ft., and the diagonal measured from the end of the first side to the end of the third side is 351 ft.; find its area.

$$\frac{237 + 261 + 351}{2} = 424.5.$$

Area of triangle $= \sqrt{424.5 \times 187.5 \times 163.5 \times 73.5}.$

$$\begin{aligned} \log 424.5 &= 2.6279 \\ \log 187.5 &= 2.2730 \\ \log 163.5 &= 2.2135 \\ \log 73.5 &= 1.8663 \\ 2)&\underline{8.9807} \\ &4.4904 \\ &= \log 30{,}925. \end{aligned}$$

$$\frac{253 + 244 + 351}{2} = 424.$$

Area of triangle

$$= \sqrt{424 \times 171 \times 180 \times 73}.$$

$$\begin{aligned} \log 424 &= 2.6274 \\ \log 171 &= 2.2330 \\ \log 180 &= 2.2553 \\ \log 73 &= 1.8633 \\ 2)&\underline{8.9790} \\ &4.4895 \\ &= \log 30{,}860. \end{aligned}$$

$30{,}860 + 30{,}925 = 61{,}785$ sq. ft. *Ans.*

58. If the four sides of a field are 237, 253, 244, and 261 ft., taken in order, and if the corner formed by the second and third sides is a square corner; find the diagonal from the beginning of the second side to the end of the third side, and also find the area of the field.

Diagonal $= \sqrt{253^2 + 244^2} = \sqrt{123545} = 351.489.$

The area is the same as in the last problem.

59. Find the area of a circle that has a radius of 10 in.; of a circle that has a diameter of 10 ft.; of a circle that has a circumference of 30 in.

Area $= \pi R^2 = 3.1416 \times 100 = 314.16$ sq. in. (1) *Ans.*

Area $= \pi R^2 = 3.1416 \times 25 = 78.54$ sq. ft. (2) *Ans.*

$$\text{Circumference} = 2\pi R.$$

$$30 = 2\pi R.$$

$$R = \frac{15}{\pi}.$$

Area $= \pi R^2 = \frac{\pi}{1} \times \frac{225}{\pi^2} = \frac{225}{3.1416} = 71.62$ sq. in. (3) *Ans.*

60. A horse is tied by a rope 28.8^{m} long; what part of a hektar can he graze?

$$\text{Area} = \pi R^2 = 3.1416 \times 27.8^2 = 3.1416 \times 772.84$$
$$= 2427.95^{qm} = 0.2428^{ha}. \quad \textit{Ans.}$$

61. How many square feet in a circle that has a diameter of $17\frac{2}{3}$ yds.?

$$17\tfrac{2}{3} \text{ yds.} = 53 \text{ ft.}$$
$$\text{Area} = \pi R^2 = 3.1416 \times 26.5^2.$$
$$\log\ 26.5^2 = 3.8464$$
$$\log 3.1416 = \underline{0.4971}$$
$$3.3435 = \log 2205.5$$
$$2205.5 \text{ sq. ft.} \quad \textit{Ans.}$$

62. How many square feet in a circle that has a circumference of 117 yds.?

$$117 \text{ yds.} = 351 \text{ ft.}$$
$$\text{Circumference} = 2\pi R.$$
$$351 = 2\pi R.$$
$$R = \frac{351}{2\pi}.$$
$$\pi R^2 = \frac{351^2}{4 \times 3.1416}.$$

$$\log 351^2 = 5.0906$$
$$\text{colog } 3.1416 = 9.5029 - 10$$
$$\text{colog } 4 = \underline{9.3979 - 10}$$
$$3.9914$$
$$= \log 9804.$$
$$9804 \text{ sq. ft.} \quad \textit{Ans.}$$

63. How many square inches in the surface of a globe that has a radius of 12.37 in.?

$$\text{Area} = 4\pi R^2$$
$$= 4\pi \times 12.37^2.$$
$$\log\ 4 = 0.6021$$
$$\log 3.1416 = 0.4971$$
$$\log 12.37^2 = \underline{2.1848}$$
$$3.2840 = \log 1923.$$
$$1923 \text{ sq. in.} \quad \textit{Ans.}$$

64. Find the area of the surface of the largest globe that can be turned out from a joist 4 in. by 6 in.

$$\text{Area} = 4\pi 2^2 = 16\pi$$
$$= 16 \times 3.1416$$
$$= 50.266 \text{ sq. in.} \quad \textit{Ans.}$$

65. How many cubic inches in a globe that has a diameter of 10 in.?

$$\text{Volume} = 0.5236 \times \text{diam.}^3$$
$$= 0.5236 \times 1000$$
$$= 523.6 \text{ sq. in.} \quad \textit{Ans.}$$

66. If a tree be round, and the girt is 17 ft. 6 in., find its diameter. Find the area of a cross-section, and find the number of cubic feet in the largest sphere that can be cut from it.

$$\text{Diameter} = \frac{17.5}{3.1416}$$
$$= 5.57 \text{ ft. (1) } \textit{Ans.}$$

$$\text{Area} = \pi R^2.$$

$$\begin{aligned} \log 3.1416 &= 0.4971 \\ \log 5.57^2 &= 1.4918 \\ \text{colog } 2^2 &= \underline{9.3979 - 10} \\ \log \text{Area} &= 1.3868 = \log 24.37. \end{aligned}$$

24.37 sq. ft. (2) *Ans.*

$$\text{Volume} = 0.5236 \times \text{diam.}^3$$

$$\begin{aligned} \log 0.5236 &= 9.7190 - 10 \\ \log 5.57^3 &= \underline{2.2377} \\ &\quad 1.9567 = \log 90.52. \end{aligned}$$

90.52 cu. ft. (3) *Ans.*

67. Find the weight in kilograms and in pounds of an iron ball 21.5^{cm} in diameter, specific gravity 7.47; of a tin ball 13^{cm} in diameter, specific gravity 7.29; of a lead ball 17.3^{cm} in diameter, specific gravity 11.35; of a silver ball 1.31^{cm} in diameter, specific gravity 10.47.

Iron.

$$V = 0.5236 \times 21.5^3$$

$$\begin{aligned} \log 0.5236 &= 9.7190 - 10 \\ \log 21.5^3 &= 3.9972 \\ \log 7.47 &= 0.8733 \\ \text{colog } 1000 &= \underline{7.0000 - 10} \\ &\quad 1.5895 = \log 38.86. \end{aligned}$$

38.86^{kg}. (1) *Ans.*

$$\begin{aligned} \log 2.205 &= 0.3434 \\ \log 38.86 &= \underline{1.5895} \\ &\quad 1.9329 = \log 85.68. \end{aligned}$$

85.68 lbs. (2) *Ans.*

Lead.

$$\begin{aligned} \log 17.3^3 &= 3.7140 \\ \log 0.5236 &= 9.7190 - 10 \\ \log 11.35 &= 1.0550 \\ \text{colog } 1000 &= \underline{7.0000 - 10} \\ &\quad 1.4880 = \log 30.76. \end{aligned}$$

30.76^{kg}. (1) *Ans.*

$$\begin{aligned} \log 2.205 &= 0.3434 \\ \log 30.76 &= \underline{1.4880} \\ &\quad 1.8314 = \log 67.83. \end{aligned}$$

67.83 lbs. (2) *Ans.*

Tin.

$$V = 0.5236 \times 13^3.$$

$$\begin{aligned} \log 0.5236 &= 9.7190 - 10 \\ \log 13^3 &= 3.3417 \\ \log 7.29 &= 0.8627 \\ \text{colog } 1000 &= \underline{7.0000 - 10} \\ &\quad 0.9234 = \log 8.383. \end{aligned}$$

8.383^{kg}. (1) *Ans.*

$$\begin{aligned} \log 2.205 &= 0.3434 \\ \log 8.383 &= \underline{0.9234} \\ &\quad 1.2668 = \log 18.48. \end{aligned}$$

18.48 lbs. (2) *Ans.*

Silver.

$$\begin{aligned} \log 1.31^3 &= 0.3519 \\ \log 0.5236 &= 9.7190 - 10 \\ \log 10.47 &= 1.0199 \\ \text{colog } 1000 &= \underline{7.0000 - 10} \\ &\quad 8.0908 - 10 \\ &= \log 0.01233. \end{aligned}$$

0.01233^{kg}. (1) *Ans.*

$$\begin{aligned} \log 2.205 &= 0.3434 \\ \log 0.01233 &= \underline{8.0908 - 10} \\ &\quad 8.4342 - 10 \\ &= \log 0.02717. \end{aligned}$$

0.02717 lbs. (2) *Ans.*

68. A slab of cast-iron 4 ft. $2\frac{1}{2}$ in. long, 17 in. wide, and $8\frac{1}{3}$ in. thick, specific gravity 7.31, is cast into 2-lb. balls. If there is a loss of 5% in melting, how many balls are obtained, and what is the diameter of each?

The slab will make $\dfrac{50.5 \times 17 \times 25 \times 0.95 \times 62.5 \times 7.31}{2 \times 3 \times 1728}$ balls.

$$
\begin{aligned}
\log 50.5 &= 1.7033\\
\log 17 &= 1.2304\\
\log 25 &= 1.3979\\
\log 0.95 &= 9.9777-10\\
\log 62.5 &= 1.7959\\
\log 7.31 &= 0.8639\\
\text{colog } 2 &= 9.6990-10\\
\text{colog } 3 &= 9.5229-10\\
\text{colog } 1728 &= \underline{6.7625-10}\\
2.9535 &= \log 898.
\end{aligned}
$$

898 balls. (1) *Ans.*

The diameter will be

$$\sqrt[3]{\frac{50.5 \times 17 \times 25 \times 0.95}{0.5236 \times 3 \times 898}}$$

$$
\begin{aligned}
\log 50.5 &= 1.7033\\
\log 17 &= 1.2304\\
\log 25 &= 1.3979\\
\log 0.95 &= 9.9777-10\\
\text{colog } 0.5236 &= 0.2810\\
\text{colog } 3 &= 9.5229-10\\
\text{colog } 898 &= \underline{7.0467-10}\\
&3\underline{)1.1599}\\
&0.3866\\
&= \log 2.436.
\end{aligned}
$$

2.436 inches. (2) *Ans.*

69. How many pounds avoirdupois would a ball of such iron 30 in. in diameter weigh?

$$
\begin{aligned}
\log 30^3 &= 4.4313\\
\log 0.5236 &= 9.7190-10\\
\log 62.5 &= 1.7959\\
\log 7.31 &= 0.8639\\
\text{colog } 1728 &= \underline{6.7625-10}\\
3.5726 &= \log 3732.5.
\end{aligned}
$$

3732.5 lbs. *Ans.*

70. If the specific gravity of ice is 0.921, find the weight and the surface of each of three spheres of ice whose diameters are 1^{cm}, 10^{cm}, and 1^{m}. Which of these spheres would roll first on a plain, in a gradually-increasing wind?

$$V = 1^3 \times 0.5236 = 0.5236^{\text{ccm}}.$$
$$0.5236^{\text{ccm}} = 523.6^{\text{mg}}.$$
$$0.921 \times 523.6^{\text{mg}} = 482.24^{\text{mg}}. \text{ (1) } \textit{Ans.}$$
$$V = 10^3 \times 0.5236 = 523.6^{\text{ccm}}.$$
$$523.6^{\text{ccm}} = 523.6^{\text{g}}.$$
$$0.921 \times 523.6^{\text{g}} = 482.24^{\text{g}}. \text{ (2) } \textit{Ans.}$$
$$V = 1^3 \times 0.5236 = 0.5236^{\text{cbm}}.$$
$$0.5236^{\text{cbm}} = 523.6^{\text{kg}}.$$
$$0.921 \times 523.6^{\text{kg}} = 482.24^{\text{kg}}. \text{ (3) } \textit{Ans.}$$

$\text{Area} = 4\pi R^2 = \pi D^2 = 3.1416 \times 1^2 = 3.1416^{\text{qcm}}$. (1) *Ans.*
$\text{Area} = 4\pi R^2 = \pi D^2 = 3.1416 \times 10^2 = 314.16^{\text{qcm}}$. (2) *Ans.*
$\text{Area} = 4\pi R^2 = \pi D^2 = 3.1416 \times 100^2 = 31{,}416^{\text{qcm}}$. (3) *Ans.*
The first, if we take no account of friction. (4) *Ans.*

71. Given a cylinder 10 in. in diameter and 12 in. long; required the area of each end, the convex area, the total area, and the contents in gallons.

Area of end $= 3.1416 \times 5^2 = 3.1416 \times 25 = 78.54$ sq. in. (1) *Ans.*
Convex area $= \pi \times 10 \times 12 = 3.1416 \times 120 = 376.99$ sq. in. (2) *Ans.*
Total surface $= 376.99 + 2 \times 78.54 = 534.07$ sq. in. (3) *Ans.*

$$V = \frac{\overset{1.02}{\cancel{78.54}} \times \overset{4}{\cancel{12}}}{\underset{77}{\cancel{231}}} \text{ gals.} = 4.08 \text{ gals. (4) } \textit{Ans.}$$

72. Find the capacity in gallons of a round cistern 13 ft. in diameter and 9 ft. deep.

$$V = \frac{3.1416 \times 6.5^2 \times 9 \times 1728}{231}.$$

$$\begin{aligned}
\log 3.416 &= 0.4971 \\
\log 6.5^2 &= 1.6258 \\
\log 9 &= 0.9542 \\
\log 1728 &= 3.2375 \\
\text{colog } 231 &= \underline{7.6364 - 10} \\
&\quad 3.9510 = \log 8935.
\end{aligned}$$

8935 gals. *Ans.*

73. What must be the diameter of a cylinder 10 in. deep, in order that it may hold 1 gallon?

$$V = \pi R^2 \times h. \qquad 231 = \pi R^2 \times 10. \qquad R = \sqrt{\frac{231}{10\pi}}.$$

$$\begin{aligned} \log 231 &= 2.3636 \\ \operatorname{colog} 10 &= 9.0000 - 10 \\ \operatorname{colog} 3.1416 &= \underline{9.5029 - 10} \\ &2)\underline{0.8665} \\ &0.4333 = \log 2.712. \end{aligned}$$

$$2.712 \text{ in.} \times 2 = 5.424 \text{ in. } \textit{Ans.}$$

74. Find the volume of a cylinder 8 in. in diameter and 11 in. high.

$$V = \pi \times 4^2 \times 11 = 3.1416 \times 16 \times 11 = 552.92 \text{ cu. in. } \textit{Ans.}$$

75. Find the dimensions of three cylinders that have the diameters equal to the heights, and hold 1 gal., 1 qt., and 1^l respectively.

$$V = \pi R^2 h = \pi \left(\frac{D}{2}\right)^2 D = \frac{\pi D^3}{4}.$$

$$231 = \frac{\pi D^3}{4}.$$

$$D = \sqrt[3]{\frac{4 \times 231}{3.1416}}.$$

$$\begin{aligned} \log 4 &= 0.6021 \\ \log 231 &= 2.3636 \\ \operatorname{colog} 3.1416 &= \underline{9.5029 - 10} \\ &3)\underline{2.4686} \\ &0.8229 = \log 6.651 \end{aligned}$$

6.65 in. (1) *Ans.*

$$V = \frac{\pi D^3}{4}.$$

$$1 \text{ qt.} = \frac{231}{4} \text{ cu. in.}$$

$$\frac{231}{4} = \pi D^3.$$

$$D = \sqrt[3]{\frac{231}{3.1416}}.$$

$$\begin{aligned} \log 231 &= 2.3636 \\ \operatorname{colog} 3.1416 &= \underline{9.5029 - 10} \\ &3)\underline{1.8665} \\ &0.6222 = \log 4.19. \end{aligned}$$

4.19 in. (2) *Ans.*

$$V = \frac{\pi D^3}{4}.$$

$$1^l = 1000^{\text{ccm}}.$$

$$1000 = \frac{\pi D^3}{4}.$$

$$D = \sqrt[3]{\frac{4000}{3.1416}}.$$

$$\begin{aligned} \log 4000 &= 3.6021 \\ \operatorname{colog} 3.1416 &= \underline{9.5029} \\ &3)\underline{3.1050} \\ &1.0350 = \log 10.84. \end{aligned}$$

10.84^{cm}. (3) *Ans.*

76. Find the volume of a triangular prism 11 in. long, the sides of the ends being 2, 3, and 4 in. long.

$$\text{Area of end} = \sqrt{4.5 \times 2.5 \times 1.5 \times 0.5} = 2.9047.$$
$$V = 11 \times 2.9047 = 31.952 \text{ cu. in. } \textit{Ans.}$$

77. Find the capacity in bushels of a bin 6 ft. long, the end of which is a square measuring 3 ft. 3 in. on a side.

$$V = 6 \times 3\tfrac{1}{4} \times 3\tfrac{1}{4} = 6 \times \tfrac{13}{4} \times \tfrac{13}{4} = \tfrac{507}{8} = 63\tfrac{3}{8} \text{ cu. ft.}$$
$$0.80356 \text{ bu.} = 1 \text{ cu. ft.}$$
$$0.80356 \times 63\tfrac{3}{8} = 50.93 \text{ bu. } \textit{Ans.}$$

78. Find the number of cubic yards in a square prism 200 ft. on a side, and 40 ft. long.

$$V = \frac{200^2 \times 40}{27} = 59259\tfrac{7}{27} \text{ cu. yds. } \textit{Ans.}$$

79. How many cubic yards in a square pyramid 210 ft. on a side, and 123 ft. high?

$$210 \text{ ft.} = 70 \text{ yds.}$$
$$123 \text{ ft.} = 41 \text{ yds.}$$
$$V = \tfrac{1}{3}(70^2 \times 41) = 66966\tfrac{2}{3} \text{ cu. yds. } \textit{Ans.}$$

80. Find the capacity of a cup, the mouth of which is a square 4 in. on a side, and the sides of which are four equilateral triangles.

$$\tfrac{1}{2} \text{ the diagonal of base} = \tfrac{1}{2}\sqrt{16 + 16} = \tfrac{1}{2}\sqrt{32}.$$
$$\text{Altitude} = \sqrt{4^2 - (\tfrac{1}{2}\sqrt{32})^2} = \sqrt{16 - 8} = \sqrt{8} = 2.8285.$$
$$V = \tfrac{1}{3} \text{ of } 16 \times 2.8285 = 15.085 \text{ cu. in. } \textit{Ans.}$$

81. The largest of the Egyptian pyramids is 147^{m} high, with a base 231^{m} square. Find its volume in cubic meters.

$$\tfrac{1}{3} \text{ of } 231^2 \times 147 = 2{,}614{,}689^{\text{cbm}}. \textit{ Ans.}$$

82. The slant depth of a conically-shaped drinking-cup is 93^{mm}, and the diameter at the top 8^{cm}. What is its capacity?

$93^{mm} = 9.3^{cm}$.

Height $= \sqrt{9.3^2 - 4^2} = 8.3958^{cm}$.

$V = \frac{1}{3}\pi \times 4^2 \times 8.3958^{cm}$

$= \frac{1}{3}$ of $3.1416 \times 16 \times 8.3958 = 140.67^{ccm} = 0.14067^{l}$. *Ans.*

83. The volume of a cone is 1^{cbm}; its height is equal to the radius of its base. Find the dimensions of the cone.

$$V = \frac{\pi R^2 h}{3}. \qquad V = \frac{\pi R^3}{3}. \qquad R^3 = \frac{3V}{\pi}. \qquad R = \sqrt[3]{\frac{3 \times 1000000}{\pi}}.$$

$\log 3,000,000 = 6.4771$

$\text{colog}\, 3.1416 = 9.5029 - 10$

$3)\underline{5.9800}$

$1.9933 = \log 98.47$.

98.47^{cm}. *Ans.*

84. Find the capacity of a wash-bowl 30^{cm} in diameter and 5^{cm} deep.

$\frac{1}{4}$ of $30^2 = \frac{1}{4}$ of $900 = 225$.

$\frac{1}{3}$ of $5^2 = \frac{1}{3}$ of $25 = 8.33$.

$225 + 8.33 = 233.33$.

$\frac{11}{7}$ of $5 \times 233.33 = 1833.31^{ccm} = 1.833^{l}$. *Ans.*

85. Find the capacity in liters of a boiler 89^{cm} in diameter and 31^{cm} deep.

$\frac{1}{4}$ of $89^2 = \frac{1}{4}$ of $7921 = 1980.25$.

$\frac{1}{3}$ of $31^2 = \frac{1}{3}$ of $961 = 320.33$.

$1980.25 + 320.33 = 2300.58$.

$\frac{11}{7}$ of $31 \times 2300.58 = 112,071.11^{ccm} = 112.07^{l}$. *Ans.*

86. Find the capacity in quarts of a bowl 10 in. in diameter and 4 in. deep.

$\frac{1}{4}$ of $10^2 = \frac{1}{4}$ of $100 = 25$.

$\frac{1}{3}$ of $4^2 = \frac{1}{3}$ of $16 = 5.333$.

$25 + 5.333 = 30.333$.

$\frac{11}{7}$ of $4 \times 30.333 = 190.66$ cu. in.

1 qt. $= 57.75$ cu. in.

$190.66 \div 57.75 = 3.3$ qts. *Ans.*

87. Find the capacity in pints of a saucer 6 in. across and $1\frac{1}{2}$ in. deep; of a bowl 7 in. across and 3 in. deep; of a bowl 8 in. across and $3\frac{1}{2}$ in. deep.

$$1 \text{ pt.} = \tfrac{1}{8} \text{ of } 231 \text{ cu. in.} = 28.875 \text{ cu. in.}$$
$$\tfrac{1}{4} \text{ of } 6^2 = \tfrac{1}{4} \text{ of } 36 = 9.$$
$$\tfrac{1}{3} \text{ of } (1\tfrac{1}{2})^2 = \tfrac{1}{3} \text{ of } \tfrac{9}{4} = 0.75.$$
$$9 + 0.75 = 9.75.$$
$$\tfrac{11}{7} \text{ of } 1.5 \times 9.75 = 22.982 \text{ cu. in.}$$
$$1 \text{ pt.} = 28.875 \text{ cu. in.}$$
$$22.982 \div 28.875 = 0.8 \text{ pt. (1) } \textit{Ans.}$$

$$\tfrac{1}{4} \text{ of } 7^2 = \tfrac{1}{4} \text{ of } 49 = 12.25.$$
$$\tfrac{1}{3} \text{ of } 3^2 = \tfrac{1}{3} \text{ of } \ 9 = 3.$$
$$12.25 + 3 = 15.25.$$
$$\tfrac{11}{7} \text{ of } 3 \times 15.25 = 71.893 \text{ cu. in.}$$
$$71.893 \div 28.875 = 2.5 \text{ pts. (2) } \textit{Ans.}$$

$$\tfrac{1}{4} \text{ of } 8^2 = \tfrac{1}{4} \text{ of } 64 = 16.$$
$$\tfrac{1}{3} \text{ of } (\tfrac{7}{2})^2 = \tfrac{1}{3} \text{ of } \tfrac{49}{4} = 4\tfrac{1}{12}.$$
$$16 + 4\tfrac{1}{12} = 20\tfrac{1}{12}.$$
$$\frac{\cancel{11}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{241}{\underset{3}{\cancel{12}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{21}{\cancel{231}}} = \frac{241}{63} = 3.8 \text{ pts. (3) } \textit{Ans.}$$

88. How many gallons will a boiler 5 ft. in diameter and 2 ft. deep hold?

$$\tfrac{1}{4} \text{ of } 5^2 = \tfrac{1}{4} \text{ of } 25 = 6.25.$$
$$\tfrac{1}{3} \text{ of } 2^2 = \tfrac{1}{3} \text{ of } \ 4 = 1.333.$$
$$6.25 + 1.333 = 7.583.$$
$$\tfrac{11}{7} \text{ of } 2 \times 7.583 = 23.832 \text{ cu. ft.}$$
$$23.832 \times 1728 \div 231 = 178.3 \text{ gals. } \textit{Ans.}$$

89. How many gallons will a boiler 30 in. in diameter and 1 ft. deep hold?

$$\tfrac{1}{4} \text{ of } 30^2 = \tfrac{1}{4} \text{ of } 900 = 225.$$
$$\tfrac{1}{3} \text{ of } 12^2 = \tfrac{1}{3} \text{ of } 144 = \ 48.$$
$$225 + 48 = 273.$$
$$\tfrac{11}{7} \text{ of } 12 \times 273 = 5148 \text{ cu. in.}$$
$$5148 \div 231 = 22.3 \text{ gals. } \textit{Ans.}$$

90. Find the capacity in pints of a cylinder 1.9375 in. in diameter, 2.4375 in. high; of a cylinder $3\frac{1}{8}$ in. in diameter, $3\frac{5}{8}$ in. high; of a cylinder $3\frac{13}{16}$ in. in diameter, $5\frac{1}{16}$ in. high.

$$1 \text{ pt.} = 28.875 \text{ cu. in.}$$

$$V = \frac{3.1416 \times 0.96875^2 \times 2.4375}{28.875}.$$

$$\begin{aligned} \log\ 3.1416 &= 0.4971 \\ \log 0.96875^2 &= 9.9724 - 10 \\ \log\ 2.4375 &= 0.3870 \\ \text{colog}\ 28.875 &= 8.5394 \\ \hline & \quad 9.3959 - 10 = \log 0.2488. \end{aligned}$$

0.249 pt. (1) *Ans.*

$$V = \frac{3.1416 \times 1.5625^2 \times 3.625}{28.875}.$$

$$\begin{aligned} \log\ 3.1416 &= 0.4971 \\ \log 1.5625^2 &= 0.3876 \\ \log\ 3.625 &= 0.5593 \\ \text{colog}\ 28.875 &= 8.5394 - 10 \\ \hline & \quad 9.9834 = \log 0.9625. \end{aligned}$$

0.963 pt. (2) *Ans.*

$$V = \frac{3.1416 \times 1.90625^2 \times 5.06}{28.875}.$$

$$\begin{aligned} \log\ 3.1416 &= 0.4971 \\ \log 1.90625^2 &= 0.5605 \\ \log\ 5.0625 &= 0.7044 \\ \text{colog}\ 28.875 &= 8.5394 - 10 \\ \hline & \quad 0.3014 = \log 2.002. \end{aligned}$$

2.002 pt. (3) *Ans.*

91. Find the capacity in pecks of a cylinder 15.865 in. in diameter, 12.5 in. high; of a cylinder 9.25 in. in diameter, 4.25 in. deep; of a cylinder 18.5 in. in diameter, 8 in. deep.

$$V = \frac{0.7854 \times 15.865^2 \times 12.5 \times 4}{2150.42}.$$

$$\begin{aligned}
\log\ 0.7854 &= 9.8950 - 10\\
\log\ 15.865^2 &= 2.4009\\
\log\ 12.5 &= 1.0969\\
\log\ 4 &= 0.6021\\
\text{colog}\ 2150.42 &= 6.6675 - 10\\
\hline
0.6624 &= \log 4.596.
\end{aligned}$$

4.596 pks. (1) *Ans.*

$$V = \frac{0.7854 \times 9.25^2 \times 4.25 \times 4}{2150.42}.$$

$$\begin{aligned}
\log\ 0.7854 &= 9.8950 - 10\\
\log\ 9.25^2 &= 1.9322\\
\log\ 4.25 &= 0.6284\\
\log\ 4 &= 0.6021\\
\text{colog}\ 2150.42 &= 6.6675 - 10\\
\hline
9.7252 - 10 &= \log 0.531.
\end{aligned}$$

0.531 pks. (2) *Ans.*

$$V = \frac{0.7854 \times 18.5^2 \times 8 \times 4}{2150.42}.$$

$$\begin{aligned}
\log\ 0.7854 &= 9.8950 - 10\\
\log\ 18.5^2 &= 2.5344\\
\log\ 8 &= 0.9031\\
\log\ 4 &= 0.6021\\
\text{colog}\ 2150.42 &= 6.6675 - 10\\
\hline
0.6021 &= \log 4.
\end{aligned}$$

4 pks. (3) *Ans.*

92. What must be the diameter of a circle, in order that it may contain 78.54 sq. ft.? to contain 314.16 sq. ft.?

$$\begin{aligned}
\text{Area} &= 0.7854 \times D^2.\\
78.54 &= 0.7854 \times D^2.\\
D^2 &= 100.\\
D &= 10 \text{ ft. (1) } \textit{Ans.}
\end{aligned}
\qquad
\begin{aligned}
\text{Area} &= 0.7854 \times D^2.\\
314.16 &= 0.7854 \times D^2.\\
D^2 &= 400.\\
D &= 20 \text{ ft. (2) } \textit{Ans.}
\end{aligned}$$

93. What must be the diameter of a circle to contain 1 A.? to contain 9 A.?

$$1 \text{ A.} = 43{,}560 \text{ sq. ft.} \qquad D = \sqrt{\frac{43560}{0.7854}}.$$
$$\text{Area} = 0.7854 \times D^2.$$

$$\begin{aligned} \log 43{,}560 &= 4.6391 \\ \operatorname{colog} 0.7854 &= \underline{0.1049} \\ 2)\underline{4.7440} \\ 2.3720 &= \log 235.5. \end{aligned}$$

235.5 ft. (1) *Ans.*

$\sqrt{9} = 3.$ $\qquad 3 \times 235.5 = 706.5$ ft. (2) *Ans.*

94. What must be the diameter of a circle to contain 1^{ha}? to contain 25^{ha}?

$$1^{ha} = 10{,}000^{qm}. \qquad D = \sqrt{\frac{10000}{0.7854}}.$$

$$\begin{aligned} \log 10{,}000 &= 4.0000 \\ \operatorname{colog} 0.7854 &= \underline{0.1049} \\ 2)\underline{4.1049} \\ 2.0525 &= \log 112.8. \end{aligned}$$

112.8^m. (1) *Ans.*

$\sqrt{25} = 5.$ $\qquad 5 \times 112.8^m = 564^m$. (2) *Ans.*

95. Find the number that exceeds its square root by 20.

On testing the square numbers exceeding 20, namely, 25, 36, etc., we see that 25 is the number; and no process of approximation is needed. 25. *Ans.*

96. How much water will a hemispherical bowl hold that is 10 in. in diameter?

$V = \frac{1}{2}$ of $0.5236 \times D^3 = \frac{1}{2}$ of $0.5236 \times 1000 = 261.8$ cu. in. *Ans.*

97. What will it cost to gild a hemispherical dome 10 ft. in diameter, at 50 cents a square foot?

$S = 2 \times 0.7854 \times 10^2 = 157.08$ sq. ft.
$157.08 \times \$0.50 = \$78.54$. *Ans.*

98. If the moon is a sphere 2170 miles in diameter, about how many million bushels would she hold if hollow? and how many yards of cloth a yard wide would it take to cover her?

$$2170 \text{ mi.} = 137{,}491{,}200 \text{ in.}$$
$$V = 0.5236 \times D^3.$$

$$\begin{array}{lrl} \log & 137{,}491{,}200^3 = & 24.4149 \\ \log & 0.5236 = & 9.7190 - 10 \\ \text{colog} & 2150.42 = & \underline{6.6675 - 10} \\ \log & V = & 20.8014 \end{array}$$

$V = 633{,}000{,}000{,}000{,}000{,}000{,}000$ bu. (1) *Ans.*

$$2170 \text{ mi.} = 3{,}819{,}200 \text{ yds.}$$
$$S = \pi D^2.$$

$$\begin{array}{lrl} \log & 3{,}819{,}200^2 = & 13.1640 \\ \log & 3.1416 = & \underline{0.4971} \\ \log & S = & 13.6611 \end{array}$$

$S = 45{,}800{,}000{,}000{,}000$ yds. (2) *Ans.*

99. If the earth is 7920 miles in diameter, and the air is 40 miles deep, how many cubic miles of air are there about the planet?

$$\begin{array}{l} 7920 + 80 = 8000 \\ \log\ 8000^3 = 11.7093 \\ \log 0.5236 = \underline{9.7190 - 10} \\ \qquad\qquad 11.4283 \\ = \log 268{,}100{,}000{,}000. \end{array}$$

$$\begin{array}{r} \log\ 7920^3 = 11.6961 \\ \log 0.5236 = \underline{9.7190 - 10} \\ 11.4151 \\ = \log 260{,}100{,}000{,}000 \\ 268{,}100{,}000{,}000 \\ \underline{260{,}100{,}000{,}000} \\ 8{,}000{,}000{,}000 \text{ cu. mi.} \end{array}$$

Ans.

100. What is the difference between 2 feet square and 2 square feet? between a foot square and a square foot? between half a foot square and 6 in. square?

"2 feet square" means a square 2 ft. on a side; "2 square feet," any surface equivalent in area to two squares each 1 foot on a side. A "foot square" is a square; while a square foot is an equivalent area in any shape. "Half a foot square" is ambiguous. Half "a foot square" is half a square foot, while "half a foot" square is 6 inches square; that is, one-fourth a square foot.

101. Find the volume of a square frustum of which the base is 3 ft. square, top 2 ft. square, and height 4 ft.

$$V = \tfrac{1}{3} \times 4\left[\sqrt{3^2 \times 2^2} + 3^2 + 2^2\right] = \tfrac{4}{3}(6 + 9 + 4) = 25\tfrac{1}{3} \text{ cu. ft. } \textit{Ans.}$$

102. Find the capacity in liquid quarts of a tin pan 10 in. in diameter at top, 8 in. in diameter at bottom, and 4 in. deep.

$$\begin{aligned}\text{Volume of whole cone} &= \tfrac{1}{3} \times 0.7854\, D^2h = \tfrac{1}{3} \times 0.7854 \times 100 \times 20\\ &= 523.6 \text{ cu. in.}\\ \text{Volume of part cut off} &= \tfrac{1}{3} \text{ of } 0.7854\, D^2h = \tfrac{1}{3} \times 0.7854 \times 64 \times 16\\ &= 268.08 \text{ cu. in.}\end{aligned}$$

$$523.6 - 268.08 = 255.52 \text{ cu. in.}$$

$$255.52 \div \tfrac{231}{4} = 4.42 \text{ qts. } \textit{Ans.}$$

103. How many hektoliters will a circular vat hold 5^m in diameter at the top, 4.57^m at the bottom, and 1.17^m deep?

$$\text{Area of top} = 5^2 \times 0.7854 = 19.635^{qm}.$$

$$\text{Area of base} = 4.57^2 \times 0.7854 = 16.403^{qm}.$$

$$\sqrt{19.635 \times 16.403} = 17.946.$$

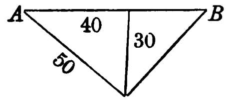
$$\tfrac{1}{3}(19.635 + 16.403 + 17.946) \times 1.17 = 21.054^{cbm} = 210.54^{hl}. \textit{ Ans.}$$

104. Find the area of an ellipse 8 in. by 11 in.; of an ellipse 15 in. by 21 in.

$$0.7854 \times 11 \times 8 = 69.115 \text{ sq. in. (1) } \textit{Ans.}$$

$$0.7854 \times 15 \times 21 = 247.401 \text{ sq. in. (2) } \textit{Ans.}$$

105. The ends of a cord 100 ft. long are fastened to stakes placed 80 ft. apart on level ground. A ring, to which a kid is tied, plays freely on the cord. How far from the straight line joining the stakes can the ring be pulled? What are the diameters of the ellipse which the kid can graze? How many square feet in the ellipse?



The cord can be pulled from AB

$$\sqrt{50^2 - 40^2} = \sqrt{900} = 30 \text{ ft. (1) } \textit{Ans.}$$

The diameters are 100 ft. and 60 ft. (2) *Ans.*

$$0.7854 \times 60 \times 100 = 4712.4 \text{ sq. ft. (3) } \textit{Ans.}$$

106. Using the same rope as in the last problem, but putting the stakes 25 ft. apart, how many per cent is the kid's pasturage increased?

The rope can now be pulled from AB a distance of

$$\sqrt{50^2 - 12.5^2} = \sqrt{2343.75} = 48.4 \text{ ft.}$$

Hence the diameters are 96.8 and 100.

Area $= 96.8 \times 100 \times 0.7854 = 7602.7$.

Hence, $\dfrac{7602.7 - 4712.4}{4712.4}$ of $100 = 61.3$. 61.3%. *Ans.*

107. A cylindrical log, 11 in. in diameter, is sawed off on such a slant that the pieces are 8 in. longer on the longest than on the shortest side. Find the dimensions of the ellipse thus made, and its area.

The shorter diameter is evidently the diameter of the log, or 11 in. The longer diameter is

$$\sqrt{11^2 + 8^2} = \sqrt{121 + 64} = \sqrt{185} = 13.6 \text{ ft.}$$

$$\text{Area} = 13.6 \times 11 \times 0.7854.$$

$$\begin{aligned} \log\ 13.6 &= 1.1335 \\ \log\ 11 &= 1.0414 \\ \log 0.7854 &= \underline{9.8951 - 10} \\ &\ 2.0700 = \log 117.5 \text{ sq. in. } \textit{Ans.} \end{aligned}$$

108. Find the length of a pendulum beating half-seconds; of a pendulum beating quarter-seconds.

The length of pendulums is inversely as the square of the number of vibrations in a given time.

Hence, a half-seconds pendulum will be $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ the length of a seconds pendulum, or $\frac{1}{4}$ of 39.138 in. = 9.784 in. (1) *Ans.*

A quarter-seconds pendulum will be $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ the length of the seconds pendulum, or $\frac{1}{16}$ of 39.138 in. = 2.446 in. (2) *Ans.*

109. How many centimeters long is a pendulum swinging 80 times a minute? A pendulum swinging 30 times a minute?

The first pendulum must be $(\frac{60}{80})^2$ of 39.138 in. = 22.014 in.

$2.54 \times 22.014 = 55.92^{\text{cm}}$. (1) *Ans*

The second pendulum must be $(\frac{60}{30})^2$ of 39.138 in. = 156.552 in.

$2.54 \times 156.552 = 397.64^{\text{cm}}$. (2) *Ans.*

110. If a cannon-ball be suspended by a fine wire 176 ft. long in the central well of the Bunker Hill Monument, how many times a minute will it swing?

$$\sqrt{2112} : \sqrt{39.138} :: 60 : x.$$

$$x = \frac{60 \times \sqrt{39.138}}{\sqrt{2112}}.$$

$$\begin{aligned} \log\ 60 &= 1.7782 \\ \log \sqrt{39.138} &= 0.7963 \\ \operatorname{colog} \sqrt{2112} &= 8.3377 - 10 \\ \hline & 0.9122 = \log 8.17. \qquad 8.17. \textit{ Ans.} \end{aligned}$$

111. Find the lifting-power of a hydraulic press, the plunger being 1^{cm} in diameter and driven with a force of 100^{kg}, if the lifting-piston is 1^{m} in diameter.

Suppose the plunger and piston square: the one would press on a surface of 1^{qcm}, the other on a surface of $10{,}000^{\text{qcm}}$. By driving the plunger in 10^{cm}, you force 10^{ccm} of water under the piston. But as this is spread under $10{,}000^{\text{qcm}}$, you raise the piston thereby only 0.001^{cm}; that is, only 0.0001 part of the way you move the plunger. Hence, by the principle of virtual velocity (what is lost in time is gained in power), you lift the piston with 10,000 times your force of 100^{kg} applied. The lifting power, in other words, is $1{,}000{,}000^{\text{kg}}$, or 1000^{t}.

112. If the plunger is $\frac{1}{2}$ in. in diameter, and is driven with a force of 1000 lbs., how much can it lift with a lifting-piston 4 ft. in diameter?

$$\tfrac{1}{2}\text{ in.} : 4\text{ ft.} = 1 : 96,$$
$$1^2 : 96^2 = 1 : 9216.$$
$$9216 \times 1000\text{ lbs.} = 9{,}216{,}000\text{ lbs. } \textit{Ans.}$$

113. If the plunger is 2 in. in diameter, and is driven with a force of 1000 lbs., how much can it lift with a lifting-piston 2 ft. in diameter?

$$2\text{ in.} : 2\text{ ft.} :: 1 : 12,$$
$$1^2 : 12^2 = 1 : 144.$$
$$144 \times 1000\text{ lbs.} = 144{,}000\text{ lbs. } \textit{Ans.}$$

114. The water stands in a fissure in a rock 10^m high and 12^m long. What pressure is exerted to split the rock on the lowest meter's width? on the highest meter's width? in the whole fissure?

$$1 \times 12 \times 9.5 = 114^{cbm} = 114\text{ t. (1) } \textit{Ans.}$$
$$1 \times 12 \times 0.5 = 6^{cbm} = 6\text{ t. (2) } \textit{Ans.}$$
$$10 \times 12 \times 5 = 600^{cbm} = 600\text{ t. (3) } \textit{Ans.}$$

115. A dam is 100 ft. long and 10 ft. deep, and the water is just flowing over it. What pressure is exerted over the lowest two feet of the dam?

$$2 \times 9 \times 100 = 1800\text{ cu. ft.}$$
$$1800 \times 62\tfrac{1}{2}\text{ lbs.} = 112{,}500\text{ lbs.}$$
$$112{,}500 \div 2000 = 56\tfrac{1}{4}\text{ t. } \textit{Ans.}$$

116. What velocity in meters a second will a cannon-ball acquire in falling three-quarters of a second? in falling three and a quarter seconds?

$$\tfrac{3}{4}\text{ of }9.806^m = 7.3545^m.\ (1)\ \textit{Ans.}$$
$$3\tfrac{1}{4}\text{ of }9.806^m = 31.869^m.\ (2)\ \textit{Ans.}$$

117. How long will it take a leaden ball, rolling off a table 29 in. high, to reach the floor?

$$29\text{ in.} = 0.7366^m.$$
$$4.903 : 0.7366 = 1^2 : x^2.$$
$$x = \sqrt{\frac{0.7366}{4.903}}.$$

$$\begin{aligned} \log 0.7366 &= 9.8673 - 10 \\ \text{colog } 4.903 &= 9.3095 - 10 \\ \hline &\ 9.1768 - 10 \\ &\ 10.0000 - 10 \\ \hline &2)\underline{19.1768 - 20} \\ &\ 9.5884 - 10 = \log 0.3876. \end{aligned}$$

0.3876 sec. *Ans.*

118. What velocity will a crowbar attain in falling endwise from a balloon 2000^m high? How long will it be in coming down?

$$4.903^m : 2000^m = 1^2 : x^2.$$

$$x = \sqrt{\frac{2000}{4.903}}.$$

$$\begin{aligned} \log\ 2000 &= 3.3010 \\ \text{colog } 4.903 &= 9.3095 - 10 \\ \hline &2)\underline{2.6105} \\ &\ 1.3053 = \log 20.2. \end{aligned}$$

20.2 sec. (2) *Ans.*

$$20.2 \times 9.806^m = 198.0812^m.$$

198^m per sec. (1) *Ans.*

119. What velocity will a crowbar attain in falling endwise from a balloon one mile and a quarter high? How long will it be coming down?

$$4.903^m = 16.086 \text{ ft.}$$

$$1\tfrac{1}{4} \text{ mi.} = 6600 \text{ ft.}$$

$$16.086 : 6600 = 1^2 : x^2.$$

$$x = \sqrt{\frac{6600}{16.086}}.$$

$$\begin{aligned} \log\ \ 6600 &= 3.8195 \\ \text{colog } 16.086 &= 8.7936 - 10 \\ \hline &2)\underline{2.6131} \\ &\ 1.3066 = \log 20.26. \end{aligned}$$

20.26 sec. (2) *Ans.*

$$2 \times 20.26 \times 16.086 \text{ ft.} = 651.80 \text{ ft.}$$

651.8 ft. per sec. (1) *Ans.*

120. If Carisbrook Well is 210 ft. deep, how long after a pebble is dropped will it be before it is heard to strike the bottom, if its velocity is reckoned at 32 ft. at the end of a second, and the velocity of sound is 1120 ft. a second?

$$16 : 210 = 1^2 : x^2$$
$$x = \sqrt{\tfrac{210}{16}}$$
$$\log 210 = 2.3222$$
$$\text{colog } 16 = \underline{8.7959 - 10}$$
$$2)\underline{1.1181}$$
$$0.5591 = \log 3.623.$$

3.623 sec. in falling.

$\frac{210}{1120} = 0.188$ sec. sound in rising.

$3.623 + 0.188 = 3.811$ sec. *Ans.*

121. On the same suppositions as in Ex. 120, how long after a pebble is dropped will it be heard to strike the bottom of a ventilating shaft 1600 ft. deep?

$$16 : 1600 = 1^2 : x^2$$
$$x = \sqrt{\tfrac{1600}{16}} = \sqrt{100} = 10 \text{ sec.}$$

$\frac{1600}{1120} = 1.429$ sec. $10 + 1.429 = 11.429$ sec. *Ans.*

122. If a pebble is dropped over a precipice, and is heard to strike the bottom in $7\frac{1}{2}$ sec., how far has it fallen, on the same suppositions as in Ex. 120?

Assume 745 ft. and 750 ft. to be the distance.

$16 : 745 = 1^2 : x^2$	$16 : 750 = 1^2 : x^2$
$x = \sqrt{\frac{745}{16}}$	$x = \sqrt{\frac{750}{16}}$
$\log 745 = 2.8722$	$\log 750 = 2.8751$
$\text{colog } 16 = 8.7959 - 10$	$\text{colog } 16 = 8.7959 - 10$
$2)\underline{1.6681}$	$2)\underline{1.6710}$
0.8341	0.8355
$= \log 6.825.$	$= \log 6.847.$
$\frac{745}{1120} = 0.665.$	$\frac{750}{1120} = 0.670.$
$6.825 + 0.665 = 7.49$ sec.	$6.847 + 0.670 = 7.517$ sec.
An error of -0.01.	An error of $+0.017$.

Difference between errors is 0.027.

Error of $745 : 5 = 0.01 : 0.027$.
Error of $745 : 5 = 10 : 27$.
Error of $745 \times 27 = 50$.
Error of $745 = \frac{50}{27} = 1.8$.
$745 + 1.8 = 746.8$ ft. 747 ft. *Ans.*

123. A pebble dropped down a shaft is heard to strike the bottom in 3 sec. after it begins to fall. Find the depth of the shaft.

Assume 130 and 140.

$16 : 130 = 1^2 : x^2$.
$x = \sqrt{\frac{130}{16}}$.
$\log 130 = 2.1139$
$2)\underline{0.9098}$
0.4549
colog $16 = 8.7959 - 10$
$= \log 2.851$.
$\frac{130}{1120} = 0.116$ sec.
$2.851 + 0.116 = 2.967$ sec.
An error of -0.033.

$16 : 140 = 1^2 : x^2$
$x = \sqrt{\frac{140}{16}}$.
$\log 140 = 2.1461$
$2)\underline{0.9420}$
0.4710
colog $16 = 8.7959 - 10$
$= \log 2.958$.
$\frac{140}{1120} = 0.125$ sec.
$2.958 + 0.125 = 3.083$ sec.
An error of $+0.083$.

Difference between errors is 0.117.
Error of $130 : 10 = 0.034 : 0.117$.
Error of $130 : 10 = 34 : 117$.
Error of $130 = \frac{10 \times 34}{117} = 2.9$ ft.
$130 + 2.9 = 132.9$ ft. 133 ft. *Ans.*

124. How long will it take a ball, rolling off a table, to drop 1^{cm}? 1 in.? 10^{cm}? 6 in.?

$4.903^m : 1^{cm} = 1^2 : x^2$.
$4.903 : 0.01 = 1 : x^2$.
$x = \sqrt{\frac{0.01}{4.903}}$.
$\log 0.01 = 8.0000 - 10$
colog $4.903 = 9.3095 - 10$
$2)\underline{17.3095 - 20}$
$8.6548 - 10$
$= \log 0.04517$.
0.04517 sec. (1) *Ans.*

$4.903^m : 1 \text{ in.} = 1^2 : x^2$.
$4.903 : 0.0254 = 1^2 : x^2$.
$x = \sqrt{\frac{0.0254}{4.903}}$.
$\log 0.0254 = 8.4048 - 10$
colog $4.903 = 9.3095 - 10$
$2)\underline{17.7143 - 20}$
$8.8572 - 10$
$= \log 0.07198$.
0.07198 sec. (2) *Ans.*

$4.903^m : 10^{cm} = 1^2 : x^2.$
$4.903 : 0.1 = 1^2 : x^2.$
$$x = \sqrt{\frac{0.1}{4.903}}.$$
$\log 0.1 = 9.0000 - 10$
$\text{colog}\, 4.903 = 9.3095 - 10$
$2)\underline{18.3095 - 20}$
$9.1548 - 10$
$= \log 0.1428.$
0.1428 sec. (3) *Ans.*

$4.903^m : 6 \text{ in.} = 1^2 : x^2.$
$4.903 : 0.1524 = 1^2 : x^2.$
$$x = \sqrt{\frac{0.1524}{4.903}}.$$
$\log 0.1524 = 9.1830 - 10$
$\text{colog}\, 4.903 = 9.3095 - 10$
$2)\underline{18.4925 - 20}$
$9.2463 - 10$
$= \log 0.1763.$
0.1763 sec. (4) *Ans.*

125. With what velocity will water flow through a hole 9 ft. below the surface?

$\sqrt{9} : \sqrt{16} = 3 : 4.$ $\frac{3}{4}$ of $32 = 24$ ft. *Ans.*

126. With what velocity will water leave a fountain having free play, and a head of 25 ft.? a head of 100 ft.?

$\sqrt{25} : \sqrt{16} = 5 : 4.$ $\frac{5}{4}$ of $32 = 40$ ft. (1) *Ans.*
$\sqrt{100} : \sqrt{16} = 10 : 4 = 5 : 2.$ $\frac{5}{2}$ of $32 = 80$ ft. (2) *Ans.*

127. If a hole in the side of a cistern 4 ft. below the surface of the water is delivering 10 gals. an hour, how many gallons would it deliver with 5 ft. more head?

$\sqrt{4} : \sqrt{9} = 10 : x.$ $2 : 3 = 10 : x.$ $x = \frac{30}{2} = 15$ gals. *Ans.*

128. If a pipe 2 in. in diameter, and 1 ft. long, inserted in a dam, the head of water being kept constant, delivers 4 gals. a minute, how many gallons a minute may be expected when another pipe of the same length, but $2\frac{1}{2}$ in. in diameter, is substituted for the two-inch pipe?

$2^2 : (2\frac{1}{2})^2 = 4 : x.$ $4 : 6\frac{1}{4} = 4 : x.$ $x = 6\frac{1}{4}$ gals. *Ans.*

129. If a one-inch pipe, 20 in. long, is substituted for the two-inch pipe, 1 ft. long, in Ex. 128, and the flow is found to be 5 pts. a minute, what part of the diminution is due to the smaller area of the orifice, and what part to the increased friction on the sides of the longer pipe?

$2^2 : 1^2 = 4 : x.$ $4 : 1 = 4 : x.$ $x = 1$ gal. $4 - 1 = 3$ gals. (1) *Ans.*
1 gal. = 8 pts. $8 - 5 = 3$ pts. (2) *Ans.*

130. A miller is using water flowing through the gate-way under 4 ft. head. How much more work could he do if the head was raised to 9 ft.? how much more if the head was raised to 25 ft.?

$$\sqrt{4^3} : \sqrt{9^3} = 1 : x. \qquad \sqrt{4^3} : \sqrt{25^3} = 1 : x.$$
$$\sqrt{64} : \sqrt{729} = 1 : x. \qquad \sqrt{64} : \sqrt{15625} = 1 : x.$$
$$8 : 27 = 1 : x. \qquad 8 : 125 = 1 : x.$$
$$x = \tfrac{27}{8} = 3\tfrac{3}{8}.\ (1)\ \textit{Ans.} \qquad x = \tfrac{125}{8} = 15\tfrac{5}{8}.\ (2)\ \textit{Ans.}$$

131. If a top 3 in. in diameter is making 200 revolutions a second, with what force does the outer layer pull away from the centre?

$1.227 \times \frac{1}{8} \times 200^2 = 6135.$
6135 times the weight of the material. *Ans.*

132. If a sling 30 in. long contains a stone, and is whirled round 80 times a minute, what is the force pulling on the string?

$30 \text{ in.} = 2\frac{1}{2} \text{ ft.}$
$1.227 \times 2\frac{1}{2} \times (1\frac{1}{3})^2 = 5.453.$
5.453 times weight when even with the hand.
$5.453 - 1 = 4.453$ times weight when highest.
$5.453 + 1 = 6.453$ times weight when lowest.

133. With what force does a locomotive running at 30 mi. an hour, on a curve of 800 ft. radius, bear against the outer rail?

30 mi. per hr. $= \frac{1}{2}$ mi. per min. $= \frac{1}{120}$ mi. per sec. $= 44$ ft. per sec.
Diameter is 1600 ft.
$3.1416 \times 1600 = 5026.56$ ft. circumference.

Hence, the locomotive makes $\dfrac{44}{5026.56}$ revolutions per sec.

$$\text{Force} = 1.227 \times 800 \times \left(\frac{44}{5026.56}\right)^2.$$

$$\begin{aligned}
\log\ 1.227 &= 0.0889\\
\log\ 800 &= 2.9031\\
\log\ 44^2 &= 3.2870\\
\text{colog}\ 5026.56^2 &= \underline{2.5974 - 10}\\
\log \text{force} &= 8.8764 - 10 = \log 0.07523.
\end{aligned}$$

0.07523 times weight. *Ans.*

134. If washed wool is put wet into a wire basket 1.2^m in diameter, and the basket is set to spinning at the rate of 180 revolutions a minute, with what force is water wrung out of the wool?

180 revolutions per min. = 3 revolutions per sec.

$4.025 \times 0.6 \times 3^2 = 21.735.$

21.735 times its weight. *Ans.*

135. If steel pens are revolved in a basket 32^{cm} in diameter, 17 revolutions a second, with what force is the oil drained from them?

$4.025 \times 0.16 \times 17^2 = 186.116.$

186.116 times weight. *Ans.*

136. How strong a horizontal pull on a chain, weighing half a pound to the yard, is required to make the lowest part curve with an 18-in. radius? with a 6-ft. radius?

18 in. = $\frac{1}{2}$ yd.	6 ft. = 2 yds.
$\frac{1}{2}$ yd. weighs $\frac{1}{4}$ lb.	2 yds. weigh 1 lb.
$\therefore$ tension = $\frac{1}{4}$. (1) *Ans.*	$\therefore$ tension = 1 lb. (2) *Ans.*

137. A $\frac{3}{4}$-in. rope, weighing $\frac{1}{4}$ of a pound to the yard, is fastened at one end to a staple, and near the other end, on the same level, runs over a pulley, and has a 25-lb. weight hung to it. What is the radius of its curvature at the middle?

The horizontal tension is 25 lbs., which represents 100 yds. of rope. $\therefore$ radius = 100 yds. *Ans.*

138. A shower wets the rope of Ex. 137, and increases its weight 40%; what does its radius now become?

The weight of the rope being $\frac{140}{100} = \frac{7}{5}$ of what it was, it takes only $\frac{5}{7}$ of 100 yds. = $71\frac{3}{7}$ yds. *Ans.*

139. A steam-tug, in attempting to move a ship, straightened her hawser until the radius of the lowest point was 1980 ft. The rope was wet, and weighed $3\frac{1}{4}$ lbs. to the yard. With what force was it stretched?

1980 ft. = 660 yds., and $3\frac{1}{4} \times 660 = 2145$ lbs. tension. *Ans.*

140. A chain 31 ft. long hangs between points on a level, and sags 4 ft.? What is the radius at the lowest point?

$$R=\frac{(\frac{1}{2}\text{ ch.}\times\text{sag})(\frac{1}{2}\text{ ch.}-\text{sag})}{2\text{ sag}}=\frac{(15.5+4)(15.5-4)}{2\times 4}$$

$$=\frac{19.5\times 11.5}{8}=28.031\text{ ft. } \textit{Ans.}$$

141. The whole chain, in Ex. 140, weighs 18 lbs. What is the horizontal tension? What is the distance of the points apart? What is the slant, or batter, of the end of the chain?

$T=$ weight of R. 1 ft. weighs $\frac{18}{31}$ lb.

Tension $=\frac{18}{31}\times 28.031=16.27$ lbs. (1) *Ans.*

From Prop. III.

$$\log\tfrac{1}{2}\text{ span}=\log 19.5+\log 11.5+\log(1.29-1.0607)+\text{colog } 4+0.0612.$$

$$1.29-1.0607=0.2293.$$

$$\begin{aligned}\log\ 19.5&=1.2900\\ \log\ 11.5&=1.0607\\ \log 0.2293&=9.3604-10\\ \text{colog}\ 4&=9.3979-10\\ &\ 0.0612\\ \hline &\ 1.1702=\log 14.79.\end{aligned}$$

$$2\times 14.79=29.58.\ (2)\ \textit{Ans.}$$

$$\text{Batter}=\frac{28.031}{15.5}=1.808.\ (3)\ \textit{Ans.}$$

142. A chain weighing 1^{kg} to the meter is suspended from points on a level; the length of chain is 31^{m}, and it sags 1.3^{m}. Find all the conditions, and find how much it falls below a level at 10^{cm} from each end.

$\frac{1}{2}$ ch. $=15.5^{m}$. sag $=1.3^{m}$.

$$\text{Radius}=\frac{(\frac{1}{2}\text{ ch.}+\text{sag})(\frac{1}{2}\text{ ch.}-\text{sag})}{2\text{ sag}}=\frac{16.8\times 14.2}{2.6}$$

$=91.75^{m}$. (1) *Ans.*

Tension $=91.75^{kg}$. (2) *Ans.*

From Prop. III.

$$\begin{aligned}
\log\ \ 16.8 &= 1.2253\\
\log\ \ 14.2 &= 1.1523\\
\log 0.0730 &= 8.8633 - 10\\
\text{colog}\ \ 1.3 &= 9.8861 - 10\\
&\ \underline{0.0612}\\
&\ 1.1882 = \log 15.42.
\end{aligned}$$

$2 \times 15.42 = 30.84$ span. (3) *Ans.*

$$\text{Batter} = \frac{91.75}{15.5} = 5.92. \text{ (4) } Ans.$$

For drop with hypotenuse of 10^{cm},

$6 : 10 :: 1 : x.$

$x = 1.667$ depression.

For drop with batter of 10,

$5.92 : 10 :: 1 : x.$

$x = 1.689.$

1.689 depression. (5) *Ans.*

143. A chain 100^m long, weighing 14 oz. to the foot, is suspended from points on a level 80^m apart. What is the sag, the batter at the ends, and the horizontal tension?

Assume that the sag is 26.5^m.

$$\begin{aligned}
\log \tfrac{1}{2}\text{ span} &= \log(\tfrac{1}{2}\text{ ch.} + \text{sag}) + \log(\tfrac{1}{2}\text{ ch.} - \text{sag}) + \log[\log(\tfrac{1}{2}\text{ ch.} + \text{sag})\\
&\quad - \log(\tfrac{1}{2}\text{ ch.} - \text{sag})] + \text{colog sag} + 0.0612\\
&= \log(50 + 26.5) + \log(50 - 26.5) + \log[\log(50 + 26.5)\\
&\quad - \log(50 + 26.5)] + \text{colog } 26.5 + 0.0612\\
&= \log 76.5 + \log 23\ 5 + \log(\log 76.5 - \log 23.5)\\
&\quad + \text{colog } 26.5 + 0.0612.
\end{aligned}$$

$$\begin{aligned}
\log\ \ 76.5 &= 1.8837\\
\log\ \ 23.5 &= 1.3711\\
\log 0.5126 &= 9.7098 - 10\\
\text{colog}\ \ 26.5 &= 8.5768 - 10\\
&\ \underline{0.0612}\\
&\ 1.6026 = \log 40.05.
\end{aligned}$$

$\tfrac{1}{2}$ span $= 40.05$. An error of $+ 0.05$.

Assume that the sag is 26.6^{m}.

$$\log \tfrac{1}{2} \text{ span} = \log 76.6 + \log 23.4 + \log(\log 76.6 - \log 23.4) + \text{colog } 26.6 + 0.0612.$$

$$\begin{aligned} \log \quad 76.6 &= 1.8842 \\ \log \quad 23.4 &= 1.3692 \\ \log 0.5150 &= 9.7118 \\ \text{colog} \quad 26.6 &= 8.5751 \\ &\underline{\ 0.0612} \\ &\ 1.6015 = \log 39.95. \end{aligned}$$

$\tfrac{1}{2}$ span $= 39.95$. Error of $26.5 : 0.1 = 0.05 : 0.1$.

An error of -0.05. $\therefore$ error of $26.5 = 0.05$.

$\tfrac{1}{2}$ sag $= 26.5 + 0.05 = 26.55$. *Ans.*

$$R = \frac{(50 + 26.55)(50 - 26.55)}{53.1} = \frac{76.55 \times 23.45}{53.1} = 33.806.$$

$$\text{Batter} = \frac{R}{\tfrac{1}{2}\text{ ch.}} = \frac{33.806}{50} = 0.6761. \textit{ Ans.}$$

$1^{m} = 3.28$ ft. 1 ft. weighs $\tfrac{7}{8}$ lb.

Tension $= 33.806 \times 3.28 \times \tfrac{7}{8}$ lb. $= 97$ lbs. $= 44^{kg}$. *Ans.*

144. Suppose the points of suspension in Ex. 143 to remain unchanged, and the chain to be shortened 5^{m}. What does the tension become?

Assume that the sag is 22.5.

$$\log \tfrac{1}{2} \text{ span} = \log 25 + \log 70 + \log(\log 70 - \log 25) + \text{colog } 22.5 + 0.0612.$$

$$\begin{aligned} \log \quad 25 &= 1.3979 \\ \log \quad 70 &= 1.8451 \\ \log 0.4472 &= 9.6505 - 10 \\ \text{colog} \quad 22.5 &= 8.6478 - 10 \\ &\underline{\ 0.0612} \\ &\ 1.6025 = \log 40.04. \end{aligned}$$

$\tfrac{1}{2}$ span $= 40.04$. An error of $+ 0.04$.

Assume that the sag is 22.6.

$$\log \tfrac{1}{2} \text{ span} = \log 70.1 + \log 24.9 + \log(\log 70 - \log 24.9) + \text{colog } 22.6 + 0.0612.$$

$$\begin{aligned}
\log\ \ 70.1 &= 1.8457\\
\log\ \ 24.9 &= 1.3962\\
\log 0.4489 &= 9.6527-10\\
\text{colog}\ \ 22.6 &= 8.6459-10\\
&\ \underline{0.0612}\\
&\ 1.6017 = \log 39.96.
\end{aligned}$$

$\frac{1}{2}$ span $= 39.96$. Error of $22.5 : 0.1 = 0.04 : 0.08$.

An error of -0.04. Hence error of 22.5 is 0.05.

$\therefore$ sag is $22.5 + 0.05 = 22.55^{\text{m}}$.

$$R = \frac{24.95 \times 70.05}{45.1}.$$

$$T = \frac{24.95 \times 70.05 \times 3.28087 \times 14}{45.1 \times 16}.$$

$$\begin{aligned}
\log\ \ 24.95 &= 1.3971\\
\log\ \ 70.05 &= 1.8454\\
\log 3.28087 &= 0.5160\\
\log\ \ 14 &= 1.1461\\
\text{colog}\ \ 45.1 &= 8.3458-10\\
\text{colog}\ \ 16 &= \underline{8.7959-10}\\
&\ 2.0463 = \log 111.3 \text{ lbs.}
\end{aligned}$$

$T = 111.3$ lbs. *Ans.*

145. How long a rope is required between points 100 ft. apart to sag 30 ft.? 20 ft.? 10 ft.?

Assume that the chain is 120.9 ft.

$$\begin{aligned}
\log \tfrac{1}{2}\text{ span} &= \log(60.45+30) + \log(60.45-30) + \log[\log(60.45+30)\\
&\quad - \log(60.45-30)] + \log 30 + 0.0612\\
&= \log 90.45 + \log 30.45 + \log(\log 90.45 - \log 30.45)\\
&\quad + \text{colog } 30 + 0.0612.
\end{aligned}$$

$$\begin{aligned}
\log\ \ 90.45 &= 1.9564\\
\log\ \ 30.45 &= 1.4836\\
\log 0.4728 &= 9.6747-10\\
\text{colog}\ \ 30 &= 8.5229-10\\
&\ \underline{0.0612}\\
&\ 1.6988 = \log 49.99.
\end{aligned}$$

$\frac{1}{2}$ span $= 49.99$. An error of -0.01.

Assume that the chain is 121 ft.

$$\begin{aligned}\log \tfrac{1}{2}\text{ span} &= \log(60.5+30)+\log(60.5-30)+\log[\log(60.5+30)\\&\quad -\log(60.5-30)]+\text{colog }30+0.0612\\&=\log 90.5+\log 30.5+\log(\log 90.5-\log 30.5)\\&\quad+\text{colog }30+0.0612.\end{aligned}$$

$$\begin{aligned}\log\ 90.5 &= 1.9566\\ \log\ 30.5 &= 1.4843\\ \log 0.4723 &= 9.6741-10\\ \text{colog}\ 30 &= 8.5229-10\\ &\ \underline{0.0612}\\ &\ 1.6991 = \log 50.01.\end{aligned}$$

$\frac{1}{2}$ span $= 50.01$. An error of $+0.01$.

Error of $120.9 : 0.1 = 0.01 : 0.02$.

$\therefore$ error of 120.9 is 0.05.

$\therefore$ chain is $120.9 + 0.05 = 120.95$ ft. (1) *Ans.*

Assume that the chain is 109.9 ft.

$$\begin{aligned}\log \tfrac{1}{2}\text{ span} &= \log(54.95+20)+\log(54.95-20)+\log[\log(54.95+20)\\&\quad -\log(54.95-20)]+\text{colog }20+0.0612\\&=\log 74.95+\log 34.95+\log(\log 74.95-\log 34.95)\\&\quad+\text{colog }20+0.0612.\end{aligned}$$

$$\begin{aligned}\log\ 74.95 &= 1.8748\\ \log\ 34.95 &= 1.5435\\ \log 0.3313 &= 9.5202-10\\ \text{colog}\ 20 &= 8.6990-10\\ &\ \underline{0.0612}\\ &\ 1.6987 = \log 49.97.\end{aligned}$$

$\frac{1}{2}$ span $= 49.97$. An error of -0.03.

Assume that the chain is 110 ft.

$$\begin{aligned}\log \tfrac{1}{2}\text{ span} &= \log(55+20)+\log(55-20)+\log[\log(55+20)\\&\quad -\log(55-20)]+\text{colog }20+0.0612\\&=\log 75+\log 35+\log(\log 75-\log 35)+\text{colog }20+0.0612\end{aligned}$$

$$\begin{aligned}\log\ 75 &= 1.8751\\ \log\ 35 &= 1.5441\\ \log 0.3310 &= 9.5198-10\\ \text{colog}\ 20 &= 8.6990-10\\ &\ \underline{0.0612}\\ &\ 1.6992 = \log 50.02.\end{aligned}$$

$\frac{1}{2}$ span $= 50.02$. An error of $+0.02$.

Error of $109.9 : 0.1 = 0.03 : 0.05$.

Error of $109.9 : 0.1 = 3.5$.

$\therefore$ error of $109.9 \quad = \dfrac{0.3}{5} = 0.06$.

$\therefore$ chain is $109.9 + 0.06 = 109.96$. (2) *Ans.*

Assume that the chain is 102.5 ft.

$$\begin{aligned}\log \tfrac{1}{2}\text{ span} &= \log(51.25+10) + \log(51.25-10) + \log[\log(51.25+10) \\ &\quad - \log(51.25-10)] + \operatorname{colog} 10 + 0.0612 \\ &= \log 61.25 + \log 41.25 + \log(\log 61.25 - \log 41.25) \\ &\quad + \operatorname{colog} 10 + 0.0612.\end{aligned}$$

$$\begin{aligned}\log\ 61.25 &= 1.7872 \\ \log\ 41.25 &= 1.6154 \\ \log 0.1718 &= 9.2350 - 10 \\ \operatorname{colog}\ 10 &= 9.0000 - 10 \\ &\ 0.0612 \\ &\ \overline{1.6988} = \log 49.98.\end{aligned}$$

$\frac{1}{2}$ span $= 49.98$. An error of -0.02.

Assume that the chain is 102.7 ft.

$$\begin{aligned}\log \tfrac{1}{2}\text{ span} &= \log(51.35+10) + \log(51.35-10) + \log[\log(51.35+10) \\ &\quad - \log(51.35-10)] + \operatorname{colog} 10 + 0.0612 \\ &= \log 61.35 + \log 41.35 + \log(\log 61.35 - \log 41.35) \\ &\quad + \operatorname{colog} 10 + 0.0612.\end{aligned}$$

$$\begin{aligned}\log\ 61.35 &= 1.7879 \\ \log\ 41.35 &= 1.6165 \\ \log 0.1714 &= 9.2340 \\ \operatorname{colog}\ 10 &= 9.0000 - 10 \\ &\ 0.0612 \\ &\ \overline{1.6996} = \log 50.07.\end{aligned}$$

$\frac{1}{2}$ span $= 50.07$. An error of $+0.07$.

Error of $102.5 : 0.2 = 0.02 : 0.09$.

Error of $102.5 : 0.2 = 2 : 9$.

$\therefore$ error of $102.5 \quad = \frac{2}{9}$ of $0.2 = 0.044$.

Hence the chain is $102.5 + 0.044 = 102.544$ ft. (3) *Ans.*

146. If 4 cu. in. of iron weigh 1 lb. avoirdupois, what is the weight of 1 cu. in. in grains? What is the specific gravity of the iron?

1 lb. = 7000 gr.

$\therefore$ 1 cu. in. of the iron weighs $\frac{1}{4}$ of 7000 = 1750 grs. (1) *Ans.*

$(1728 \div 4) \times 16 = 6912$ oz. $\quad 6912 \div 1000 = 6.912$. (2) *Ans.*

147. If 4 cu. in. of iron weigh 1 lb., what is the diameter of a 6-lb. ball? of a 32-lb. ball?

$$V = 4 \times 6 = 24 \text{ cu. in.}$$
$$V = 0.5236\,D^3.$$
$$24 = 0.5236\,D^3.$$
$$D = \sqrt[3]{\frac{24}{0.5236}}.$$

$$\begin{array}{r} \log 24 = 1.3802 \\ \text{colog}\, 0.5236 = \underline{0.2810} \\ 3\underline{)1.6612} \\ 0.5537 \\ = \log 3.578. \end{array}$$

3.578 in. (1) *Ans.*

$$V = 32 \times 4 = 128 \text{ cu. in.}$$
$$V = 0.5236\,D^3.$$
$$128 = 0.5236\,D^3.$$
$$D = \sqrt[3]{\frac{128}{0.5236}}.$$

$$\begin{array}{r} \log 128 = 2.1072 \\ \text{colog}\, 0.5236 = \underline{0.2810} \\ 3\underline{)2.3882} \\ 0.7961 \\ = \log 6.253. \end{array}$$

6.253 in. (2) *Ans.*

148. If a block of iron (with all its corners square) measures 17.36 in. by 8.7 in. by 1.76 in., what does it weigh at $\frac{1}{4}$ lb. to the cubic inch? and what would be the diameter if cast into a ball, if 11% is allowed for waste?

$\frac{1}{4}$ of $17.36 \times 8.7 \times 1.76 = 66.45$ lbs. (1) *Ans.*

$265.82 - 0.11$ of $265.82 = 236.58$.

$$\begin{array}{r} \log 236.58 = 2.3739 \\ \text{colog}\, 0.5236 = \underline{0.2810} \\ 3\underline{)2.6549} \\ 0.8850 = \log 7.673. \end{array}$$

7.673 in. (2) *Ans.*

149. Answer the same questions as in the last example, for a block of which the dimensions are 71.4 in. by $8\frac{2}{3}$ in. by $3\frac{1}{3}$ in.

$71.4 \times 3\frac{1}{3} = 238.$

$238 \times 8\frac{2}{3} = 2062.67.$

$\frac{1}{4}$ of $2062.67 = 515.67$ lbs. (1) *Ans.*

$$\log 2062.67 = 3.3145$$
$$\log \quad 0.89 = 9.9494 - 10$$
$$\text{colog}\, 0.5236 = 0.2810$$
$$3)\underline{3.5449}$$
$$1.1816 = \log 15.19.$$

15.19 in. (2) *Ans.*

150. What is the diameter of a cylinder 11 in. long that holds 2 gals.?

$$2 \text{ gals.} = 462 \text{ cu. in.}$$
$$V = \tfrac{1}{4}\pi D^2 \times h.$$
$$462 = 0.7854\, D^2 \times 11.$$
$$D = \sqrt{\frac{462}{0.7854 \times 11}} = \sqrt{\frac{42}{0.7854}}.$$
$$\log 42 = 1.6232$$
$$\text{colog}\, 0.7854 = 0.1049$$
$$2)\underline{1.7281}$$
$$0.8641 = \log 7.313.$$

7.313 in. *Ans.*

151. What is the diameter of a cylinder 9 in. long that holds 2 gals.?

$$462 = 0.7854\, D^2 \times 9.$$
$$D = \sqrt{\frac{462}{0.7854 \times 9}}.$$
$$\log \quad 462 = 2.6646$$
$$\text{colog}\, 0.7854 = 0.1049$$
$$\text{colog} \quad 9 = 9.0458 - 10$$
$$2)\underline{1.8153}$$
$$0.9077 = \log 8.086.$$

8.086 in. *Ans.*

152. What is the diameter of a cylinder 30^{cm} long that holds 1^l?

$$10^l = 10000^{ccm}.$$
$$10000 = 0.7854\,D^2 \times 30.$$
$$D = \sqrt{\frac{1000}{0.7854 \times 3}}.$$
$$\begin{aligned} \log\ 1000 &= 3.0000 \\ \text{colog}\ 0.7854 &= 0.1049 \\ \text{colog}\ 3 &= \underline{9.5229 - 10} \\ &\ \ 2)\underline{2.6278} \\ &\ \ \ 1.3139 = \log 20.6. \end{aligned}$$

20.6^{cm}. *Ans.*

153. What is the circumference of a globe if the square centimeters of its surface are three times the cubic centimeters of its volume?

$$V = 0.5236\,D^3.$$
$$S = 3.1416\,D^2.$$
$$3.1416\,D^2 = 3 \times 0.5236\,D^3.$$

Divide both sides by $3 \times 0.5236\,D^2$.

$$2 = D.$$

Hence, circumference $= 2 \times 3.1416 = 6.2832^{cm}$. *Ans.*

154. Find the diameter of a circle of which the number of inches in its circumference is equal to the square feet of its area.

$$\text{Area} = \tfrac{1}{4}\,\pi\,D^2 \text{ sq. ft.}$$
$$\text{Circumference} = \pi\,D \text{ ft. or } 12\pi D \text{ in.}$$
$$12\pi D = \tfrac{1}{4}\pi D^2.$$

Multiply each side by 4.

$$48\pi D = \pi\,D^2.$$

Divide each side by πD.

$$48 = D.$$
$$\therefore\ D = 48 \text{ ft. } \textit{Ans.}$$

155. How many times does a carriage-wheel 3 ft. 2 in. in diameter turn in going a mile on a smooth road?

The circumference of the wheel is

$$3\tfrac{1}{6} \times 3.1416 = 9.9484 \text{ ft.}$$
$$1 \text{ mi.} = 5280 \text{ ft.}$$

$\therefore$ the wheel turns $\frac{5280}{9.9484} = 530.7$ times. *Ans.*

156. The top of a wheel is at each instant moving with twice the velocity of the carriage, and is moving in a curve whose centre, at the instant, is as far below ground as the point is above ground. What, then, is the force exerted to separate the mud from the top of a wheel 3 ft. 2 in. in diameter, when the carriage is moving at the rate of 10 mi. an hour?

The carriage going at 10 mi., the top of the wheel goes at 20 mi. per hour, or $29\frac{1}{3}$ ft. per second. The radius of the curve is $2 \times 3\frac{1}{6} = 6\frac{1}{3}$ ft., the total circumference of which would be $12\frac{2}{3} \times 3.1416 = 39.7936$ ft. $\therefore$ the force is

$$1.227 \times 6\frac{1}{3} \times \left(\frac{29.3333}{39.7936}\right)^2 = 4.224 \text{ times the weight.}$$

$$\begin{aligned} \log \quad 1.227 &= 0.0889 \\ \log \quad 6\tfrac{1}{3} &= 0.8016 \\ \log \quad (29\tfrac{1}{3})^2 &= 2.9348 \\ \text{colog}\,(39.7936)^2 &= \underline{6.8004 - 10} \\ 0.6257 &= \log 4.224. \quad \textit{Ans.} \end{aligned}$$

157. A point in the tire, as a spike-head, moves, while the wheel rolls over once, just four times the diameter of the wheel. How far does the point travel while the wheel, 3 ft. 2 in. in diameter, travels 1 mi.?

In going one mile the wheel turns 530.7 times.

$\therefore$ the point travels $3\frac{1}{6} \times 4 \times 530.7 = 6722.2$ ft. *Ans.*

158. An oil-can is formed of two cylinders connected by a frustum of a cone. The upper cylinder, or neck, is 6^{cm} in diameter, and 75^{mm} high; the lower cylinder is 13^{cm} in diameter, and 153^{mm} high; the total length of the can is 30^{cm}. Find its capacity in liters.

A square shaft to enclose the neck would contain

$6 \times 6 \times 7.5 = 270^{ccm}$;

one to enclose the body would contain

$13 \times 13 \times 15.3 = 2585.7^{ccm}$.

The frustum to enclose the remainder would contain

$(\sqrt{6 \times 6 \times 13 \times 13} + 169 + 36)\frac{1}{3}$ of $7.2 = 679.2^{ccm}$.

$(270 + 2585.7 + 679.2) \times 0.7854 = 2776^{ccm} = 2.776^{l}$. *Ans.*

159. A common tunnel is formed of a frustum of a cone terminated with a cylinder. The height of the frustum is 14^{cm}, and the diameters of the two bases are 175^{mm} and 16^{mm} respectively. The cylinder is 8^{cm} long. Find the capacity of the tunnel in liters.

$16^{mm} = 1.6^{cm}$.

The cylinder holds $1.6^2 \times 8 \times 0.7854 = 20.48 \times 0.7854^{ccm}$.

The frustum holds $\frac{14}{3}(17.5^2 + 1.6^2 + 17.5 \times 1.6) \times 0.7854$
$= 1571.78 \times 0.7854^{ccm}$.

Both together hold $(1571.78 + 20.48) \times 0.7854^{ccm}$
$= 1592.26 \times 0.7854^{ccm} = 1250^{ccm} = 1.25^{l}$. *Ans.*

160. A pan is in the form of a frustum of a cone. The interior measurement is 10^{cm} deep, 12^{cm} across the bottom, and 23^{cm} across the top. Find the capacity of the pan in liters.

$$\begin{aligned}\tfrac{10}{3}(12^2 + 23^2 + 12 \times 23) \times 0.7854 &= \tfrac{10}{3}(144 + 529 + 276) \times 0.7854\\ &= \tfrac{10}{3} \text{ of } 949 \times 0.7854\\ &= 3163\tfrac{1}{3} \times 0.7854 = 2484.5^{ccm}\\ &= 2.4845^{l}. \textit{ Ans.}\end{aligned}$$

161. A stove-pipe is 4^{m} long, 26^{cm} in diameter, and 1^{mm} thick. Find how many square centimeters of sheet-iron it has taken to make it, if the edges lap one centimeter; and give the weight of the pipe, if the specific gravity of the sheet-iron is 7.8.

$4^{m} = 400^{cm}$; $1^{mm} = 0.1^{cm}$.

$S = \pi Dh = (3.1416 \times 26 + 1) \times 400 = 33{,}072.64^{qcm}$. (1) *Ans.*

$0.1 \times 33{,}072.64 = 3307.26^{ccm}$.

$7.8 \times 3307.26 = 25{,}796.659^{g} = 25.8^{kg}$. (2) *Ans.*

162. A spherical bomb is 32^{cm} in diameter, and the sides 38^{mm} thick. The specific gravity of the metal of which it is made is 7.2. Find its weight and interior capacity.

Inside diameter is $32 - 2 \times 3.8 = 24.4^{cm}$.

$V = 0.5236\, D^3 = 0.5236 \times 24.4^3$.

$$\begin{aligned}\log 5236 &= 9.7190 - 10\\ \log 24.4^3 &= \underline{4.1622}\\ & 3.8812 = \log 7607.\end{aligned}$$

$7607^{ccm} = 7.607^{l}$. (1) *Ans.*

$$V = 0.5236 \times 32^3.$$
$$\log\ 32^3 = 4.5153$$
$$\log 0.5236 = 9.7190 - 10$$
$$4.2343 = \log 17{,}150.$$
$$17{,}150^{\text{ccm}} = 17.15^{\text{l}}.$$
$$7.2 \times (17.15 - 7.607) = 68.71^{\text{kg}}.\ (2)\ \textit{Ans.}$$

163. The diameters of a lamp-shade are 25^{cm} and 7^{cm}; its slant height is 134^{mm}. Give its curved surface in square centimeters.

$$134^{\text{mm}} = 13.4^{\text{cm}}.$$
$$\frac{25+7}{2} = 16, \text{ average diameter.}$$
$$3.1416 \times 16 \times 13.4 = 673.6^{\text{qcm}}.\ \textit{Ans.}$$

164. A niche is formed like a half-cylinder surmounted by a quarter of a sphere. The height of the cylinder is 1.2^{m}, the diameter 0.8^{m}. Find the volume of the niche, and the area of its interior surface.

The volume of half-cylinder $= \frac{1}{2}(0.8^2 \times 0.7854 \times 1.2) = 301.59^{\text{l}}$.
The volume of quarter-sphere $= \frac{1}{4}(0.8^3 \times 0.5236) = 67.02^{\text{l}}$.
$301.59 + 67.02 = 368.61^{\text{l}}$. (1) *Ans.*

The surface of cylinder is $\frac{1}{2} \times 0.8 \times 3.1416 \times 1.2 = 1.5080^{\text{qm}}$; of quarter-sphere, $\frac{1}{4} \times 4 \times 0.8^2 \times 0.7854 = 0.5027^{\text{qm}}$; of floor, $\frac{1}{2}(0.8^2 \times 0.7854) = 0.2513^{\text{qm}}$.
$1.5080 + 0.5027 + 0.2513 = 2.2620^{\text{qm}}$. (2) *Ans.*

165. A steam-boiler is formed of a cylinder terminated at each end by a hemispherical cap of the same diameter. The length of the cylinder is 3.4^{m}, interior diameter 0.8^{m}. Find the number of hektoliters of water required to fill the boiler half full.

The volume of cylinder is
$$3.4 \times 0.8^2 \times 0.7854 = 1.709^{\text{cbm}} = 17.09^{\text{hl}}.$$
The two caps form a sphere, the volume of which is
$$0.8^3 \times 0.5236 = 0.268^{\text{cbm}} = 2.68^{\text{hl}}.$$
$$\tfrac{1}{2}(2.68 + 17.09) = 9.89^{\text{hl}}.\ \textit{Ans.}$$

166. A hill 482 ft. high is 8 mi. from the shore. How many miles out at sea is it visible?

$$\tfrac{1}{2}\log 482 = 1.3415$$
$$\underline{0.1215}$$
$$1.4630 = \log 29.04.$$
$$29.04 - 8 = 21.04 \text{ mi. } \textit{Ans.}$$

167. A sailor at the topmast 80 ft. above the sea can just see a sailor at the topmast of a similar ship. How many miles apart are the vessels?

$$\tfrac{1}{2}\log 80 = 0.9516$$
$$\underline{0.1215}$$
$$1.0731 = \log 11.83.$$
$$2 \times 11.83 \text{ mi.} = 23.66 \text{ mi. } \textit{Ans.}$$

168. A vessel approaching Valparaiso at daybreak just makes out the peak of Aconçagua, 23,000 ft. high and 140 mi. back from the coast. How far is the vessel from land if the eye of the observer is 30 ft. above the water?

$$\sqrt{\tfrac{7}{4} \text{ of } 30} = \sqrt{52.5} = 7.25 \text{ mi.}$$
$$\sqrt{\tfrac{7}{4} \text{ of } 23000} = \sqrt{40250} = 200.6 \text{ mi.}$$
$$200.6 - 140 = 60.6.$$
$$60.6 + 7.25 = 67.85 \text{ mi. } \textit{Ans.}$$

169. If Mount Washington is 6240 ft. high and 76 mi. in an air-line from Cape Elizabeth, how far out from the Cape will its peak be visible in the ordinary state of the atmosphere?

$$\sqrt{\tfrac{7}{4} \text{ of } 6240} = \sqrt{10920} = 104.5 \text{ mi.}$$
$$104.5 \text{ mi.} - 76 \text{ mi.} = 28.5 \text{ mi. } \textit{Ans.}$$

170. How many acres of water can a man see, standing on a raft with his eyes just 6 ft. above water, and no land in sight?

$$\begin{array}{rl}
\tfrac{1}{2}\log 6 = & 0.3891 \\
 & \underline{0.1215} \\
 & 0.5106 \\
 & \underline{2} \\
 & 1.0212 \\
\log 3.1416 = & 0.4971 \\
\log \quad 640 = & \underline{2.8062} \\
 & 4.3245 = \log 21{,}110. \qquad 21{,}110 \text{ A. } \textit{Ans.}
\end{array}$$

171. How far would a mountain 30,000 ft. high be visible? one of 5000 ft. high? one of 1000 ft. high?

$$\begin{aligned} \tfrac{1}{2}\log 30{,}000 &= 2.2386 \\ &\underline{\ 0.1215} \\ &\ 2.3601 = \log 229.2. \qquad 229.2 \text{ mi. (1) } \textit{Ans.} \end{aligned}$$

$$\begin{aligned} \tfrac{1}{2}\log 5{,}000 &= 1.8495 \\ &\underline{\ 0.1215} \\ &\ 1.9710 = \log 93.55. \qquad 93.55 \text{ mi. (2) } \textit{Ans.} \end{aligned}$$

$$\begin{aligned} \tfrac{1}{2}\log 1{,}000 &= 1.5000 \\ &\underline{\ 0.1215} \\ &\ 1.6215 = \log 41.83. \qquad 41.83 \text{ mi. (3) } \textit{Ans.} \end{aligned}$$

172. How high must a mountain be in order to be visible at sea-level 50 mi.? 100 mi.? 150 mi.?

$$\tfrac{4}{7} \text{ of } 50^2 = \tfrac{4}{7} \text{ of } 2{,}500 = 1{,}429 \text{ ft. (1) } \textit{Ans.}$$
$$\tfrac{4}{7} \text{ of } 100^2 = \tfrac{4}{7} \text{ of } 10{,}000 = 5{,}714 \text{ ft. (2) } \textit{Ans.}$$
$$\tfrac{4}{7} \text{ of } 150^2 = \tfrac{4}{7} \text{ of } 22{,}500 = 12{,}860 \text{ ft. (3) } \textit{Ans.}$$

173. How far is a mountain 1000^{m} high visible? 2000^{m} high?

$$\sqrt{15 \times 1000} = \sqrt{15000} = 122.5^{\text{km}}. \text{ (1) } \textit{Ans.}$$
$$\sqrt{15 \times 2000} = \sqrt{30000} = 173.2^{\text{km}}. \text{ (2) } \textit{Ans.}$$

174. How far can a man see from the shore, if he stands on a bluff that raises his eye 11^{m} above the sea?

$$\sqrt{15 \times 11} = \sqrt{165} = 12.85^{\text{km}}. \ \textit{Ans.}$$

175. If in steaming away from a mountainous island a sailor estimates his distance at 171^{km}, when the island disappears beneath the wave, how high shall he estimate the mountains?

$$\begin{aligned} 2 \times \log 171 &= 4.4660 \\ \text{colog } 15^2 &= \underline{8.8239 - 10} \\ &\ 3.2899 = \log 1950. \qquad 1950^{\text{m}}. \ \textit{Ans.} \end{aligned}$$

176. The flash of a gun is seen $7\frac{1}{2}$ sec. before the report of the gun is heard; there is no wind, and the thermometer is 73° F. How far off was the gun?

$$73 - 57 = 16.$$
$$16 + 1114 = 1130 \text{ ft. per sec.}$$
$$7\tfrac{1}{2} \times 1130 = 8475 \text{ ft. } \textit{Ans.}$$

177. A meteor was seen to burst; the report followed in 4 min. 17 sec. What was its distance if the average temperature of the intervening air was 50° F.?

$$4 \text{ min. } 17 \text{ sec.} = 257 \text{ sec.}$$
$$57 - 50 = 7.$$
$$1114 - 7 = 1107 \text{ ft. per sec.}$$
$$257 \times 1107 \div 5280 = 53.88 \text{ mi. } \textit{Ans.}$$

178. Find the distances of the last two examples by § 461.

$$73° \text{ F.} = 22.8 \text{ C.}$$
$$22.8 - 21 = 1.8.$$
$$1.8 \times 0.53 = 0.95.$$
$$7\tfrac{1}{2}(345 + 0.95) = 2594.6^{\text{m}} = 2.5946^{\text{km}}. \text{ (1) } \textit{Ans.}$$
$$50° \text{ F.} = 10° \text{ C.}$$
$$0.53 \times (21 - 10) = 5.83.$$
$$257 \times (345 - 5.83) = 87{,}166^{\text{m}} = 87.16^{\text{km}}. \text{ (2) } \textit{Ans.}$$

179. How long will it take for an explosion at the equator to be heard at the antipodes of the place, if the circumference of the earth at the equator be reckoned at $40{,}000^{\text{km}}$, and the average temperature at the equator at 23° C.?

$$0.53 \times (23 - 21) = 1.06^{\text{m}}.$$
$$345 + 1.06 = 346.06^{\text{m}} = 0.34606^{\text{km}}$$
$$20{,}000 \div 0.34606 = 57{,}793 \text{ sec.}$$
$$57{,}793 \text{ sec.} = 16 \text{ hrs. } 3.2 \text{ min. } \textit{Ans.}$$

180. If an explosion at the equator occurs at sunset, and the average temperature east of the spot is 22° C., and that to the west 24° C., how far from the antipodes would the sounds meet?

The velocity at 22° C. is 345.53^m per sec.
The velocity at 24° C. is 346.59^m per sec.

692.12^m per sec. is the velocity with which the two sounds are approaching each other.

$$692.12 : 346.59 : : 40,000 : x.$$

$$x = \frac{346.59 \times 40000}{692.12} \text{ km.} = 20,030.63^{km}.$$

$$20,030.63 - 20,000 = 30.63^{km}. \textit{ Ans.}$$

181. How far off is the lightning when the thunder follows in 13 sec., the temperature being 76° F.?

$$76 - 57 + 1114 = 1133 \text{ ft.}$$

$$13 \times 1133 \div 5280 = 2.79 \text{ mi. } \textit{Ans.}$$

182. How long would it take sound to go through a whispering-tube 3 mi. long, temperature 61° F.?

$$61 - 57 + 1114 = 1118 \text{ ft.}$$

$$3 \text{ mi.} = 3 \times 5280 = 15,840 \text{ ft.}$$

$$15,804 \div 1118 = 14.16 \text{ sec. } \textit{Ans.}$$

183. Sound travels in iron about $10\frac{1}{2}$ times as fast as in air. How long, then, after seeing the blow of a sledge-hammer given on the other end of an iron pipe $1\frac{1}{2}$ mi. long, may I expect to hear the sound by the iron; and how long after, to hear the sound through the air in the pipe; thermometer 63° F.?

$$63 - 57 + 1114 = 1120 \text{ ft. per sec.}$$

$$1\tfrac{1}{2} \text{ mi.} = 7920 \text{ ft.}$$

$$7920 \div 1120 = 7.07 \text{ sec. through air. (1) } \textit{Ans.}$$

$$7.07 \div 10.5 = 0.67 \text{ sec. through iron. (2) } \textit{Ans.}$$

184. Two gunners fire at each other simultaneously from forts $1\frac{1}{2}$ mi. apart; the wind, at 70° F., blows steadily from one fort to the other, at 11 mi. an hour. How soon will each hear the report of the other's gun? Suppose one ball flies on the average 987 ft. a second, the other 818 ft. a second; when will each receive the other's shot?

$$70 - 57 + 1114 = 1127 \text{ ft. per sec.}$$

$$1\tfrac{1}{2} \text{ mi.} = 7920 \text{ ft.}$$

$$11 \text{ mi. per hr.} = 16.1 \text{ ft. per sec}$$

The velocity of the sound with the wind will be

$$1127 + 16.1 = 1143.1.$$

The velocity of the sound against the wind will be

$$1127 - 16.1 = 1110.9.$$

$7920 \div 1143.1 = 6.93$ sec. (1st sound). *Ans.*

$7920 \div 1110.9 = 7.13$ sec. (2d sound). *Ans.*

$7920 \div 987 = 8.02$ sec. (1st ball). *Ans.*

$7920 \div 818 = 9.68$ sec. (2d ball). *Ans.*

185. Sound travels in water about 4.26 times as fast as in air. How many seconds sooner would the sound of a torpedo exploded under water 2 mi. off reach you by water than by air, at 68° F.?

$68 - 57 + 1114 = 1125$ ft. per sec.

2 mi. $= 10{,}560$ ft.

$10{,}560 \div 1125 = 9.387$ sec. in air.

$9.387 \div 4.26 = 2.203$ sec. in water.

$9.387 - 2.203 = 7.18$ sec. *Ans.*

186. Required the expense of painting the walls and ceiling of a room 22 ft. 6 in. long, 13 ft. 6 in. wide, and 10 ft. high, at 30 cents a square yard.

$2(22\frac{1}{2} + 13\frac{1}{2}) \times 10 = 720$ sq. ft., walls.

$22\frac{1}{2} \times 13\frac{1}{2} = 303.75$ sq. ft., ceiling.

$\frac{1}{9}(720 + 303.75) \times \$0.30 = \$34.13$. *Ans.*

187. In what time would a cistern be filled by three pipes whose diameters are $\frac{1}{2}$ in., $\frac{3}{4}$ in., and 1 in., when the largest alone would fill it in 40 min.; the rates of flow being proportional to the squares of the diameters?

The smallest alone would fill it in $(\frac{2}{1})^2$ of 40 min. $= 160$ min.
The other alone would fill it in $(\frac{4}{3})^2$ of 40 min. $= 71\frac{1}{9}$ min.
Hence, in 1 min., the largest fills $\frac{1}{40}$ of the cistern,
the smallest fills $\frac{1}{160}$ of the cistern,
the other fills $\frac{9}{640}$ of the cistern,
and all three together fill $\frac{1}{40} + \frac{1}{160} + \frac{9}{640} = \frac{29}{640}$ of it.
Hence, it will take $1 \div \frac{29}{640} = 22\frac{2}{3}$ min. *Ans.*

188. How many gallons of water are contained in a length of of 50 yds. of a canal, if its width at the top is 8 yds. and at the bottom 7 yds., and its depth 5 ft.?

The average width is $7\frac{1}{2}$ yds. = 270 in.; 50 yds. = 1800 in.; 5 ft. = 60 in.

$$\text{Volume} = \frac{1800 \times 60 \times 270}{231} = 126{,}233.8 \text{ gals. } \textit{Ans.}$$

189. Find the weight supported by each of two men, A and B, who carry a hundred-weight suspended on a pole 6 ft. long, at a point 2 ft. 3 in. from A's end, if the pole weighs 16 lbs.

$100 + 16 = 116$ lbs., total weight.
$6 - 2.25 = 3.75$ ft. from B's end.
$6 : 3.75 = 100 : x.$

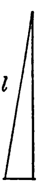
$$x = \frac{100 \times 3.75}{6} = 62.5 \text{ lbs.}$$

$62.5 + 8 = 70.5$, A. (1) *Ans.*
$116 - 70.5 = 45.5$, B. (2) *Ans.*

190. A man who rows 4 mi. an hour in still water takes 1 hr. 12 min. to row the same distance up a river. How long will it take him to row down again?

In still water, the man could row $4 \times 1.2 = 4.8$ miles in 1 hr. 12 min. Hence, the stream bears him down 0.8 mi. in 1.2 hrs., or flows at the rate of $\frac{2}{3}$ miles per hour. When he rows with the stream he will row $4\frac{2}{3}$ miles per hour, and row 4 miles in $\frac{6}{7}$ hr. = $51\frac{3}{7}$ min. *Ans.*

191. How long a ladder will be required to reach a window 40 ft. from the ground, if the distance of the foot of the ladder from the wall is one-fifth of the length of the ladder.



l

$\frac{1}{5}l$

Let l be the length of the ladder.
$\frac{1}{5}l$ is the distance of the foot from the wall.

The height of the wall is $\sqrt{l^2 - \left(\frac{l}{5}\right)^2}$.

But 40 is the height of the wall.

Therefore $\sqrt{l^2 - \left(\frac{l}{5}\right)^2} = 40.$

Square both sides, $\frac{24}{25}l^2 = 1600.$

Divide both sides by $\frac{24}{25}$, $l^2 = 1666.67.$

$$\therefore\ l = \sqrt{1666.67} = 40.8 \text{ ft. } \textit{Ans.}$$

192. If 3 oz. of gold 15 carats fine are mixed with 7 oz. 12 carats fine, what will be the fineness of the compound? What must be the fineness of 11 oz. so that, when added to this compound, the whole may be 14 carats fine?

$$\begin{array}{r} 3 \times 15 = 45 \\ 7 \times 12 = \underline{84} \\ 10)\underline{129} \\ 12.9 \end{array} \text{ carats. (1) } \textit{Ans.}$$

$$14 - 12.9 = 1.1 \text{ carats.}$$
$$1.1 \times 10 = 11 \text{ carats.}$$
$$11 + 11 + 14 = 15 \text{ carats. (2) } \textit{Ans.}$$

193. Find the surface of each face of a cube whose volume is 14 cu. ft. 705.088 cu. in.

$$14 \text{ cu. ft. } 705.088 \text{ cu. in.} = 24{,}897.088 \text{ cu. in.}$$

$$\begin{array}{rr|l} & & 24\dot{8}97.08\dot{8}\ \underline{|\ 29.2} \\ & & 8 \\ 3 \times 20^2 = & 1200 & 16897 \\ 3(20 \times 9) = & 540 & \\ 9^2 = & \underline{81} & \\ & 1821 & \underline{16389} \\ 3 \times 290^2 = & 252300 & 508088 \\ 3(290 \times 2) = & 1740 & \\ 2^2 = & \underline{4} & \\ & 254044 & \underline{508088} \end{array}$$

$$(29.2)^2 = 852.64 \text{ sq. in. } \textit{Ans.}$$

194. What must be the length of spermaceti candles $\frac{7}{8}$ of an inch in diameter, that 6 of them may weigh a pound, if the specific gravity of spermaceti is 0.943?

$V = 0.7854 \times \frac{7^2}{8^2} \times h \times 6$ cu. in.

1 lb. is the weight of $\frac{1728}{0.943 \times 62.5}$ cu. in. of spermaceti.

Hence $0.7854 \times \frac{7^2}{8^2} \times h \times 6 = \frac{1728}{0.943 \times 62.5}$.

$$h = \frac{1728 \times 8^2}{0.7854 \times 7^2 \times 6 \times 0.943 \times 62.5}.$$

$$\begin{aligned}
\log\ 1728 &= 3.2375 \\
\log\ 8^2 &= 1.8062 \\
\text{colog}\ 0.7854 &= 0.1049 \\
\text{colog}\ 7^2 &= 8.3098 - 10 \\
\text{colog}\ 6 &= 9.2218 - 10 \\
\text{colog}\ 0.943 &= 0.0255 \\
\text{colog}\ 62.5 &= \underline{8.2041 - 10} \\
& \ 0.9098 = \log 8.124.
\end{aligned}$$

8.124 in. *Ans.*

195. A cylinder 10 in. across and 10 in. high contains 0.3927 of a cubic foot of water. How many shot 0.1 in. in diameter must be poured in to raise the water to the top?

The volume of the cylinder is

$$0.785398 \times 10^2 \times 10 = 785.398 \text{ cu. in.}$$

$$0.3927 \text{ cu. ft.} = 678.5856 \text{ cu. in.}$$

$$785.398 - 678.5856 = 106.8124 \text{ cu. in.}$$

$$0.1^3 \times 0.5236 = 0.0005236 \text{ cu. in., vol. of 1 shot.}$$

$$106.8124 \div 0.0005236 = 203{,}997. \textit{ Ans.}$$

196. Determine the depth of conical-shaped wine-glasses $2\frac{1}{2}$ in. across the top, that 60 of them may hold a gallon.

$\frac{1}{60}$ of 231 = 3.85 cu. in., the capacity of a glass.

$2.5 \times 2.5 \times 0.7854 = 4.90875$, the area of the top.

$3 \times \frac{3.85}{4.90875} = 2.353$ in. *Ans.*

197. A train approaching a station at the rate of 25 mi. an hour passes over two signals placed half a mile apart. Find the interval between the times at which their reports are heard at the station, sound travelling at the rate of 1090.2 ft. per second.

25 mi. an hr. is $\frac{1}{2}$ mi. in $1\frac{1}{5}$ min. = 72 sec.

When the train reaches the second signal, the sound of the first signal is $72 \times 1090.2 - 2640$ ft. ahead of it; and this distance, divided by 1090.2, is 72 sec. $-$ 2.4 = 69.6 sec. *Ans.*

198. The apparent intensity of light varies inversely as the square of its distance. Find the point between two lamps 50 yds. apart at which one appears twice as bright as the other.

The distances are in the proportion $\sqrt{1} : \sqrt{2} = 1 : 1.4142$,

and $2.4142 : 50 = 1 : x$.

$x = 20.71$ yds., smaller distance. (1) *Ans.*

$50 - 20.71 = 29.29$ yds., greater distance. (2) *Ans.*

199. How deep may a round cistern 4 ft. across be made so as to be lined with the same lead as a cubical cistern 4 ft. each way? Compare their capacities.

$4 \times 4 \times 4 = 64$ cu. ft. in cubical cistern.

$5 \times 16 = 80$ sq. ft. of lead to line it.

$0.7854 \times 16 = 12.5664$ sq. ft. in bottom of round cistern.

$80 - 12.5664 = 67.4336$.

$\frac{67.4336}{4 \times 3.1416} = 5.366$. (1) *Ans.*

$16 \times 0.7854 \times 5.366 = 67.43$ cu. ft. in round cistern. (2) *Ans.*

200. The material for lining a cubical cistern cost \$10; find the cost of material for lining two similar cisterns which shall each hold one-half as much.

The cost is proportional to $(\sqrt[3]{1})^2 : (\sqrt[3]{\tfrac{1}{2}})^2 = 1^2 : 0.7937^2 = 1 : 0.63$.

$1 : 0.63 = \$10 : x$.

$x = \frac{\$10 \times 0.63}{1} = \$6.30$.

$2 \times \$6.30 = \$12.60$. *Ans.*

201. What distance can be seen from the top of a mountain 4 mi. high?

$$\begin{aligned} 4 \text{ mi.} &= 21{,}120 \text{ ft.} \\ \tfrac{1}{2}\log 21{,}120 &= 2.1622 \\ &\ \underline{0.1215} \\ &\ 2.2837 = \log 192.2. \end{aligned}$$

192.2 mi. *Ans.*

202. If 5 excavators sink a circular shaft 8 ft. in diameter and 125 fathoms deep in 100 dys. of 10 hrs. each, how many nights of 7 hrs. each will 4 excavators be in sinking a shaft 6 ft. in diameter and 75 fathoms deep; if the difficulty of working by night is one-seventh greater than by day, and the hardness of the ground in the smaller shaft is to that in the larger shaft as 7 is to 5?

$$\begin{array}{r|l} 4 & 5 \\ 8^2 & 6^2 \\ 125 & 75 : : 100 : x. \\ 7 & 10 \\ 7 & 8 \\ 5 & 7 \end{array}$$

$$\frac{\cancel{5} \times 6 \times 6 \times \overset{3}{\cancel{75}} \times 10 \times \cancel{8} \times \cancel{8} \times \cancel{7} \times \overset{\overset{5}{\cancel{25}}}{\cancel{100}}}{\cancel{4} \times \cancel{8} \times \underset{\cancel{5}}{\cancel{125}} \times \cancel{7} \times 7 \times \cancel{5}} = 96\tfrac{3}{7} \text{ nights. } \textit{Ans.}$$

203. Find the length of a pendulum that oscillates 80 times a minute.

$$\begin{aligned} 80^2 : 60^2 &: : 39.138 : x. \\ 4^2 : \ 3^2 &: : 39.138 : x. \\ 16 : 9 &= 39.138 : x. \\ \frac{9 \times 39.138}{16} &= 22.015 \text{ in. } \textit{Ans.} \end{aligned}$$

204. A man whose weight is 160 lbs., wishing to raise a rock, leans with his whole weight on a horizontal crowbar 5 ft. long, which is propped at the distance of 4 in. from the end in contact with the rock. Find what force he exerts on the rock, and what pressure the prop has to sustain, if the weight of the crowbar is not reckoned.

$$4 : 56 = 160 : x.$$

$$\frac{56 \times \overset{40}{\cancel{160}}}{\cancel{4}} = 2240 \text{ lbs. (1) } \textit{Ans.}$$

$$2240 + 160 = 2400 \text{ lbs. (2) } \textit{Ans.}$$

205. A child weighing 56 lbs. is at one end of a plank 16 ft. long, and a child weighing 72 lbs. is at the other end. Find the distance of each child from the fulcrum when the plank is used for a see-saw.

$56 : 72 = 7 : 9.$ 9 ft. and 7 ft. *Ans.*

206. In a pair of nut-crackers if the nut be placed at a distance of 1 in. from the hinge, and the hand presses at a distance of 8 in. from the hinge, find the pressure upon the nut for every ounce of pressure exerted by the hand.

$1 : 8 = 1$ oz. : what? 8 oz. *Ans.*

207. A substance is weighed from both arms of a false balance, and its apparent weights are 9 lbs. and 4 lbs. Find the true weight.

$4 : x = x : 9.$ $x^2 = 36.$ $x = 6$ lbs. *Ans.*

208. A rope passes over a single pulley. How much force is required to raise 180 lbs. attached to one end of a rope if 1% of the force applied is required to overcome friction?

$\frac{100}{99}$ of $180 = 181.82$ lbs. *Ans.*

209. A cask of lamp-black weighing 160^{kg} is attached to a rope wound on an axle 19^{cm} in diameter; at one end of the axle is a wheel 175^{cm} in diameter. With what force must a man pull down on a rope passing over the wheel to raise the cask if $1\frac{3}{4}\%$ of the pull is required to overcome friction?

$$175 : 19 = 160^{kg} : x.$$

$$\frac{19 \times \overset{32}{\cancel{160}}}{\underset{35}{\cancel{175}}} = 17.3714^{kg}.$$

$$17.3714 \div 0.9825 = 17.69^{kg}. \textit{ Ans.}$$

210. The circumference of the circle described by the end of the power-arm of a screw is 24 in., and by one turn of the screw the head advances half an inch. If the power is 3 lbs., find the weight lifted.

$\frac{1}{2} : 24 = 3 : x.$ $2 \times 3 \times 24 = 144$ lbs.
Less than 144 lbs. *Ans.*

211. In a screw used to raise a load of 10 t. the power is 50 lbs., acting by an arm 4 ft. long. Find the distance between two consecutive threads.

$4 \text{ ft.} = 48 \text{ in.}$
$3.1416 \times 2 \times 48 = 301.5936.$
$20{,}000 : 50 = 301.6 : x.$
$400 : 1 = 301.6 : x.$
$301.6 \div 400 = 0.754.$
Less than 0.754 in. *Ans.*

212. What per cent of water is oxygen? what per cent hydrogen?

$(2 \times 1) + 16 = 2 + 16 = 18.$
$\frac{2}{18}$ of $100 = 11\frac{1}{9}\%$ H. (1) *Ans.*
$100 - 11\frac{1}{9} = 88\frac{8}{9}\%$ O. (2) *Ans.*

213. What per cent of quicklime, CaO, is oxygen?

$40 + 16 = 56.$ $\frac{16}{56}$ of $100 = 28\frac{4}{7}\%$. *Ans.*

214. What per cent of water, H_2O, in slacked lime, CaO_2H_2?

$$\begin{array}{rr} Ca = & 40 \\ O_2 = & 32 \\ H_2 = & 2 \\ \hline & 74 \end{array} \qquad \begin{array}{rr} H_2 = & 2 \\ O = & 16 \\ \hline & 18 \end{array}$$

$\frac{18}{74}$ of $100 = 24.32\%$. *Ans.*

215. What per cent of pure marble, $CaCO_3$, is oxygen?

$40 + 12 + 48 = 100.$ $\frac{48}{100}$ of $100 = 48\%$. *Ans.*

216. What per cent of gypsum (plaster of Paris), $CaSO_4 + 2\,H_2O$, is sulphur?

$40 + 32 + 64 + 2(2 + 16) = 136 + 36 = 172.$
$\frac{32}{172}$ of $100 = 18\frac{26}{43}\%$. *Ans.*

217. What per cent of washing-soda, $Na_2CO_3 + 10\,H_2O$, is carbon?

$46 + 12 + 48 + 10(2 + 16) = 106 + 180 = 286.$
$\frac{12}{286}$ of $100 = 4\frac{28}{143}\%$. *Ans.*

218. In 118 lbs. of Glauber salts, $Na_2SO_4 + 10\,H_2O$, how many ounces of sulphur?

$46 + 32 + 64 + 10(2 + 16) = 142 + 180 = 322.$
$\frac{32}{322}$ of $118 \times 16 = 187.6$ oz. *Ans.*

219 How many ounces of soda, $Na_2O + H_2O$, in 7 lbs. of borax, $Na_2B_4O_7 + 10\,H_2O$?

$$46 + 16 + 18 = 80.$$
$$46 + 44 + 112 + 10(2 + 16) = 202 + 180 = 382.$$
$$7 \text{ lbs.} = 112 \text{ oz.}$$
$$\tfrac{80}{382} \times 112 = 23.46 \text{ oz. } \textit{Ans.}$$

220. What per cent of pure alcohol, C_2H_6O, is carbon? What per cent of pure white marble, $CaCO_3$, is carbon?

$24 + 6 + 16 = 46.$ $\quad 40 + 12 + 48 = 100.$

$\tfrac{24}{46}$ of $100 = 52\tfrac{4}{23}\%$. (1) *Ans.* $\quad \tfrac{12}{100}$ of $100 = 12\%$. (2) *Ans.*

221. What per cent of pure acetic acid (the acid of vinegar) is carbon, the formula being $C_2H_4O_2$?

$24 + 4 + 32 = 60.$ $\quad \tfrac{24}{60}$ of $100 = 40\%$. *Ans.*

222. How much acetic acid can be obtained from 12 lbs. of alcohol, C_2H_6O, if there is no waste?

$C_2H_4O_2 = 60$ acid. $\quad 24 + 6 + 16 = 46$ alcohol.

As alcohol contains $\tfrac{16}{46}$ of O, and acid demands $\tfrac{32}{60}$, 12 lbs. of alcohol will yield by combustion of H and C

$$\tfrac{16}{46} \times \tfrac{60}{32} \times 12 \text{ lbs.} = 7.82 \text{ lbs. } \textit{Ans.}$$

223. How many grains of carbon in 1 oz. avoirdupois of oxalic acid, $C_2H_2O_4 + 2\,H_2O$?

$$24 + 2 + 64 + 2(2 + 16) = 90 + 36 = 126.$$

$$\frac{\overset{\overset{}{\cancel{3}}}{\cancel{24}}}{\underset{\underset{21}{\cancel{63}}}{\cancel{126}}} \text{ of } \frac{\overset{\overset{1750}{\cancel{3500}}}{\cancel{7000}}}{\underset{\cancel{2}}{\cancel{16}}} \text{ gr.} = \tfrac{1750}{21} = 83\tfrac{1}{3} \text{ grs. } \textit{Ans.}$$

224. How many milligrams of carbon in 3^g of tartaric acid, $C_4H_6O_6$?

$24 + 3 + 48 = 75.$

$3^g = 3000^{mg}.$

$$\frac{24}{\cancel{75}} \times \frac{\overset{40}{\cancel{3000}}}{1} = 960^{mg}. \textit{ Ans.}$$

225. How many kilograms of carbon in 95^{kg} of white sugar, $C_{12}H_{22}O_{11}$?

$$144 + 22 + 176 = 342.$$

$$\frac{\overset{8}{\cancel{144}}}{\underset{19}{\cancel{342}}} \times \frac{\overset{5}{\cancel{95}}}{1} = 40^{kg}. \textit{ Ans.}$$

226. The formula of camphor is $C_{10}H_{16}O$. How many grams of carbon in 14^{kg} of camphor?

$$120 + 16 + 16 = 152.$$

$$\frac{\overset{15}{\cancel{120}}}{\underset{19}{\cancel{152}}} \times \frac{14000}{1} = \frac{210000}{19} = 11{,}052.6^{g}. \textit{ Ans.}$$

227. In 20^{kg} of oil of vitriol, H_2SO_4, how many grains of sulphur?

$$2 + 32 + 64 = 98.$$

$$\frac{\overset{16}{\cancel{32}}}{\underset{49}{\cancel{98}}} \times \frac{20000}{1} = \frac{320000}{49} = 6530.6^{g}. \textit{ Ans.}$$

228. What per cent of oil of vitriol is water? what per cent sulphuric acid, SO_3?

$H_2SO_4 = 98.$ $\qquad$ $\frac{18}{98}$ of $100 = 18.37\%$ water.

$H_2O = 18.$ $\qquad$ $100 - 18.37 = 81.63\%$ sulphuric acid.

229. In 3.5^{g} of black oxide of iron, FeO, how many milligrams of iron?

$3.5^{g} = 3500^{mg}.$ $\qquad$ $56 + 16 = 72.$

$$\frac{\overset{7}{\cancel{56}}}{\underset{9}{\cancel{72}}} \text{ of } \frac{3500}{1} = \frac{24500}{9} = 2722\tfrac{2}{9}^{mg}. \textit{ Ans.}$$

230. Red iron-rust consists of 70% iron and 30% oxygen. Find its formula.

$Fe = 56$ and $O = 16$. $16 : 56 = 2 : 7$. $30 : 70 = 3 : 7$.

First seek multiples of 16 and 56 in the proportion of 30 to 70; that is, of 3 to 7. $\therefore$ $Fe : O = 2 : 3$.

Formula $= Fe_2O_3$. *Ans.*

231. The choking vapor of burning sulphur is equal parts sulphur and oxygen. What is its chemical formula?

$S = 32$. $O = 16$. $O_2 = 32$. $\therefore$ SO_2. *Ans.*

232. If the composition of copperas is sulphuric acid 28.9%, oxide of iron 25.7%, water 45.4%, what is its formula?

Water being 18, oxide of iron being 72, and sulphuric acid 80, first seek multiples of 72 and 80, in the proportion of 25.7 to 28.9; that is, of 0.8892 to 1. But it is evident that 72 and 80 are in almost exactly that proportion. This gives $FeSO_4$ + water; and the remaining question is to find a multiple of 18 which is to 152 as 45.4 is to 54.6; that is, which is 0.8315 of 152, or 126.4. But $7 \times 18 = 126$; and the addition of 7 parts of water gives as the complete formula, $FeSO_4 + 7\,H_2O$. *Ans.*

233. Spirits of turpentine is 11.76% hydrogen and 88.24% carbon. Give a chemical formula which will express that fact. Compare now with camphor, $C_{10}H_{16}O$. What per cent of oxygen combined with spirits of turpentine are required to make camphor?

Hydrogen being 1 and carbon 12, and 88.24 being almost exactly 7.5 times 11.76, we seek the smallest multiple of 12 that is 7.5 times a whole number. This is evidently 5 times 12, equal to 7.5 times 8. Therefore the formula is C_5H_8. (1) *Ans.*

Double it and add O, and we have $C_{10}H_{16}O$, the formula of camphor; containing $16 \div 136 = 11.76\%$ of oxygen added to spirits of turpentine. (2) *Ans.*

234. Out of the ten digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, how many numbers each consisting of two figures can be formed?

The numbers actually formed are those from 10 to 99; 90 in all. To show how many can be formed, consider that each digit may be followed by itself or by any of the other 9, giving 100 sequences of 2 figures; but 10 of these begin with 0, and are to be rejected, leaving 90. *Ans.*

235. How many odd numbers consisting of two figures can be formed with the ten digits?

As 10 is even and 99 odd, one-half the 90, viz. 45, are odd. *Ans.*

236. With the digits 0, 1, 2, 3, 4, 5, how many numbers between 1000 and 4000 can be formed?

The digit in the thousands' place must be 1, 2, or 3. Each of these may be followed in the hundreds' place by each of the 6 given digits, making 18 different hundreds. Each of these may be followed in the tens' place by each of the 6, giving 108 different tens; each of which may have 6 digits, giving 648 numbers if we include the 1000. *Ans.*

237. How many dominoes are there in a set numbered from double-blank to double-nine?

By a domino is meant two numbers, side by side. Double-blank is 00. The next pieces introducing 1 are 01, 11; those introducing 2 are 02, 12, 22. Every number introduced requires new pieces exceeding by one the number introduced. Thus, as we have seen that to go to double-2 requires 6, to go to double-3 will require $6 + 4 = 10$ pieces; double-4, $10 + 5 = 15$ pieces. The ordinary set is $15 + 6 + 7 = 28$ pieces; and for double-9's we should require $28 + 8 + 9 + 10 = 55$ pieces. *Ans.*

238. How many different amounts can be weighed with 1-lb., 2-lb., 4-lb., 8-lb., and 16-lb. weights?

With a single piece, 5 amounts; with 2 pieces, $4 + 3 + 2 + 1 = 10$ amounts; with 3 pieces, 10 amounts; with 4 pieces, 5 amounts; with all 5 at once, 1 amount; with the 5 in their various combinations, therefore, $5 + 10 + 10 + 5 + 1 = 31$ amounts. *Ans.*